EL-MOASSER
By a group of supervisors

Question Bank
& Practice Exams

PURE MATHEMATICS

Differential & Integral Calculus

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Preface

Thanks to God who helped us to introduce one of our famous series "El Moasser" in mathematics.

We introduce this book to our colleagues.

We also introduce it to our students to help them study mathematics.

In fact, this book is the outcome of more than thirty years experience in the field of teaching mathematics.

This book will make students aware of all types of questions.

We would like to know your opinions about the book hoping that it will win your admiration.

We will be grateful if you send us your recommendations and your comments.

The Authors

Contents

- Summary for differential & integral calculus.
- Multiple choice question bank.
- Practice exams.
- School book examinations.
- Egypt exams (2017 : 2020 first and second sessions).
- Al-Azhar exams (2019 , 2020 first and second sessions).

Summary

for

Differential & Integral calculus

Summary Differential & Integral calculus

Important notes related to (e)

The Nabier's constant e is an irrational number , $2 < e < 3$,

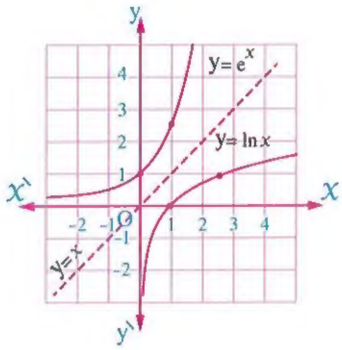
$$e = 1 + \frac{1}{\underline{|1}} + \frac{1}{\underline{|2}} + \frac{1}{\underline{|3}} + \cdots = \sum_{n=0}^{\infty} \frac{1}{\underline{|n}} \text{ (Taylor's series)}$$

$e = 2.71828$

- **The natural exponential function :**

$\boxed{f : \mathbb{R} \longrightarrow \mathbb{R}^+ \text{ where } f(x) = e^x}$

is exponential function whose base is e and it is one - to - one its domain = $\mathbb{R}$, its range = $\mathbb{R}^+$, its curve passes through (0 , 1) , (1 , e)



- **The natural logarithmic function :**

$\boxed{f : \mathbb{R}^+ \longrightarrow \mathbb{R} \text{ where } f(x) = \ln x}$

is logarithmic function of base "e" ($\log_e x = \ln x$) and it is one - to - one function its domain $\mathbb{R}^+$, its range = $\mathbb{R}$ its curve passes through (1 , 0) , (e , 1)

Remarks

From the previous we get : $\lim\limits_{x \to \infty} \ln x = \infty$, $\lim\limits_{x \to 0^+} \ln x = -\infty$

Some properties of natural logarithm :

if $x , y \in \mathbb{R}^+$, $n \in \mathbb{R}$ under condition the base of the logarithm $\in \mathbb{R}^+ - \{1\}$, then :

1. $\ln e = 1$

2. $\ln 1 = \text{zero}$

3. $\ln x^n = n \ln x$

4. $e^{\ln x} = x$

5. $\ln xy = \ln x + \ln y$

6. $\ln \frac{x}{y} = \ln x - \ln y$

7. $\log_y x = \frac{\ln x}{\ln y}$

8. $\ln x \times \log_x e = 1$

Limits of functions related to the number e

1. $\boxed{\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{x} = e}$ and $\boxed{\lim_{x \to 0} \left(1 + x\right)^{\frac{1}{x}} = e}$

Remarks

1. $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{a x} = e^{a}$

2. $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{x + a} = e$

3. $\lim_{x \to \infty} \left(1 + \frac{a}{x}\right)^{x} = e^{a}$

4. $\lim_{x \to \infty} \left(1 - \frac{a}{x}\right)^{x + k} = e^{-a}$

5. $\lim_{x \to \infty} \left(1 + \frac{a}{x}\right)^{\frac{x}{a}} = e$

2. $\boxed{\lim_{x \to 0} e^{x} = 1}$

3. $\boxed{\lim_{x \to 0} \frac{\log_a (1 + x)}{x} = \log_a e}$ and $\boxed{\lim_{x \to 0} \frac{\ln (1 + x)}{x} = 1}$

4. $\boxed{\lim_{x \to 0} \frac{a^{x} - 1}{x} = \ln a}$ and $\boxed{\lim_{x \to 0} \frac{e^{x} - 1}{x} = 1}$

Derivative

Derivative of exponential and logarithmic functions :

1. If $y = e^{x}$, then $\frac{dy}{dx} = e^{x}$

2. If $y = a^{x}$, then $\frac{dy}{dx} = a^{x} \ln a$

3. If $y = \ln x$, then $\frac{dy}{dx} = \frac{1}{x}$

4. If $y = \log_a x$, then $\frac{dy}{dx} = \frac{1}{x} \log_a e$

In general :

1. If $y = e^{f(x)}$, then $\frac{dy}{dx} = f'(x)\, e^{f(x)}$

2. If $y = a^{f(x)}$, then $\frac{dy}{dx} = f'(x)\, a^{f(x)} . \ln a$

3. If $y = \ln f(x)$, then $\frac{dy}{dx} = \frac{f'(x)}{f(x)}$

4. If $y = \log_a f(x)$, then $\frac{dy}{dx} = \frac{f'(x)}{f(x)} \times \log_a e$

Summary

Notice that

- If $y = \ln | f(x) |$, then $\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
- If $y = \log_a | f(x) |$, then $\frac{dy}{dx} = \frac{f'(x)}{f(x)} \log_a e$

Differentiation of trigonometric functions :

1. If $y = \sin x$, then $\frac{dy}{dx} = \cos x$

2. If $y = \cos x$, then $\frac{dy}{dx} = -\sin x$

3. If $y = \tan x$, then $\frac{dy}{dx} = \sec^2 x$

4. If $y = \cot x$, then $\frac{dy}{dx} = -\csc^2 x$

5. If $y = \sec x$, then $\frac{dy}{dx} = \sec x \tan x$

6. If $y = \csc x$, then $\frac{dy}{dx} = -\csc x \cot x$

In general :

1. If $y = \sin \left(f(x)\right)$, then $\frac{dy}{dx} = f'(x) \cos \left(f(x)\right)$

2. If $y = \cos \left(f(x)\right)$, then $\frac{dy}{dx} = -f'(x) \sin \left(f(x)\right)$

3. If $y = \tan \left(f(x)\right)$, then $\frac{dy}{dx} = f'(x) \sec^2 \left(f(x)\right)$

4. If $y = \cot \left(f(x)\right)$, then $\frac{dy}{dx} = -f'(x) \csc^2 \left(f(x)\right)$

5. If $y = \csc \left(f(x)\right)$, then $\frac{dy}{dx} = -f'(x) \csc \left(f(x)\right) \cot \left(f(x)\right)$

6. If $y = \sec \left(f(x)\right)$, then $\frac{dy}{dx} = f'(x) \sec \left(f(x)\right) \tan \left(f(x)\right)$

Implicit differentiation

The derivative of a relation between two (or more) variables with respect to one of them without separating them.

Notice $\frac{dx^n}{dx} = n x^{n-1}$ but $\frac{dy^n}{dx} = n y^{n-1} \cdot \frac{dy}{dx}$

Parametric differentiation

If $y = f(t)$, $x = g(t)$ are the parametric equations of a curve , then :

$$\boxed{\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}}$$

Higher derivatives of the function

The derivatives starting from the second derivative are called higher derivatives and the n^{th} derivative is denoted as $y^{(n)} = \frac{d^n y}{d x^n} = f^{(n)}(x)$, where n is a positive integer.

Notice that

- $\frac{dy}{dx} \times \frac{dx}{dy} = 1$ but $\frac{d^2y}{dx^2} \times \frac{d^2x}{dy^2} \neq 1$
- $\frac{dy}{dz} \times \frac{dz}{dx} = \frac{dy}{dx}$ but $\frac{d^2y}{dz^2} \times \frac{dz^2}{dx^2} \neq \frac{d^2y}{dx^2}$

"Chain rule can not be applied to find second derivative"

- $\frac{d}{dx}\left[y \frac{dy}{dx}\right] = 1^{st} \times$ derivative of the $2^{nd} + 2^{nd} \times$ derivative of the 1^{st}

$$= y \frac{d}{dx}\left(\frac{dy}{dx}\right) + \frac{dy}{dx} \times \frac{dy}{dx} = y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2$$

- The rate of change of the slope of the tangent to the curve $y = f(x)$ equals $\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$
- If $y = (f \circ g)(x) = f(g(x))$, then $\frac{dy}{dx} = \grave{f}(g(x)) \cdot \grave{g}(x)$
- If $y = \sin a x$, then $y^{(n)} = a^n \sin\left(a x + \frac{n\pi}{2}\right)$
- If $y = \cos a x$, then $y^{(n)} = a^n \cos\left(a x + \frac{n\pi}{2}\right)$
- If $\boxed{y = \sin a x}$ or $\boxed{y = \cos a x}$, then $\boxed{y^{(n)} = a^n y}$ where n is divisible by 4

Applications on first derivative (The equations of the tangent and the normal to a curve)

If $A(x_1, y_1)$ is a point on the curve $y = f(x)$, then :

1. The slope of the tangent to the curve at A

$$= \left(\frac{dy}{dx}\right)_{(x_1, y_1)}$$

2. The slope of the normal to the curve at A

$$= \frac{-1}{\left(\frac{dy}{dx}\right)_{(x_1, y_1)}}$$

3. The tangent equation at A : $\boxed{y - y_1 = m(x - x_1)}$

and the normal equation : $\boxed{y - y_1 = \frac{-1}{m}(x - x_1)}$

Remark

- The slope of the curve at a point on it is the slope of the tangent to the curve at this point.
- The normal to the curve is the straight line perpendicular to the tangent at the point of tangency.

Summary

Related time rates

- If we have a relation between several variables x , y , z , then the derivative of this relation with respect to time (t) gives the relation between the related rates of these variables : $\boxed{\frac{dx}{dt} , \frac{dy}{dt} , \frac{dz}{dt}}$

- If the variable x is
  - Increasing / moving away / expanding pouring into / accumulating — as the time increase then $\frac{dx}{dt}$ → positive
  - Decreasing / approaching / shrinking / leaking / melting — as the time increase then $\frac{dx}{dt}$ → negative

Remarks

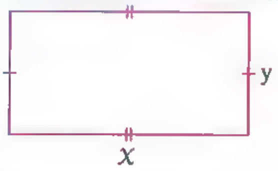
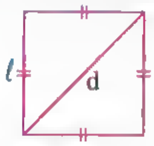
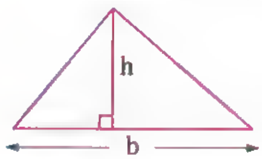
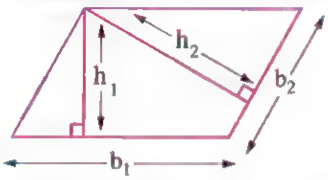
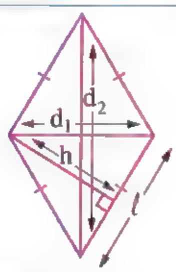
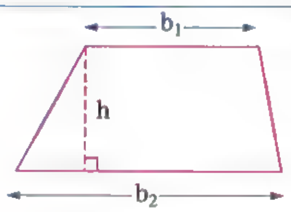
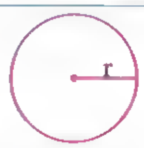
- Let x_o be the intial value of variable $x_{(t=0)}$, and is the rate of change of x with respect to time is constant (**i.e.** $\frac{dx}{dt}$ = constant value) , then after time (t) the magnitude of the variable x is given by : $\boxed{x = x_o + \frac{dx}{dt} \times t}$
- The distance between any two points : (x_1 , y_1) , (x_2 , y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- The volume of the bounded part between two concentric spheres , their radii lengths are r_1 , $r_2 = \frac{4}{3} \pi \left(r_2^3 - r_1^3\right)$
- If k = x y z (Three variables) , then $\frac{dk}{dt} = \frac{dx}{dt} \times yz + \frac{dy}{dt} \times xz + \frac{dz}{dt} \times xy$
- The distance between the point (x_1 , y_1) and the straight line

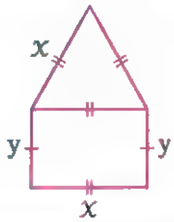
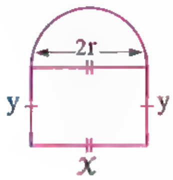
$$ax + by + c = 0 \text{ is } \frac{|a x_1 + b y_1 + c|}{\sqrt{a^2 + b^2}}$$

- If the measure of angle x in radian , then :

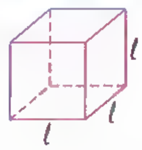
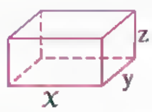
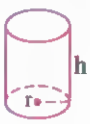
1. $\frac{d(\sin x)}{dt} = \cos x . \frac{dx}{dt}$
2. $\frac{d(\cos x)}{dt} = (-\sin x) \times \frac{dx}{dt}$
3. $\frac{d(\tan x)}{dt} = \sec^2 x . \frac{dx}{dt}$

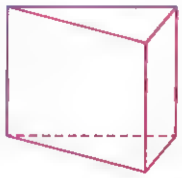
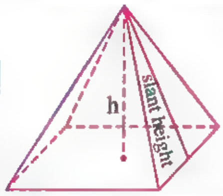
Area and perimeter of some geometrical figures

Rectangle		Perimeter $= 2\,(x + y)$ Area $= x \times y$
Square		Perimeter $= 4\,\ell$ Area $= \ell^2 = \frac{1}{2}\,d^2$
Triangle		Perimeter = sum of side lengths Area $= \frac{1}{2} \times b \times h$ $= \frac{1}{2}$ the product of any two sides $\times$ sine of included angle
Parallelogram		Perimeter $= 2\,(b_1 + b_2)$ Area $= b_1 \times h_1 = b_2 \times h_2$
Rhombus		Perimeter $= 4\,\ell$ Area $= \ell \times h$ $= \frac{1}{2}\,d_1 \times d_2$
Trapezium		Perimeter = sum of its side lengths Area $= \frac{1}{2}\,(b_1 + b_2) \times h$
Circle		Perimeter $= 2\,\pi\,r$ Area $= \pi\,r^2$
Sector		Perimeter $= 2\,r + \ell$ Area $= \frac{1}{2}\,\ell\,r = \frac{1}{2}\,\theta^{rad}\,r^2$ where $\theta^{rad} = \frac{\ell}{r}\,,\ \frac{x^\circ}{180^\circ} = \frac{\theta^{rad}}{\pi}$

Window in form of rectangle and equilateral triangle on its top		Perimeter $= 3x + 2y$ Area $= xy + \frac{1}{2}x^2 \sin 60°$ $= xy + \frac{\sqrt{3}}{4}x^2$
Window in form of rectangle and semi-circle on its top		$r = \frac{1}{2}x$ Perimeter $= x + 2y + \pi r$ $= x + 2y + \frac{1}{2}\pi x$ Area $= xy + \frac{1}{2}\pi r^2$ $= xy + \frac{1}{8}\pi x^2$
Regular polygon	Where n is the number of sides and x is the side length.	Perimeter $= nx$ Area $= \frac{1}{4}n\,x^2 \cot\frac{\pi}{n}$

Remember volume and lateral area and total surface area of some solids

Solid		Lateral area	Total area	Volume
Cube		$4\ell^2$	$6\ell^2$	ℓ^3
Cuboid		$2(x + y) \times z$	$2(xy + yz + zx)$	$x \times y \times z$
Right circular cylinder		$2\pi r h$	$2\pi r h + 2\pi r^2$ $= 2\pi r(h + r)$	$\pi r^2 h$
Sphere		------	$4\pi r^2$	$\frac{4}{3}\pi r^3$
Right cone		$\pi r \ell$	$\pi r \ell + \pi r^2$	$\frac{1}{3}\pi r^2 h$

Prism		Base perimeter $\times$ height	Lateral area + sum of areas of its two bases	Area of its base $\times$ height
Regular pyramid		$\frac{1}{2}$ Base perimeter $\times$ slant height	Lateral area + area of its base	$\frac{1}{3}$ base area $\times$ height

The critical points

If the function f continuous in the interval $]a , b[$, then it has a critical point $\left(c , f(c)\right)$ where $c \in]a , b[$ if $\boxed{f'(c) = 0}$ or $\boxed{f'(c) \text{ is not exist}}$

The critical point at $x = a$ must belong to the domain of the function

i.e. $f(a)$ is defined

Increasing and decreasing of the function

1. The function is increasing on an interval if the slope of the tangent to its curve at any point on it in this interval is positive.

   i.e. If $f'(x) > 0$ for all the values of $x \in]a , b[$, then f is increasing on this interval.

2. The function is decreasing on an interval if the slope of the tangent to its curve at any point on it in this interval is negative.

   i.e. If $f'(x) < 0$ for all the values of $x \in]a , b[$, then f is decreasing on this interval.

So we use the first derivative in steps of investigation of increasing and decreasing functions as follow :

1. Determine the domain of the function.
2. Find $f'(x)$
3. Find the critical point. (the points at which $f'(x) = 0$ or $f'(x)$ is not exist)
4. Determine the intervals of the domain by which these points divide the domain.
5. Determine the sign of $f'(x)$ in each of these intervals and so the increasing intervals where $[f'(x) > 0]$ and the decreasing intervals where $[f'(x) < 0]$

Summary

Local maxima and minima of function

Using the first derivative to identify the local maxima and minima

If (c , f (c)) is a critical point of the function f which is continuous at c and there is an open interval around c where :

1. $f'(x) > 0$ at $x < c$, $f'(x) < 0$ at $x > c$, then f (c) is a local maximum value.
2. $f'(x) < 0$ at $x < c$, $f'(x) > 0$ at $x > c$, then f (c) is a local minimum value.
3. If the sign of $f'(x)$ on both sides of c does not change , then the function has no local maximum or local minimum value at c

Using the second derivative :

If f is differentiable function twice on an open interval contains c where $f'(c) = 0$ and

1. $f''(c) < 0$, then f (c) is local maximum value.
2. $f''(c) > 0$, then f (c) is local minimum value.
3. $f''(c) = 0$, then the 2^{nd} derivative test failed to determine the kind of the point (c , f (c)) if it is local maximum or local minimum value , in this case we use the first derivative test.

Remarks

1. The local maximum points and local minimum points are critical points but the converse is not always true.
2. If the function f is only increasing (or decreasing) on an interval , so the function has no local maximum or local minimum in this interval.
3. The critical point at which the first derivative = 0
   i.e. The tangent is horizontal at this point sometimes is called stationary point.
4. The polynomial function of n^{th} degree has at most $(n - 1)$ local maximum or minimum values.

Steps to study the existence of the local maximum and local minimum values of continuous function not including the constant function :

1. Determine the domain of the function.
2. Calculate $f'(x)$
3. Find out the critical points (i.e. The points at which $f'(x) = 0$ or not exist) , let the x-coordinate of one of them be x_1
4. Determine the type of the critical point if it is maximum or minimum by one of the next two methods.

Applications of maxima and minima

To solve these questions , express the variable wanted to find its maximum or minimum value as a function in one variable , using the givens in the problem , then find the maximum or minimum of this function as explained before.

Remarks

- To find the greatest volume (v) , put $\frac{dv}{dx} = 0$, and make sure that $\frac{d^2 v}{dx^2} < 0$
- To find the smallest cost (c) put $\frac{dc}{dx} = 0$, and make sure that $\frac{d^2 c}{dx^2} > 0$ and so on.

Absolute maxima and minima (Absolute extrema) on a closed interval

Studying the absolute maximum and minimum values on a closed interval [a , b]

If f is a continuous function on the interval [a , b] :

1. Determine the critical points at which $f'(x)$ = zero or not exist and belongs to the interval $[a , b]$
2. Find the values of the function at the critical points and the endpoints $f(a)$ and $f(b)$
3. Compare among the previous values , then the greatest value is the absolute maximum value on $[a , b]$ and the smallest value is the absolute minimum value on $[a , b]$

Remark

If the function f is defined on the interval $[a , b]$ and if :

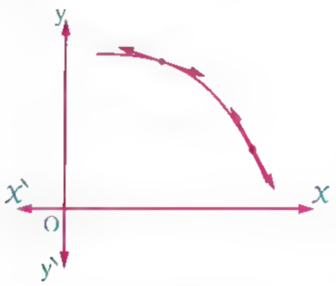
1. $f'(x) > 0$ **i.e.** The function is increasing on the same interval , then :
   - * The absolute minimum value = $f(a)$
   - * The absolute maximum value = $f(b)$
2. $f'(x) < 0$ **i.e.** The function is decreasing on the same interval , then :
   - * The absolute minimum value = $f(b)$
   - * The absolute maximum value = $f(a)$

Summary

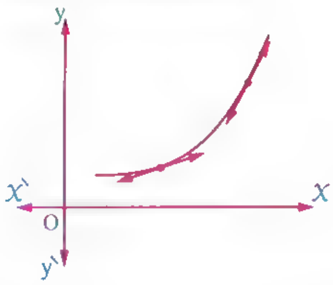
Convexity of curves and inflection points

A continuous part of a curve is said to be :

- Convex upwards "concave downwards" If the curve lies below all its tangents.



- Convex downwards "concave upwards" If the curve lies above all its tangents.



1. If f is a differentiable function on the interval $]a , b[$, then the curve of the function f

(a) Convex downwards If f' is increasing on $]a , b[$

(b) Convex upwards If f' is decreasing on $]a , b[$

2. Let f be a differentiable function twice on the interval $]a , b[$

(a) If $f''(x) > 0$ for all the values of $x \in]a , b[$, then the curve of f is convex downwards on the interval $]a , b[$

(b) If $f''(x) < 0$ for all the values of $x \in]a , b[$, then the curve of f is convex upwards on the interval $]a , b[$

The point of inflection :

The point C (c , f (c)) is an inflection point of a curve of a function f if the following is satisfied.

1. The curve of the function f is continuous at C

2. We can draw one tangent to the curve of the function at C Where :

(a) $f'(c) \in \mathbb{R}$ i.e. the tangent is inclined or horizontal.

(b) $f'(c) = \pm\infty$ (denominator of f' at the point C = zero) i.e. the tangent is vertical.

3. The sign of $f''(x)$ changes before and after the point C i.e. $[f''(c) = 0$ or not exist$]$

Steps to study convexity intervals and inflection points :

1. Find $f''(x)$, then find the values of x which make $f'' = 0$ or not exist.

2. Determine the sign of $f''(x)$ to determine the intervals over which the function is convex upwards where $[f''(x) < 0]$ and the intervals over which the function is convex downwards where $[f''(x) > 0]$

3. Determine the inflection points from the obtained points at which the sign of $f''(x)$ changes around it. unless $f''(x)$ changes around any of these points , then it is not be considered an inflection point.

Remarks

1. The inflection point at $X = a$ must belong to the domain of the function
   i.e. f (a) is defined
2. The inflection points are the points which separated between convex upwards and convex downwards regions.
3. The tangent at the inflection point intersects the curve of the function.
4. The critical point of the function f is the point at which $f'(X) = 0$ or not exist and if the sign of $f'(X)$ changes around this point , then it is a local maximum or minimum.

   Also we find that :

   The critical point of the function f' is the point at which $f''(X) = 0$ or not exist and if the sign of $f''(X)$ changes around this point , then it is an inflection point of the function f
5. The inflection points of a function f which is differentiable twice is a local maximum or minimum of the function f'
6. If the point (c , h) $\in$ the curve of a function which is differentiable twice and
   (a) (c , h) is an inflection point , then $f''(c) = 0$, f (c) = h
   (b) (c , h) is a local maximum or a local minimum or critical point ,
   then $f'(c) = 0$, f (c) = h

Curve sketching of polynomial functions

⊙ Steps of drawing a curve of function f (where f is a polynomial of 3^{rd} degree or less) :

1. Determine the domain of the function , then determine the symmetric of the function f if exist where :
   (a) $f(-X) = f(X)$ for every $X \in$ the domain
   $\therefore$ The function is even and so the curve is symmetric about y-axis.
   (b) $f(-X) = -f(X)$ for every $X \in$ the domain
   $\therefore$ The function is odd and so the function is symmetric about origin point.
2. Find out $f'(X)$, $f''(X)$
3. Use $f'(X)$ to determine :
   (a) The increasing interval where $[f'(X) > 0]$, decreasing interval where $[f'(X) < 0]$
   (b) The points of local maximum and local minimum (if exist) where $f'(X) = 0$
   (notice that the function is differentiable) and the sign of $f'(X)$ changes before and after this point.

Summary

4. Use $f''(x)$ to determine :

(a) The intervals where the curve convex upwards where $[f''(x) < 0]$, the intervals where the curve convex downwards where $[f''(x) > 0]$

(b) The inflection point (if exist) where $f''(x) = 0$ (notice that the function is differentiable twice) and the sign of $f''(x)$ changes before and after the point.

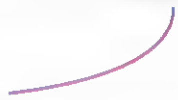
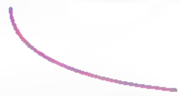
5. Determine some assisting points in sketching as :

(a) The point / points of intersection with x-axis

(b) The point / points of intersection with y-axis

(c) Some extra points by substitution by some values of x and get the corresponding values of $f(x)$

6. Arrange the points we get in a table and represent them graphically then join these points taking in consideration the following :

Sign of $f'(x)$, $f''(x)$	Properties of the curve of function f	Shape of the curve
$f'(x) > 0$, $f''(x) > 0$	Increasing , convex downwards	
$f'(x) > 0$, $f''(x) < 0$	Increasing , convex upwards	
$f'(x) < 0$, $f''(x) > 0$	Decreasing , convex downwards	
$f'(x) < 0$, $f''(x) < 0$	Decreasing , convex upwards	

Indefinite integration

• Some fundamental integrations (standard)

1. $\int k \, dx = kx + c$ (where k is constant)

2. $\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$ (where $n \neq -1$)

3. $\int (ax + b)^n \, dx = \frac{(ax + b)^{n+1}}{a(n+1)} + c$ (where $n \neq -1$)

4. $\int e^x \, dx = e^x + c$

5. $\int e^{kx+b} \, dx = \frac{e^{kx+b}}{k} + c$

6. $\int a^{x}\, dx = \frac{a^{x}}{\ln a} + c$

7. $\int a^{kx+b}\, dx = \frac{e^{kx+b}}{k \ln a} + c$

8. $\int \frac{1}{x}\, dx = \ln|x| + c$ (where $x \neq 0$)

- **Important rules of integration :**

1. $\int (f(x))^{n}\, f'(x)\, dx = \frac{(f(x))^{n+1}}{n+1} + c$

2. $\int e^{f(x)}\, f'(x)\, dx = e^{f(x)} + c$

3. $\int a^{f(x)}\, f'(x)\, dx = \frac{a^{f(x)}}{\ln a} + c$

4. $\int \frac{f'(x)}{f(x)}\, dx = \ln|f(x)| + c$

5. $\int \frac{f'(x)}{\sqrt{f(x)}}\, dx = 2\sqrt{f(x)} + c$

- **Some properties of the indefinite integration :**

1. $\int a\, f(x)\, dx = a \int f(x)\, dx$ (where a is a constant $\neq 0$)

2. $\int [f(x) \pm g(x)]\, dx = \int f(x)\, dx \pm \int g(x)\, dx$

3. $\frac{d}{dx} \int f(x)\, dx = f(x)$

4. $\int \frac{d}{dx} [f(x)]\, dx = f(x) + c$

5. $\int f(x)\, dx - \int f(x)\, dx =$ constant (not necessary $= 0$)

Results

1. $\int \sin(ax+b)\, dx = -\frac{1}{a} \cos(ax+b) + c$

2. $\int \cos(ax+b)\, dx = \frac{1}{a} \sin(ax+b) + c$

3. $\int \sec^2(ax+b)\, dx = \frac{1}{a} \tan(ax+b) + c$

4. $\int \csc^2(ax+b)\, dx = \frac{-1}{a} \cot(ax+b) + c$

5. $\int \sec(ax+b) \tan(ax+b)\, dx = \frac{1}{a} \sec(ax+b) + c$

6. $\int \csc(ax+b) \cot(ax+b)\, dx = \frac{-1}{a} \csc(ax+b) + c$

Summary

Remember

1. $\int \tan x \, dx = \int \frac{\sin x}{\cos x} dx = -\int \frac{-\sin x}{\cos x} dx$

$= \boxed{-\ln|\cos x| + c} = \boxed{\ln|\sec x| + c}$

Notice that
The numerator is the derivative of the denominator.

2. $\int \cot x \, dx = \int \frac{\cos x}{\sin x} dx$

$= \boxed{\ln|\sin x| + c} = \boxed{-\ln|\csc x| + c}$

Notice that
The numerator is the derivative of the denominator.

3. $\int \sec x \, dx = \int \frac{\sec x (\sec x + \tan x)}{(\sec x + \tan x)} dx$

(by multiplying numerator and denominator by $(\sec x + \tan x)$)

$= \int \frac{\sec^2 x + \sec x \,.\, \tan x}{\sec x + \tan x} dx$

$= \ln|\sec x + \tan x| + c$

Notice that
The numerator is the derivative of the denominator.

4. $\int \csc x \, dx = \int \frac{\csc x (\csc x + \cot x)}{(\csc x + \cot x)} dx$

(by multiplying numerator and denominator by $(\csc x + \cot x)$)

$= -\int \frac{-\csc^2 x - \csc x \cot x}{\csc x + \cot x} dx$

$= -\ln|\csc x + \cot x| + c$

Notice that
The numerator is the derivative of the denominator.

Generally

1. $\int \sin\big(f(x)\big) \times f'(x) \, dx = -\cos\big(f(x)\big) + c$
2. $\int \cos\big(f(x)\big) \times f'(x) \, dx = \sin\big(f(x)\big) + c$
3. $\int \sec^2\big(f(x)\big) \times f'(x) \, dx = \tan\big(f(x)\big) + c$
4. $\int \csc^2\big(f(x)\big) \times f'(x) \, dx = -\cot\big(f(x)\big) + c$
5. $\int \sec\big(f(x)\big) \times \tan\big(f(x)\big) \times f'(x) \, dx = \sec\big(f(x)\big) + c$
6. $\int \csc\big(f(x)\big) \times \cot\big(f(x)\big) \times f'(x) \, dx = -\csc\big(f(x)\big) + c$

Methods of integration :

1. Integration by substitution :

- If the given integration is given in the form $\int f\left(g(x)\right) . \grave{g}(x) . dx$

  use the substitution $\boxed{g(x) = z}$

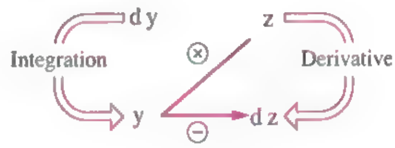
- If the given integration contains the n^{th} root of a function as $\sqrt[n]{g(x)}$, use the

  substitution $\boxed{g(x) = z^n}$ or $\boxed{g(x) = z}$

- In some questions you can use an appropriate substitution in order to simplify the integration and rewrite it in standard form.

2. Integration by parts :

$$\boxed{\int z\, dy = y\, z - \int y\, dz}$$



The definite integral

If the function f is continuous on the interval $[a , b]$, and F is any anti derivative to the function f on the same interval , then : $\boxed{\int_a^b f(x)\, dx = F(b) - F(a)}$

Properties of definite integral :

1. If f is a continuous on $[a , b]$, $c \in]a , b[$, then :

1. $\int_b^a f(x)\, dx = -\int_a^b f(x)\, dx$

2. $\int_a^a f(x)\, dx = \text{zero}$

3. $\int_a^b f(x)\, dx = \int_a^c f(x)\, dx + \int_c^b f(x)\, dx$

2. If f is a continuous odd function on the interval $[-a , a]$, then $\boxed{\int_{-a}^a f(x)\, dx = \text{zero}}$

3. If f is a continuous even function on the interval $[-a , a]$

, then $\boxed{\int_{-a}^a f(x)\, dx = 2\int_0^a f(x)\, dx}$

4. If f , g are two continuous function on the interval $[a , b]$

1. $\int_a^b [f(x) \pm g(x)]\, dx = \int_a^b f(x)\, dx \pm \int_a^b g(x)\, dx$

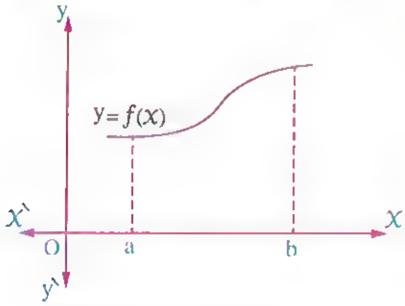
2. $\int_a^b k\, f(x)\, dx = k \int_a^b f(x)\, dx$ (where $k \in \mathbb{R}$)

Definite integral and areas in the plane

First The area of a region bounded by the curve of the function f and X-axis in the interval $[a , b]$

1. $f(X) \geq 0$

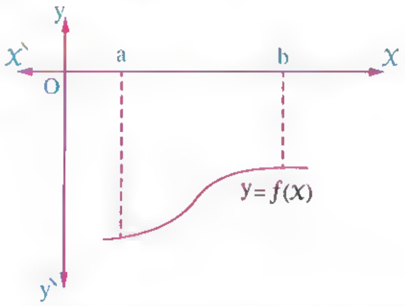
i.e. "The region above X-axis"



, then $A = \int_a^b f(X)\, dX$

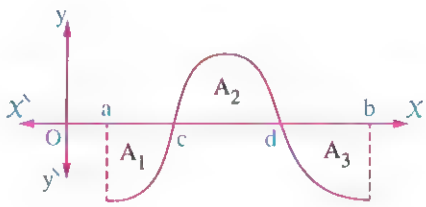
2. $f(X) \leq 0$

i.e. "The region below X-axis"



, then $A = -\int_a^b f(X)\, dX = \left| \int_a^b f(X)\, dX \right|$

- If the curve of the function f intersects the X-axis at $X = c$ and $X = d$ where c and d belong to $[a , b]$ as in the opposite figure :



We find that : $f(X) \geq 0$ for all $X \in [c , d]$

, $f(X) \leq 0$ for all $X \in [a , c]$ and $X \in [d , b]$

$\therefore$ The shaded area (A) $= A_1 + A_2 + A_3$

i.e. $A = \left| \int_a^c f(X)\, dX \right| + \int_c^d f(X)\, dX + \left| \int_d^b f(X)\, dX \right|$

Notice that

The absolute value to the two areas A_1 , A_3 because they are below X-axis.

Remarks

1. It is favorable to graph the curve of the function to identify the region above or below X-axis.
2. Sometimes it is difficult to sketch some curves then , its favorable to find zeroes of the function (even the limits of the integral are given) which divides the domain of the function $[a , b]$ if exist into intervals and determine the sign of the function in each part and so you know if the region is above or below X-axis.
3. The value of the integration , could be positive or negative but the area is always positive.
4. In general , the area of the included region between any continuous function : $y = f(X)$ and the X-axis and the two straight lines $X = a$, $X = b$ is $\boxed{A = \int_a^b |f(X)| \, dX}$

Second The area of plane region bounded by two curves

If f , g are two continuous function on the interval $[a , b]$ and $f(X) \geq g(X)$ for every $X \in [a , b]$, then the area bounded by the region between the two curves $y_1 = f(X)$, $y_2 = g(X)$ and the two straight lines $X = a$, $X = b$ is given by the relation.

$$\boxed{A = \int_a^b [f(X) - g(X)] \, dX}$$

and that's because the area between the two curves $y_1 = f(X)$, $y_2 = g(X)$

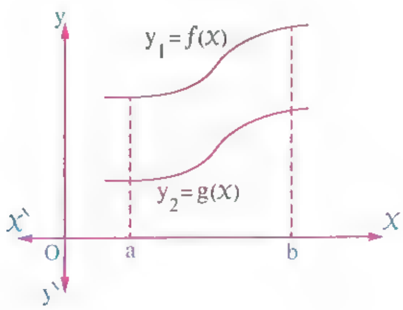
$= [$The area under $f(X)$ and above X-axis$]$

$- [$The area under $g(X)$ and above X-axis$]$

$$= \int_a^b y_1 \, dX - \int_a^b y_2 \, dX$$

$$= \int_a^b f(X) \, dX - \int_a^b g(X) \, dX$$

$$= \int_a^b [f(X) - g(X)] \, dX$$



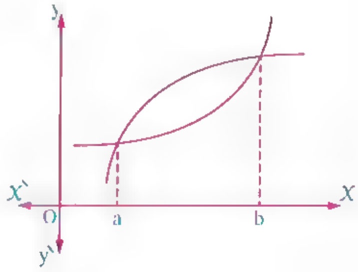
Summary

Remarks

1. To identify the higher function $y_1 \geq y_2$ for every $x \in [a , b]$ by using the graph or by choosing an arbitrary value of $x \in [a , b]$ and substitution in the equations of the two curves , or by using the absolute value as follow :

The area (A) = $\left| \int_a^b \right.$ (any of the two functions – the other function) d $\left. x \right|$

2. When a region included between two curves , then the terms of integral with respect to x are the x-coordinates of their points of intersection and it will be found by solving the two equations algebraically.



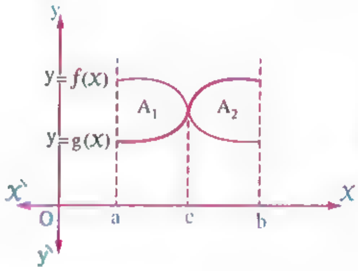
3. If the two curves intersect at one point $c \in]a , b[$

and $f(x) \geq g(x)$ for every $x \in [a , c]$

and $g(x) \geq f(x)$ for every $x \in [c , b]$

, then $A = A_1 + A_2$

$= \int_a^c [f(x) - g(x)]\, dx + \int_c^b [g(x) - f(x)]\, dx$



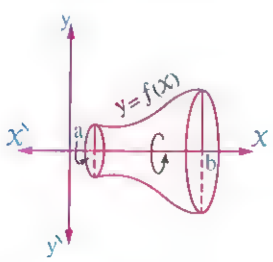
Volumes of revolution solids

Revolution Solid

* It is a solid generated by revolving a plane area a complete revolution about a fixed straight line in its plane called «axis of revolution»

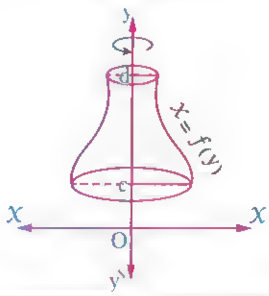
The volume of a solid produced by revolving a region about an axis.

The x-axis



$v = \pi \int_a^b y^2\, dx = \pi \int [f(x)]^2\, dx$

The y-axis



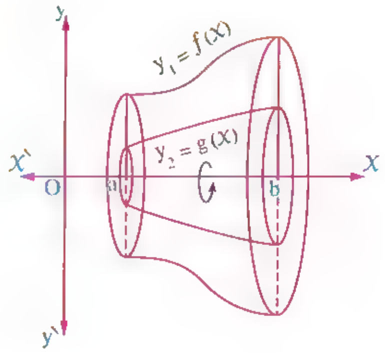
$v = \pi \int_c^d x^2\, dy = \pi \int_c^d [f(y)]^2\, dy$

Volume of the solid generated by revolving

oregion bounded by two curves

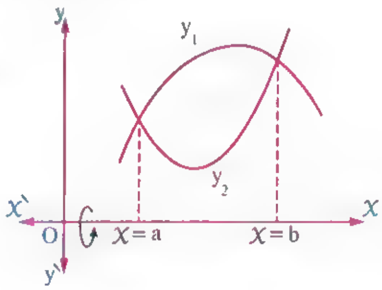
$$v = \pi \int_a^b (y_1^2 - y_2^2)\, d X$$

$$= \pi \int_a^b [(f(X))^2 - (g(X))^2]\, d X$$



Remarks

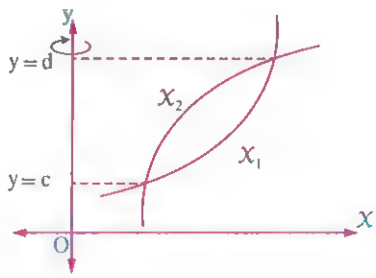
1. If a region bounded by two intersecting curves $y_1 = f(X)$, $y_2 = g(X)$ where $y_1 \geq y_2$ for every $X \in [a , b]$ revolve a complete revolution about X-axis , then the X-coordinates of the two intersecting points of the two curves are the terms of integration a , b , where a < b and ,



$$v = \pi \int_a^b \left[(y_1^2 - y_2^2)\right] d X$$

i.e. $v = \pi \int_a^b y_1^2\, d X - \pi \int_a^b y_2^2\, d X$

2. If a region bounded by two intersecting curves $X_1 = f(y)$, $X_2 = g(y)$ where $X_1 \geq X_2$ for every $y \in [c , d]$ revolve a complete revolution about y-axis , then the y-coordinates of the two intersecting points of the two curves are the terms of integration c , d where c < d and



$$v = \pi \int_c^d \left[(X_1^2 - X_2^2)\right] d y$$

i.e. $v = \pi \int_c^d X_1^2\, d y - \pi \int_c^d X_2^2\, d y$

Multiple Choice Question Bank

Differential & Integral calculus

Multiple choice question bank

Differential & Integral calculus

First Questions on limits

Choose the correct answer from the given ones :

1. $\lim\limits_{x \to \infty} \left(1 + \frac{1}{x}\right)^{2x}$ equals ....

(a) 1 (b) 2 (c) e (d) e^2

2. $\lim\limits_{x \to 0} (1 + x)^{\frac{1}{3x}}$ equals ....

(a) $\frac{1}{3}$ (b) e^3 (c) $e^{\frac{1}{3}}$ (d) $\frac{e}{3}$

3. $\lim\limits_{x \to \infty} \left(1 + \frac{1}{1+x}\right)^{x}$ = ...

(a) e (b) $\frac{1}{1+e}$ (c) e + 1 (d) e^{-1}

4. $\lim\limits_{x \to \infty} \left(\frac{x+5}{x+3}\right)^{x}$ = ...

(a) e (b) e^2 (c) $\frac{1}{e}$ (d) $\frac{2}{e}$

5. $\lim\limits_{x \to \infty} \left(\frac{x+7}{x+3}\right)^{x+4}$ = ...

(a) e^4 (b) e^3 (c) e^2 (d) e^5

6. $\lim\limits_{x \to 0} \frac{e^{4x} - 1}{5x}$ = ...

(a) $\frac{4}{5}$ (b) $e^{\frac{4}{5}}$ (c) $e^{\frac{5}{4}}$ (d) ln 5

7. $\lim\limits_{x \to 0} \frac{e^{x} - \sin x - 1}{x}$ = ...

(a) 2 (b) 1 (c) e – 1 (d) zero

8. $\lim\limits_{x \to 0} \frac{5^{x} - 1}{3^{x} - 1}$ = ...

(a) $\ln \frac{5}{3}$ (b) $\frac{\log 5}{\log 3}$ (c) $\log \frac{5}{3}$ (d) $\frac{5}{3}$

9 $\lim\limits_{x \to 0} \frac{e^{-x} - 1}{x} = \dots\dots\dots$

(a) e (b) -1 (c) $-e$ (d) e^{-1}

10 $\lim\limits_{x \to 0} \frac{2^{x} - 1}{3x} = \dots\dots\dots$

(a) $3 \ln 2$ (b) $\frac{1}{3} \ln 2$ (c) $\ln \frac{2}{3}$ (d) $2 \ln 3$

11 If $\lim\limits_{x \to a} \left(1 + \frac{1}{x}\right)^{2x} = e^2$, then a = ……………

(a) zero (b) e (c) 1 (d) ∞

12 $\lim\limits_{x \to 0} (e^{x} - 4) = \dots\dots\dots$

(a) -4 (b) -3 (c) e (d) e^4

13 $\lim\limits_{x \to 1} \left(\frac{\ln x}{x - 1}\right)$ equals ……………

(a) zero (b) 1 (c) e (d) e^{-1}

14 $\lim\limits_{x \to 0} \frac{1 - e^{2x}}{1 - e^{x}} = \dots\dots\dots$

(a) 1 (b) -1 (c) 2 (d) -2

15 $\lim\limits_{x \to 0} \left(1 + \frac{x}{a}\right)^{\frac{a}{x}} = \dots\dots\dots$

(a) e^{-1} (b) $-e$ (c) $-e^{-1}$ (d) e

16 $\lim\limits_{x \to 0} \frac{\ln (1 + x^2)}{x^2} = \dots\dots\dots$

(a) 1 (b) $\log_a e$ (c) $\ln a$ (d) 2

17 $\lim\limits_{x \to 0} \frac{\log (1 + 2x)}{5^{x} - 1} = \dots\dots\dots$

(a) $2 \ln 10 \ln 5$ (b) $\frac{2}{5}$ (c) $\log_5 2e$ (d) $2 \log e \log_5 e$

18 If $\lim\limits_{x \to 0} \frac{\ln (1 + a x)}{b x} = -1$, then a + b = ……………

(a) -1 (b) zero (c) 1 (d) 2

19. $\lim\limits_{x \to 0} \dfrac{e^{x} - e^{\sin x}}{x - \sin x} = \dots\dots\dots$

(a) $e^{\sin x}$ (b) e (c) 1 (d) zero

20. $\lim\limits_{x \to 6} \dfrac{e^{x} - e^{6}}{x - 6} = \dots\dots\dots$

(a) e (b) e^{6} (c) ln 6 (d) e^{12}

21. $\lim\limits_{x \to \frac{\pi}{2}} (1 + \cos x)^{\frac{\sec x}{4}} = \dots\dots\dots$

(a) 4 e (b) e^{4} (c) $\frac{1}{4}$ e (d) $e^{\frac{1}{4}}$

22. $\lim\limits_{x \to 0} (1 + 3\tan^2 x)^{\cot^2 x} = \dots\dots\dots$

(a) e^{3} (b) e^{-3} (c) 3 e (d) – 3 e

23. $\lim\limits_{x \to 0} \left(4\sin^3 x + \sin\dfrac{\pi}{2}\right)^{\csc^3 x} = \dots\dots\dots$

(a) e^{-4} (b) 4 e (c) e^{2} (d) e^{4}

24. $\lim\limits_{x \to \infty} \left(\dfrac{x^2 - 7x + 12}{x^2 - 8x + 16}\right)^{2x - 4} = \dots\dots\dots$

(a) 1 (b) e (c) e^{2} (d) e^{4}

25. $\lim\limits_{x \to 0} (1 + 2x + x^2)^{\frac{1}{2x}} = \dots\dots\dots$

(a) e^{2} (b) $\frac{1}{2}$ e (c) e (d) $e^{\frac{1}{2}}$

26. $\lim\limits_{x \to 0} \dfrac{a^{x} + b^{x} + c^{x} - 3}{x} = \dots\dots\dots$

(a) ln (a b c) (b) log a + log b + log c

(c) ln a . ln b . ln c (d) 1

27. $\lim\limits_{n \to \infty} n\left(\ln(n+1) - \ln n\right) = \dots\dots\dots$

(a) zero (b) e (c) $\frac{1}{e}$ (d) 1

28. $\lim\limits_{x \to \infty} \left(\dfrac{x}{x + k}\right)^{x} = e$, then k = $\dots\dots\dots$

(a) 1 (b) – 1 (c) – 2 (d) 2

29 $\lim\limits_{x \to 0} \dfrac{(10)^{\sin x} - 1}{\tan x} = \text{..............}$

(a) $\ln 10$ (b) $\log x$ (c) zero (d) 1

30 $\lim\limits_{x \to 0} \dfrac{\ln (1 - x)}{x} = \text{..............}$

(a) e (b) $-$ e (c) -1 (d) 1

31 $\lim\limits_{x \to 3} \dfrac{\ln (x - 2)}{x - 3} = \text{..............}$

(a) e (b) 1 (c) e^2 (d) -1

32 $\lim\limits_{x \to 0} \dfrac{\ln (x^2 + 3x + 1)}{\ln (x^2 + 5x + 1)} = \text{..............}$

(a) 1 (b) $\frac{3}{5}$ (c) $\frac{5}{3}$ (d) $\ln \frac{3}{5}$

33 If $\lim\limits_{x \to 0} \dfrac{e^{ax} - e^{5ax}}{3x} = 2$, then a = ... where $a \in \mathbb{R}^+$

(a) 2 (b) 3 (c) 5 (d) 6

34 If $\lim\limits_{x \to 0} \dfrac{e^{ax} - 1}{\ln (1 + 3x)} = 2$, then a = ...

(a) 1 (b) 2 (c) 3 (d) 6

35 If $\lim\limits_{x \to 0} \dfrac{x e^{x} - x}{1 - \cos 2x} = \text{..............}$

(a) 1 (b) $\frac{1}{2}$ (c) e^2 (d) $e^{\frac{1}{2}}$

36 $\lim\limits_{x \to 0} \dfrac{(10)^{x} - 2^{x} - 5^{x} + 1}{x \sin x} = \text{..............}$

(a) $\ln 10$ (b) $\ln \frac{5}{2}$ (c) $\ln 5 \times \ln 2$ (d) $\log_2 5$

37 If $f(x) = e^{\tan x}$, then $\lim\limits_{x \to \frac{\pi}{4}} \dfrac{f(x) - f\left(\frac{\pi}{4}\right)}{x - \frac{\pi}{4}} = \text{..............}$

(a) e (b) 2 e (c) e^2 (d) $2e^2$

38 $\lim\limits_{h \to 0} \dfrac{\cot (x + h) - \cot (x)}{h} = \text{..............}$

(a) $-\csc^2 x$ (b) $\sec^2 x$ (c) $\cot^2 x$ (d) $\tan^2 x$

39 $\lim\limits_{h \to 0} \dfrac{\sec\left(\frac{\pi}{4} + h\right) - \sec\left(\frac{\pi}{4}\right)}{h} = \dots\dots\dots$

(a) $\sqrt{2}$ (b) zero (c) $\frac{1}{\sqrt{2}}$ (d) undefined.

40 If $f(x)$ is the rule of a polynomial , then $\lim\limits_{h \to 0} \dfrac{f'(x+h) - f'(x)}{h} = \dots\dots\dots$

(a) $f'(x)$ (b) $f'(h)$ (c) $f''(x)$ (d) $f''(h)$

41 If $f'(x) = \cos^3 x$ and $f(0) = 0$, then $\lim\limits_{x \to 0} \dfrac{f(x)}{x} = \dots\dots\dots$

(a) -1 (b) zero (c) 1 (d) does not exist

42 If $e^x = \sum\limits_{n=0}^{\infty} \dfrac{x^n}{\lfloor n}$, then $\lim\limits_{x \to 0} \dfrac{e^x - 1 - x - \frac{x^2}{2}}{x^3} = \dots\dots\dots$

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{6}$ (d) $e^3 - 1$

Second Questions on differentiation - Implicit and parametric differentiation - Higher derivatives

Choose the correct answer from the given ones :

1 If $f(x) = \cot(5x - \pi)$, then $f'\left(\frac{\pi}{4}\right) = \dots\dots\dots$

(a) $5\sqrt{2}$ (b) $-5\sqrt{2}$ (c) 10 (d) -10

2 If $y = \csc 2x$, then $\frac{dy}{dx} = \dots\dots\dots$ at $x = \frac{\pi}{6}$

(a) $\frac{3}{4}$ (b) $\frac{-4}{3}$ (c) $\frac{1}{2}$ (d) $\sqrt{3}$

3 If $y = \sec\left(\frac{\pi}{4} x\right)$, then $\frac{dy}{dx} = \dots\dots\dots$

(a) $\frac{\pi}{4} \sec\left(\frac{\pi}{4} x\right) \tan\left(\frac{\pi}{4} x\right)$ (b) $\frac{\pi}{4} x$

(c) $\frac{\pi}{4}$ (d) $\sqrt{2}$

4 If $y = \left(\sec \frac{\pi}{4}\right) x$, then $\frac{dy}{dx} = \dots\dots\dots$

(a) $\frac{\pi}{4} \sec\left(\frac{\pi}{4} x\right) \tan\left(\frac{\pi}{4} x\right)$ (b) $\frac{\pi}{4} x$

(c) $\frac{\pi}{4}$ (d) $\sqrt{2}$

5 If $y = \csc\sqrt{x}$, then $\frac{dy}{dx} = \ldots\ldots$

(a) $\frac{-1}{2\sqrt{x}} \csc\sqrt{x} \cot\sqrt{x}$
(b) $-\csc\sqrt{x} \times \cot\sqrt{x}$
(c) $-\csc^2 \sqrt{x}$
(d) $-\frac{1}{2\sqrt{2}} \csc^2 \sqrt{x}$

6 If $y = \cot(x^2 + 3)$, then $\frac{dy}{dx} = \ldots\ldots$

(a) $-2x\csc^2(x^2 + 3)$
(b) $2x\csc^2(x^3 + 3)$
(c) $-2x\cot(x^2 + 3)\csc(x^2 + 3)$
(d) $\csc^2(x^2 + 3)$

7 If $f(x) = (5 - 2\cot x)^3$, then $\acute{f}\left(\frac{\pi}{4}\right) = \ldots\ldots$

(a) -108
(b) 27
(c) 54
(d) 108

8 If $f(x) = \frac{x - 4}{2\sqrt{x}}$, then $\acute{f}(0) = \ldots\ldots$

(a) zero
(b) 1
(c) not exist
(d) -1

9 If $g(x) = |x|$, then $\acute{g}(-5) = \ldots\ldots$

(a) 5
(b) -5
(c) 1
(d) -1

10 If $y = \sqrt{f(x)}$, and $\acute{f}(2) = 4$, $f(2) = 9$, then $\frac{dy}{dx}$ when $x = 2$ equals $\ldots\ldots$

(a) $\frac{4}{3}$
(b) $\frac{2}{9}$
(c) $\frac{1}{6}$
(d) $\frac{2}{3}$

11 If $x^3 y^2 = 1$, then $\left[\frac{dy}{dx}\right]_{y=1} = \ldots\ldots$

(a) $-\frac{2}{3}$
(b) $-\frac{3}{2}$
(c) $\frac{2}{3}$
(d) $\frac{3}{2}$

12 If $f(5x) = x^2 + x$, then $\acute{f}(2) = \ldots\ldots$

(a) 5
(b) 1
(c) $\frac{3}{25}$
(d) $\frac{9}{25}$

13 If $xy = 1$, then $\frac{dy}{dx}$ equals each of the following except $\ldots\ldots$

(a) x^{-2}
(b) $\frac{-1}{x^2}$
(c) $\frac{-y}{x}$
(d) $-y^2$

14. If $\frac{x}{y} + \frac{y}{x} = a$ where a is constant , then $\frac{dy}{dx} = \dots\dots\dots$

(a) $\frac{x}{y}$ (b) $\frac{y}{x}$ (c) $\frac{ax}{y}$ (d) $\frac{ay}{x}$

15. $\frac{d}{dx}\left(2 \cot \frac{\pi}{4}\right) = \dots\dots\dots$

(a) $-2 \csc^2 \frac{\pi}{4}$ (b) $-2 \cot \frac{\pi}{4} \csc \frac{\pi}{4}$ (c) 2 (d) zero

16. If $\frac{2}{\sqrt{x}+\sqrt{y}} = 9$, then $\left(\frac{dy}{dx}\right)^2 = \dots\dots\dots$

(a) $\frac{x}{y}$ (b) $\frac{y}{x}$ (c) $\frac{2y}{x}$ (d) $\frac{1}{2\sqrt{xy}}$

17. If $y = 8x^3$, then $dy = \dots\dots\dots$

(a) $24x^2\,dx$ (b) $24x^2 + c$ (c) $2x^4 + c$ (d) $24x^2$

18. If $m = 4\pi r^2$ which of the following is equal to differential m ?

(a) 4π (b) $2r\,dr$ (c) $8\pi r\,dr$ (d) $8\pi\,dr$

19. $\frac{d}{dx}(2\cos^2 x - 1) = \dots\dots\dots$

(a) $\sin 2x$ (b) $\cos 2x$ (c) $-2\sin 2x$ (d) $-2\cos 2x$

20. If $f(x) = \sin x \sec 2x$, then $f'(0) = \dots\dots\dots$

(a) -1 (b) zero (c) 1 (d) 2

21. $\frac{d}{dx}\left[x^2 + \frac{d}{dx}(x + \sec x)\right] = \dots\dots\dots$

(a) $2x + 1 + \sec x \tan x$ (b) $2x + 2\sec^3 x - \sec x$

(c) $2 + \sec^3 x - \sec x$ (d) $2x + \sec^2 x \tan^2 x$

22. If $f(x) = 8x \sin x \cos x \cos 2x$, then $f'\left(\frac{\pi}{8}\right) = \dots\dots\dots$

(a) -2 (b) zero (c) 1 (d) 2

23. $\frac{d}{dx}(\cos x \csc x) = \dots\dots\dots$

(a) zero (b) $\csc^2 x$ (c) $\sec^2 x$ (d) $-\csc^2 x$

24 If $y = \sin X \sec\left(\frac{\pi}{2} - X\right)$ where X is an acute angle , then $\frac{dy}{dX} =$ ....

(a) zero (b) 1
(c) $\cos X \csc X + \sin X \sec X$ (d) $\sin X \cos X + \sec X \cot X$

25 If $y + \cot X = 0$, then $\frac{dy}{dX} =$ ....

(a) $1 + y$ (b) $1 + y^2$ (c) $y^2 - 1$ (d) y^2

26 If $y = (\csc X - \cot X)(\csc X + \cot X)$, then $\frac{dy}{dX} =$ ....

(a) zero (b) 1 (c) $-y \csc X$ (d) $-y \cot X$

27 If $f(X) = \tan(X - \theta) . \cot(X + \theta)$, then $f'(X) =$ .... at $X = \theta$

(a) $\cot \theta$ (b) $\tan 2\theta$ (c) $\cot 2\theta$ (d) $\tan \theta + \cot \theta$

28 If $f(X) = X^3 + 5X - 3$, then $\frac{d}{dX}[f'(4)] =$ ....

(a) zero (b) 4 (c) 24 (d) 53

29 If $f(X) = \sec X \cos X + \csc X \sin X$, then $f\left(\frac{13\pi}{4}\right) + f'\left(\frac{13\pi}{4}\right) =$ ....

(a) $\frac{-13\pi}{4}$ (b) $\frac{13\pi}{4}$ (c) 2 (d) undefined.

30 If $y = \sec X(\sin X + \cos X)$, then $\frac{dy}{dX} =$ ....

(a) $1 - \tan^2 X$ (b) $\cot^2 X$ (c) $\sec^2 X$ (d) $\csc^2 X$

31 If $X^2 + 4Xy - 3y^2 - 2X = 3$, then $\frac{dy}{dX}$ could be equal .... at $X = 3$

(a) 3 (b) $\frac{1}{3}$ (c) $-\frac{1}{3}$ (d) -3

32 If $f(X) = X^3 - 5X^2 + 9X - 3$, then $f''(0) =$ ....

(a) -20 (b) -10 (c) 0 (d) 10

33 If $f(n) = X^4$ where X is constant , then $f''(n) =$ ....

(a) $12X^2$ (b) $4X^3$ (c) zero (d) $12n^2$

34 If $f(x) = a x^3 + 3x^2 + 4x + 1$ and $f''(1) = 6$, then $a =$ ....

(a) zero (b) $\frac{-4}{3}$ (c) -1 (d) $\frac{-2}{3}$

35 If $f(x) = \sin 2x$, then $f''\left(\frac{\pi}{4}\right) =$ ....

(a) zero (b) -2 (c) -4 (d) -6

36 If $f(x) = \sin^2 x + \cos^2 x$, then $f''(-1) =$ ....

(a) -1 (b) zero (c) 1 (d) 2

37 If $f(x) = \cot x$, then $f''\left(\frac{\pi}{4}\right) =$ ....

(a) $\frac{-4}{9}$ (b) $\frac{4}{9}$ (c) 4 (d) $\frac{9}{2}$

38 If $f(x) = \sin 2x \cos 2x$, then $f''\left(\frac{\pi}{3}\right) =$ ....

(a) -4 (b) zero (c) $4\sqrt{3}$ (d) 8

39 $\frac{d^2}{dx^2}\left(\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}\right) =$ ....

(a) $-\cos x$ (b) $\cos x$ (c) $-\sin x$ (d) $\cos x - \sin x$

40 $\frac{d}{dx}\left(\frac{dy}{dz}\right) = \frac{d^2y}{dz^2} \times$ ....

(a) $\frac{dz}{dx}$ (b) $\frac{dy}{dx}$ (c) $\frac{dx}{dz}$ (d) $\frac{dy}{dz}$

41 If $y = -\sin x$, then $\frac{d^2y}{dx^2} + y =$ ....

(a) -4 (b) 2 (c) 4 (d) zero

42 If $y = x^2 + 2x$, and $\frac{d^2y}{dx^2} - y - 3 = 0$, then $x =$ ....

(a) 1 (b) 2 (c) -1 (d) -2

43 If $f(x) = (x-3)^{-1}$, then $f''(3) =$ ....

(a) zero (b) -6 (c) undefined (d) 2

44 If f is a polyonomial function of fifth degree , then the fifth derivative of the function f equals ....

(a) zero (b) non zero constant (c) x (d) $5x$

45 The third derivative of the quadratic function is a .... function.

(a) linear (b) quadratic (c) constant (d) zero

46 The second derivative of the cubic function is a .... function.

(a) linear (b) quadratic (c) constant (d) zero

47 If $f(x) = \frac{x}{x-2}$, then $f'''(3) =$ ....

(a) 12 (b) -8 (c) -12 (d) -16

48 $\frac{d}{dx}[y \frac{dy}{dx}] = y \frac{d^2y}{dx^2} +$ ....

(a) $\frac{d^2y}{dx^2}$ (b) $\left(\frac{dy}{dx}\right)^2$ (c) $\frac{dy}{dx}$ (d) y

49 If $y = a \sin(mx) + b \cos(mx)$, $\frac{d^2y}{dx^2} =$ ....

(a) $m^2 y$ (b) $-m^2 y$ (c) my (d) $-my$

50 If $y = \sin 3x + \cos 3x$, then $\frac{d^4y}{dx^4} =$ ....

(a) gy (b) $81y$

(c) $3 \cos 3x - 3 \sin 3x$ (d) $81x$

51 If $f(x) = \cos 3x \cos x - \sin 3x \sin x$, then $f'''\left(\frac{\pi}{4}\right) =$ ....

(a) 2 (b) 4 (c) 8 (d) 16

52 $\frac{d^3}{dx^3}(\sin x \cot x) =$ ....

(a) $-\sin x$ (b) $-\cos x$ (c) $\sin x$ (d) $\cos x$

53 If $x = (1-y)(1+y)(1+y^2)(1+y^4)$, then $\frac{d^2y}{dx^2} =$ ....

(a) $\frac{-1}{8} y^{-7}$ (b) $-56 y^6$ (c) $\frac{-7}{64} y^{-15}$ (d) $\frac{7}{8} y^6$

54 If $(x+y)^5 = 3$, then $\frac{d^2 y}{d x^2} + \frac{d y}{d x} =$ ....

(a) -1 (b) zero (c) $20 (x+y)^4$ (d) 1

55 If $y = f(x)$ satisfies the relation $\frac{d^4 y}{d x^4} = y$, then y can be qual to ....

(a) $(x+1)^4$ (b) $\sin x$

(c) $\tan x$ (d) constant value.

56 If $f(x+1) = x^2 + 2x + 1$, then $f'(3) =$ ....

(a) 1 (b) 2 (c) 3 (d) 4

57 If $x^2 - y^2 = 9$ and $\frac{d^2 y}{d x^2} = \frac{a}{y^3}$, then a = ...

(a) 1 (b) -1 (c) 9 (d) -9

58 If $f(x) = (x-3)(x-4)(x-k)$ and $f'(3) = 2$ where $k \in \mathbb{R}$, then k = ...

(a) zero (b) 1 (c) 2 (d) 3

59 If $y = x^5 - 10x^4 + 60x^2 + 120$, then $\frac{d^2 y}{d x^2} =$ .... at $\frac{d^4 y}{d x^4} = 0$

(a) 300 (b) 200 (c) -200 (d) -300

60 If $f(x) = 2 \sin \frac{x}{2} \cos \frac{x}{2}$, then the 1000th derivative of this function = ...

(a) $[\sin x]^{1000}$ (b) $\sin x$ (c) $-\sin x$ (d) $-\cos x$

61 If $f(x) = e^{3x}$, then $f''(x)$ equals ....

(a) e^{2x} (b) $3 e^{3x}$ (c) $9 e^{3x}$ (d) $3 e^{2x}$

62 If $y = e^{ax}$, then $\frac{d^4 y}{d x^4} =$ ....

(a) a^4 (b) $a^4 y$ (c) $a e^{ax}$ (d) $a^2 e^{ax}$

63 If $f(x) = x^2 - 3 \ln 5x$, then $f'(2) =$ ....

(a) -1 (b) 1 (c) $\frac{5}{2}$ (d) 6

64 If $f(x) = a e^{x}$, then $\acute{f}(-2)$ equals ....

(a) $-f(2)$ (b) $-\acute{f}(2)$ (c) $-f(-2)$ (d) $f(-2)$

65 If $y = \ln(\sec x + \tan x)$, then $\frac{dy}{dx} =$ ....

(a) $\tan x$ (b) $\sec x$ (c) $\tan^2 x$ (d) $\csc x$

66 If $y = \ln(\csc x - \cot x)$, then $\frac{dy}{dx} =$ ....

(a) $\csc x$ (b) $\cot x$ (c) $\frac{1}{\csc x - \cot x}$ (d) $\sin x - \tan x$

67 If $y = (e^{-x} \ln x)$, then $\frac{dy}{dx} =$ ....

(a) $e^{-x}\left(\frac{1}{x} - \ln x\right)$ (b) $e^{x}\left(\frac{1}{x} - \ln x\right)$

(c) $\frac{e^{-x}}{x} - \ln x$ (d) $e^{-x}\left(\frac{1}{x} + \ln x\right)$

68 If $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = e^{x} + e^{-x}$, then $f(1) + \acute{f}(1) =$ ....

(a) $-2e$ (b) $-e$ (c) e (d) $2e$

69 If $y = x^6 + 6^{x}$, then $\frac{dy}{dx} =$ ....

(a) $12x$ (b) $x + 6$

(c) $6x^5 + 6^{x} \ln 6$ (d) $6x^5 + x \times 6^{x-1}$

70 If $y = \ln|x^2 - 1|$, then $\frac{dy}{dx} =$ ....

(a) $2x|x^2 - 1|$ (b) $\frac{2x}{x^2 - 1}$ (c) $\ln\left(\frac{2x}{x^2 - 1}\right)$ (d) $\frac{1}{x^2 - 1}$

71 If $y = \ln(\tan x)$, then $\frac{dy}{dx} =$ ....

(a) $2 \sec x \tan x$ (b) $2 \csc 2x$

(c) $2 \cot 2x$ (d) $-2 \csc x \cot x$

72 If $y = e^{\pi}$, then $\frac{dy}{dx} =$ ....

(a) zero (b) e^{π} (c) πe^{π} (d) $\frac{1}{\pi} e^{\pi}$

73 If $f(x)$ is an even function , $\grave{f}(0)$ is exist , then $\grave{f}(0) =$ ....

(a) zero (b) -1 (c) 1 (d) otherwise

74 If $f(x)$ is an odd function and is differentiable in the interval $]-\infty, \infty[$, $\grave{f}(3) = 2$, then $\grave{f}(-3) =$ ....

(a) zero (b) 1 (c) 2 (d) 4

75 $\frac{d}{dx}(1 + \tan^2 x)^3 =$ ....

(a) $6 \sec^5 x \tan x$ (b) $3 \sec^2 x \tan x$

(c) $6 \sec^6 x \tan x$ (d) $3 \sec^4 x \tan x$

76 $\frac{d}{dx}[(\sec x - 1)(\sec x + 1)] =$ ....

(a) $\sec^2 x \tan^2 x$ (b) $2 \sec^2 x \tan x$ (c) $\sec^2 x \tan x$ (d) $\sec^4 x$

77 If $y = \sec 3x + \tan 3x$, then $\frac{dy}{dx} =$ ....

(a) $3 y \sec 3x$ (b) $3 y \tan 3x$ (c) $3 y \sec^2 3x$ (d) $3 y \tan^2 3x$

78 If $y = \cot a x$ and $\frac{dy}{dx} + 4(1 + y^2) = 0$, then $a =$ ....

(a) 1 (b) -2 (c) 4 (d) -4

79 If $y = x \sin y$, then $\frac{dx}{dy} =$ ....

(a) $\frac{1 - \sin y}{x \cos y}$ (b) $\frac{1 - x \sin y}{x \cos y}$ (c) $\frac{1 - x \cos y}{\sin y}$ (d) $\frac{1 - x \cos y}{x \sin y}$

80 If $y \in]0, \frac{\pi}{4}[$, $x = \frac{2 \tan y}{1 - \tan^2 y}$, then $\frac{dy}{dx} =$ ....

(a) $\frac{1}{2} \cos^2 2y$ (b) $2 \sec^2 2y$ (c) $\sin^2 2y$ (d) $\cot 2y$

81 If $y = \frac{2 \cot x}{\cot^2 x - 1}$, then $\frac{dy}{dx} =$ .... where $x \in]0, \frac{\pi}{6}[$

(a) $2 \sec^2 2x$ (b) $2 \cot^2 2x$ (c) $4 \csc^2 2x$ (d) $\tan 2x$

82 If $y = \frac{\sec x \csc x}{\csc^2 x - \sec^2 x}$, then $\frac{dy}{dx} =$ ....

(a) $\sec^2 (2x)$ (b) $\frac{1}{2} \tan (2x)$ (c) $\sec x \csc x$ (d) $2 \sin^2 (2x)$

83 If $y = x \sec x$, then $\frac{dy}{dx} =$ ....

(a) $y \tan x + y x^{-1}$ (b) $\sec x \tan x$ (c) $x \sec x \tan x$ (d) $y (\tan x + 1)$

84 If $y = \csc x + \cot x$, then $\frac{dy}{dx} =$ ....

(a) $y \csc x$ (b) $-y \csc x$ (c) $y \cot x$ (d) $-y \cot x$

85 If $y = \sec^n (x)$, then $\frac{dy}{dx} =$ ....

(a) $n y \sec x$ (b) $n y \tan x$ (c) $n y \sec^2 x$ (d) $n y \tan^2 x$

86 If $y = x \sin x$, then $\ddot{y} + y =$ ....

(a) $-\sin x$ (b) x

(c) $2 \cos x$ (d) $-x \sin x + 2 \cos x$

87 If $x = \sin y$, $0 < x < 1$, y is an acute angle , then $\frac{dy}{dx} =$ ....

(a) $\sqrt{1-x^2}$ (b) $\frac{1}{\sqrt{1-x^2}}$ (c) $\sqrt{x^2-1}$ (d) $\frac{1}{\sqrt{x^2-1}}$

88 $\frac{d^3}{dx^3} (\sin^2 x) =$ ....

(a) $-4 \sin x$ (b) $-4 \sin 2x$ (c) $2 \cos 2x$ (d) $-\sin 2x$

89 $\frac{d^2}{dx^2} (\cos^4 x + \sin^4 x) =$ ....

(a) zero (b) $-2 \sin 2x$ (c) $-4 \cos 4x$ (d) 1

90 If $y = \tan x + \frac{1}{3} \tan^3 x$, then $\frac{dy}{dx} = (..............)^4$

(a) $\csc x$ (b) $\sec x$ (c) $\tan x$ (d) $\sec^4 x$

91 If $y = \tan x$, then $\frac{d^2 y}{dx^2} =$ ....

(a) $y + y^3$ (b) $\sec^2 x$ (c) $2y(1 + y^2)$ (d) $\sec x \tan x$

92 If $y = x \tan \frac{x}{2}$, then $(1 + \cos x)\frac{dy}{dx} - \sin x =$ ....

(a) xy (b) y (c) zero (d) x

93 If $y = e^{x} \sin x$, then $2\frac{dy}{dx} - \frac{d^2y}{dx^2} =$ ....

(a) $2y$ (b) $4y$ (c) $5y$ (d) $8y$

94 If $y = \ln\left(\frac{e^{4x}}{1 + e^{4x}}\right)$, then $\frac{dy}{dx} =$ ....

(a) $\frac{-1}{1 + e^{4x}}$ (b) $\frac{2}{1 + e^{4x}}$ (c) $\frac{-3}{1 + e^{4x}}$ (d) $\frac{4}{1 + e^{4x}}$

95 If $y = \frac{z + 1}{z - 1}$, $x = \frac{z - 1}{z + 1}$, then $\frac{dy}{dx} =$ .... at $x = 2$

(a) $\frac{-1}{8}$ (b) $\frac{-1}{4}$ (c) $\frac{1}{4}$ (d) -4

96 If $y = 3t^2 + 1$, $z = 2t - 5$, then $\frac{dy}{dz} =$ ....

(a) $\frac{t}{3}$ (b) $3t$ (c) 3 (d) $\frac{3}{t}$

97 If $x = 2t^2 + 3$, $y = \sqrt{t^3}$, then $\frac{dy}{dx}$ at $t = 1$ equals ....

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$ (c) $\frac{3}{8}$ (d) $\frac{8}{3}$

98 If $x = at^2$, $y = 2at$, then $\frac{dy}{dx} =$ ....

(a) $2at$ (b) $2a$ (c) t (d) $\frac{1}{t}$

99 If $y = \cot\left(\frac{\pi}{6} z\right)$, $z = 3\sqrt{x}$, then $\frac{dy}{dx} =$ .... at $x = 1$

(a) $-\frac{\pi}{3}$ (b) $\frac{\pi}{36}$ (c) $-\frac{\pi}{6}$ (d) $-\frac{\pi}{4}$

100 If $y = e^{x}$, $z = \sin x$, then $\frac{dy}{dz} =$ ....

(a) $\frac{e^{x}}{\sin x}$ (b) $e^{x} \tan x$ (c) $e^{x} \cos x$ (d) $\frac{e^{x}}{\cos x}$

101 If $x = \sin 2\pi\theta$, $y = \cos 2\pi\theta$, then $\frac{dy}{dx} =$ .... at $\theta = \frac{1}{6}$

(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) $\frac{1}{\sqrt{2}}$ (d) $-\sqrt{3}$

102 If $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$, then all the following are true except ....

(a) $\frac{dx}{d\theta} = y$ (b) $\frac{dy}{dx} = \cot\frac{\theta}{2}$

(c) $\frac{dy}{d\theta} + \frac{dx}{d\theta} = 1$ (d) $y\frac{dy}{dx} = a\sin\theta$

103 If $x = a\left(t - \frac{1}{t}\right)$, $y = a\left(t + \frac{1}{t}\right)$, then $\frac{dy}{dx} =$ ....

(a) $\frac{-x}{y}$ (b) $\frac{x}{y}$ (c) $\frac{-y}{x}$ (d) $\frac{y}{x}$

104 If $y = t^3 - t$, $t = \frac{1}{z^2} + z$, $z = 2x - 1$, then $\left(\frac{dy}{dx}\right)_{x=1} =$ ....

(a) -22 (b) -20 (c) -18 (d) -15

105 If $y = (5x - 4)(x + 3)$, $z = 3x^2 - 4x + 17$, then $\frac{d^2y}{dx^2} + \frac{d^2z}{dx^2} =$ ....

(a) 12 (b) 14 (c) 16 (d) 18

106 If $x = \frac{t}{t+1}$, $y = \frac{t+1}{t}$, then $\frac{d^2y}{dx^2} = \frac{\text{..............}}{x^2}$

(a) $\frac{2}{x}$ (b) $2x^{-3}$ (c) $-x^{-2}$ (d) zero

107 If $x = \sec z$, $\sqrt{y} = \tan z$, then $\frac{d^2y}{dx^2} =$ ....

(a) $2\tan z\sec z$ (b) $\sec^2 z\tan^2 z$ (c) 3 (d) 2

108 If $\frac{dz}{dx} = 2x - 3$, $\frac{dy}{dx} = x^2 + 1$, then $\frac{d^2z}{dy^2}$ at $x = 1$ equals ....

(a) 1 (b) $\frac{3}{4}$ (c) $\frac{3}{2}$ (d) $\frac{4}{3}$

109 If $y = x^2 + 3x + 2$, $z = 3x^2 - 5x + 4$, then $\frac{d^2y}{dz^2}$ at $x = 2$ equals ....

(a) $\frac{-4}{7}$ (b) $\frac{-4}{49}$ (c) 8 (d) 56

110 If $x = 2t^3 + 3$, $y = t^4$, then $\frac{d^2y}{dx^2} =$ .... at $t = 1$

(a) $\frac{2}{3}$ (b) $\frac{1}{9}$ (c) 4 (d) $\frac{1}{3}$

111 If $y = x^9 - 14x^7 - x^2 + 3$, then $\frac{d^{10}y}{dx^{10}} =$ ....

(a) $\lfloor 9$ (b) $\lfloor 10$ (c) zero (d) 9

112 If $y = x^7$, then $\frac{d^7 y}{d x^7} = \cdots\cdots$

(a) $7x^6$ (b) $42x^5$ (c) 49 (d) $|\underline{7}$

---

113 If $y = x^n$ where $x \neq 0$, then smallest value of n to make $\frac{d^3 y}{d x^3} \neq$ zero is $\cdots\cdots$

(a) 1 (b) 2 (c) 3 (d) 4

---

114 If $y = x^n$ where n is a natural number and $\frac{d^4 y}{d x^4} = 360\, x^{n-4}$, then the value of n = $\cdots\cdots$

(a) 7 (b) 13 (c) 5 (d) 6

---

115 If $f(x) = 20\, x^{n-1}$ and $f'''(x) = c$ where $c \in \mathbb{R}$, $n \in \mathbb{Z}^+$, then $n + c = \cdots\cdots$

(a) 104 (b) 123 (c) 124 (d) 125

---

116 If $f(x) = \frac{x^{25}}{|\underline{25}}$, then $f^{(25)}(x) = \cdots\cdots$

(a) $|\underline{25}$ (b) 25 (c) 1 (d) zero

---

117 If $f(x) = x \ln x$ and $\frac{d^8 y}{d x^8} = \frac{k}{x^7}$, then k = $\cdots\cdots$

(a) $|\underline{5}$ (b) $|\underline{6}$ (c) $|\underline{7}$ (d) $|\underline{8}$

---

118 If y is a function in x, then $\frac{d}{dx}[y^{(4)}] = \cdots\cdots$

(a) $4\, y^{(3)}$ (b) $4\, y^{(3)} \frac{dy}{dx}$ (c) $y^{(5)}$ (d) $\frac{1}{5}\, y^{(5)}$

---

119 If $y = x^{2021}$, then $\cdots\cdots$ = zero

(a) $y^{(2019)}$ (b) $y^{(2020)}$ (c) $y^{(2021)}$ (d) $y^{(2022)}$

---

120 If $y = x^{2021}$, then $\cdots\cdots = |\underline{2021}$

(a) $y^{(2019)}$ (b) $y^{(2020)}$ (c) $y^{(2021)}$ (d) $y^{(2022)}$

---

121 If $y = x^{n+1} + n\, x^{n-1} + 1$, then $\frac{d^n y}{d x^n} = \cdots\cdots$

(a) $|\underline{n+1}$ (b) $x\, |\underline{n+1}$ (c) $x\, |\underline{n}$ (d) $x^{-1}\, |\underline{n}$

122 If $y = \ln x$, then $\frac{d^{10} y}{d x^{10}} =$ ....

(a) $\frac{\lfloor 9}{-x^{10}}$ (b) $\frac{\lfloor 10}{-x^{9}}$ (c) $\frac{\lfloor 9}{x^{10}}$ (d) $\frac{\lfloor 10}{x^{9}}$

123 If $y = \ln x$, n is positive integer , then $\frac{d^n (y)}{d x^n} = $ ....

(a) $\left(\frac{-e}{x}\right)^n$ (b) $(n-1)\, x^{-n}$

(c) $(n+1)\, x^{-n-1}$ (d) $(-1)^{n-1} \lfloor n-1 \, x^{\;\;n}$

124 If $x y = a$ (where (a) is a positive real number) and $\frac{d^2 y}{d x^2} \times \frac{d^2 x}{d y^2} > \frac{d y}{d x} \times \frac{d x}{d y}$, then $a \in$ ....

(a) $]2 , \infty[$ (b) $]0 , 4[$ (c) $]4 , \infty[$ (d) $]0 , 2[$

125 If $y = \sin (a x)$, then $y^{(2017)} = $ ....

(a) $a^{2017}\, y$ (b) $-a^{2017}\, y$

(c) $a^{2017} \cos (a x)$ (d) $-a^{2017} \cos (a x)$

126 If $y = \cos (a x)$, then $y^{(2020)} = $ ....

(a) $a^{2020}\, y$ (b) $-a^{2020}\, y$

(c) $a^{2020} \sin (a x)$ (d) $-a^{2020} \sin (a x)$

127 If $y = \sin 5 x$ and $\frac{d^{20} y}{d x^{20}} = a \sin 5 x$, then $a = $ ....

(a) 20 (b) 5^{20} (c) -5^{20} (d) $\lfloor 20$

128 If (x , y) is any point on the unit circle , then ....

(a) $y\ddot{y} - 2 (\dot{y})^2 + 1 = 0$ (b) $y\ddot{y} + (\dot{y})^2 + 1 = 0$

(c) $y\ddot{y} + (\dot{y})^2 - 1 = 0$ (d) $y\ddot{y} + 2 (\dot{y})^2 + 1 = 0$

129 If $f (2 x + 1) = x . h (2 x - 5)$ and $h (1) = 2$, $\acute{h} (1) = 4$, then $\acute{f} (7) = $ ....

(a) 4 (b) 7 (c) 11 (d) 13

130 If $f (x) + \acute{f} (x) = x^3 + 5 x^2 + x + 2$, then f is a polynomial function , then $f (x) = $ ....

(a) $x^3 + 2 x^2 - 3 x$ (b) $3 x^2 + 4 x - 3$

(c) $x^3 + 2 x^2 + 5$ (d) $x^3 + 2 x^2 - 3 x + 5$

131 If $f(x) = e^{x} g(x)$, $g(0) = 2$, $g'(0) = 1$, then $f'(0) =$ ....

(a) 1 (b) 3 (c) 2 (d) zero

132 If $y = f(x)$ and $f(x + h) - f(x) = 5x^2 h + h^2$, then $\frac{d^3 y}{d x^3} =$ ....

(a) 10 (b) $10x$ (c) $5x^2$ (d) zero

133 The rate of change of the volume of a sphere with respect to its surface area when r = 2 cm. is ....

(a) 1 (b) 2 (c) 3 (d) 4

134 If $f(x) = \begin{cases} x^2 & \text{when } x < 2 \\ 4x & \text{when } x \geq 2 \end{cases}$, then $f'(2) =$ ....

(a) 2 (b) 4 (c) 1 (d) doesnot exist

135 If f is an even function and f' exists , then $f'(h) + f'(-h)$ .... zero

(a) = (b) < (c) > (d) ≠

136 If $f : f(x) = \begin{cases} a x^2 + b x + c & x < 0 \\ 2x + 5 & x \geq 0 \end{cases}$ is differentiable twice at $x = 0$, then a = ...

(a) zero (b) 2 (c) 5 (d) 1

137 If $y = \sqrt{e^{a x}}$ and $y'' + 4y' + 4y = 0$, then a = ...

(a) 4 (b) −4 (c) 16 (d) −16

138 If $x = a e^{\theta} (\sin \theta - \cos \theta)$, $y = a e^{\theta} (\sin \theta + \cos \theta)$, then $\left(\frac{dy}{dx}\right)_{\theta = \frac{\pi}{4}} =$ ....

(a) 1 (b) 2 (c) $\frac{2}{\pi}$ (d) $\frac{1}{\pi}$

139 If $f(x)$, $g(x)$ are two differentiable functions at $x = 2$ and $h(x) = f(x) \cdot g(x)$ where $f(2) = 2$ $f'(2) = 3$ $f''(2) = 12$, $g(2) = 3$ $g'(2) = 4$ $g''(2) = 6$, then $h''(2) =$ ....

(a) 66 (b) 144 (c) zero (d) 60

140 If $f'(x) = x f(x)$, $f(3) = -5$, then $f''(3) =$ ....

(a) −50 (b) −40 (c) 15 (d) 27

141 If $f(\sin x) = \sin^2 x$, then $f'(1) =$ ....

(a) 1 (b) 2 (c) π (d) $\frac{\pi}{2}$

142 If $y = 1 + \frac{x}{\underline{|1}} + \frac{x^2}{\underline{|2}} + \frac{x^3}{\underline{|3}} + \cdots$ to ∞ , then $y' + y'' =$ ....

(a) x (b) 1 (c) y (d) 2 y

143 The rate of change of e^{x^3} respect to $\ln x$ equals ....

(a) $3x^2 e^{x^3} + 3x^2$ (b) e^{x^3} (c) $3x^3 e^{x^3}$ (d) $3x^2 e^{x^3}$

144 The rate of change of $\sin x^3$ respect to $\cos x^3$ equals ....

(a) $-\cot x^3$ (b) $\cot x^3$ (c) $\tan x^3$ (d) $-\tan x^3$

145 If $x = \sin^3 \theta$, $y = \cos^3 \theta$, then $\frac{d^2 y}{d x^2} =$ .... at $\theta = \frac{\pi}{4}$

(a) $\frac{2}{3}$ (b) $\frac{8}{3}$ (c) $\frac{4\sqrt{3}}{3}$ (d) $\frac{4\sqrt{2}}{3}$

146 The rate of change of $(x - \sin x)$ respect to $(1 - \cos x)$ at $x = \frac{\pi}{3}$ equals ....

(a) $\frac{\sqrt{3}}{3}$ (b) $2\sqrt{3}$ (c) $\sqrt{3}$ (d) $\frac{2}{3}$

147 $\frac{d}{dx}\left(\sum_{n=0}^{\infty} \frac{1}{\underline{|n}}\right) =$ ....

(a) zero (b) 1 (c) $\underline{|n}\, x$ (d) $\underline{|n+1}$

148 If $y = \frac{e^x + 1}{e^x - 1}$, then $\frac{y^2}{2} + \frac{dy}{dx} =$ ....

(a) 1 (b) -1 (c) $-\frac{1}{2}$ (d) $\frac{1}{2}$

149 If $f(x) = \ln\left(x + \sqrt{x^2 + 1}\right)$, then $f'(x) =$ ....

(a) $\sqrt{x^2+1}$ (b) $\frac{x}{\sqrt{x^2+1}}$ (c) $1 + \frac{x}{\sqrt{x^2+1}}$ (d) $\frac{1}{\sqrt{x^2+1}}$

150 If $y = \ln\sqrt{\tan x}$, then $\frac{dy}{dx} =$ .... when $x = \frac{\pi}{4}$

(a) 1 (b) zero (c) $\frac{1}{2}$ (d) ∞

151 If $f(x) = e^{x} \sin x$, $g(x) = e^{x} \cos x$ Which of the following statements is not true ?

(a) $f'(x) = f(x) + g(x)$
(b) $g'(x) = g(x) - f(x)$
(c) $f''(x) = 2\,g(x)$
(d) $g''(x) = 2\,f(x)$

152 If $y = \frac{1 + \tan x}{1 - \tan x}$, then $\frac{dy}{dx} = \ldots\ldots\ldots\ldots$

(a) $\cos^2(x + 45^\circ)$
(b) $\sec^2(x + 45^\circ)$
(c) $\sin^2(x + 45^\circ)$
(d) $\csc^2(x + 45^\circ)$

153 If $y = \ln(\sin x)$, then $\frac{d^2y}{dx^2} = \ldots\ldots\ldots\ldots$

(a) $-\csc^2 x$
(b) $\sec x$
(c) $-\csc x \cot x$
(d) $\sec x \tan$

154 If $f(x) = e^{\ln x}$, then $f'(x) = \ldots\ldots\ldots\ldots$

(a) $[\ln x]\, e^{\log x}$
(b) $e^{\ln x}$
(c) $(\ln x)\, e^{x - \ln x}$
(d) 1

155 If $f(x) = e^{\ln(x^3 - 2x + 1)}$, then $f'(0) = \ldots\ldots\ldots\ldots$

(a) -4
(b) -2
(c) zero
(d) 2

156 If $y = x^2 \ln e^{x}$, then $\frac{dy}{dx} = \ldots\ldots\ldots\ldots$

(a) $3x^2$
(b) x^3
(c) $\ln x^3$
(d) zero

157 If $y = e^{(1 + \ln x)}$, then $\frac{dy}{dx} = \ldots\ldots\ldots\ldots$

(a) x
(b) $e\,x$
(c) e
(d) 1

158 If $f(x) = \ln(\sin x) - \ln(\cos x)$, then $f'\left(\frac{\pi}{4}\right) = \ldots\ldots\ldots\ldots$

(a) 2
(b) -2
(c) 1
(d) -1

159 If $f(x) = (\cos x)^{\cos x}$, then $f'(\text{zero}) = \ldots\ldots\ldots\ldots$

(a) -3
(b) -2
(c) -1
(d) zero

160 If $y = \sqrt{x + \sqrt{x + \sqrt{x + \sqrt{x + \cdots}}}}$, then $\frac{dy}{dx} = \ldots\ldots\ldots\ldots$

(a) $2y - 1$
(b) $\frac{1}{2y - 1}$
(c) $x^2 + y$
(d) $1 + x + x^2 + \cdots$

161 If $a^{y} = b^{x}$ where a , b $\in \mathbb{R}^{+}$, then $\frac{dy}{dx} =$ ....

(a) $\log \frac{a}{b}$ (b) $\log_a b$ (c) $\log_b a$ (d) $\log \frac{b}{a}$

162 If $y = x^{x}$, $x > 0$, then $\frac{dy}{dx} =$ ....

(a) $\ln x$ (b) $2 + \ln x$ (c) $x^{x} \ln x$ (d) $x^{x} (1 + \ln x)$

163 If $y^{x} = x^{y}$, then $\frac{dy}{dx} =$ ....

(a) $\frac{y}{x}$ (b) $\frac{y (x \ln y - y)}{x (y \ln x - x)}$

(c) $\frac{x}{y}$ (d) $\frac{x \ln y}{y \ln x}$

164 If $y = x e^{xy}$, then $\frac{dy}{dx} =$ ....

(a) $e^{xy} + x$ (b) $e^{xy} + x e^{xy}$

(c) $x e^{xy} + y e^{xy}$ (d) $e^{xy} + x e^{xy} (y + x \acute{y})$

165 If $f : \mathbb{R}^{+} \longrightarrow \mathbb{R}^{+}$ where $f (x) = x^{2x}$, then $\acute{f} (e) =$ ....

(a) $4 e^{2e}$ (b) $2 e^{2e}$ (c) $2 e^{e}$ (d) $4 e^{e}$

166 If $y = \left(\frac{x^{a}}{x^{b}}\right)^{a+b} \times \left(\frac{x^{b}}{x^{c}}\right)^{b+c} \times \left(\frac{x^{c}}{x^{a}}\right)^{c+a}$, then $\frac{dy}{dx} =$ ....

(a) 1 (b) zero (c) a + b + c (d) 2 (a + b + c)

167 If $f (x) = 2 x^{3} + 1$, $g (x) = x^{2}$, then $\acute{f} \big(g (3)\big) =$ ....

(a) zero (b) 9 (c) 486 (d) 2916

168 If $f (x) = \frac{2}{x + 1}$, $g (x) = 3 x$, then $\frac{d}{dx} \big[(f \circ g) (x)\big] =$ .... at $x = -2$

(a) $\frac{-3}{25}$ (b) 6 (c) $\frac{1}{25}$ (d) $\frac{-6}{25}$

169 If $f (x) = 3 x^{2} - 2$, then $(f \circ f)\grave{} (-1) =$ ....

(a) -36 (b) -18 (c) zero (d) 18

170 If f , g are two functions where $f(x) = x^2$, $g(2) = 3$, $g'(2) = -2$, $g''(2) = 5$, then $(f \circ g)''(2) = \dots\dots$

(a) 16 (b) 24 (c) 32 (d) 38

---

171 If $\lim\limits_{h \to 0} \dfrac{f'(1) - f'(1+h)}{2h} = 21$ where $f(x) = 2x^4 - a x^3$, then a = $\dots\dots$

(a) 11 (b) 12 (c) 13 (d) 14

---

172 If $\sin x = e^y$ where $0 < x < \pi$, then $\dfrac{dy}{dx} = \dots\dots$

(a) $\tan x$ (b) $\cot x$ (c) $-\tan x$ (d) $-\cot x$

---

173 If $y = 4^{\log_2 \sin x} + 9^{\log_3 \cos x}$, then $\dfrac{dy}{dx} = \dots\dots$

(a) zero (b) 1 (c) $\sin x + \cos x$ (d) -1

---

174 If k , m $\in \mathbb{R}$, then $f(x) = x e^x$ and $f^{(10)}(x) = k e^x + m x e^x$, then k + m = $\dots\dots$

(a) 9 (b) 10 (c) 11 (d) 12

---

175 If $f(x) = \begin{vmatrix} 2x & 7 & 6 \\ 0 & 3x^2 & 4 \\ 0 & 0 & x^3 \end{vmatrix}$, then $f'(1) = \dots\dots$

(a) 1 (b) 6 (c) 36 (d) 72

---

176 If $f(x) = \begin{vmatrix} g(x) & h(x) \\ I(x) & J(x) \end{vmatrix}$, then $f'(x) = \dots\dots$

(a) $\begin{vmatrix} g(x) & h(x) \\ I(x) & J(x) \end{vmatrix} + \begin{vmatrix} g(x) & h(x) \\ I'(x) & J(x) \end{vmatrix}$

(b) $\begin{vmatrix} g'(x) & h'(x) \\ I(x) & J(x) \end{vmatrix} + \begin{vmatrix} g(x) & h(x) \\ I'(x) & J'(x) \end{vmatrix}$

(c) $\begin{vmatrix} g(x) & h'(x) \\ I(x) & J'(x) \end{vmatrix} + \begin{vmatrix} g'(x) & h(x) \\ I(x) & J'(x) \end{vmatrix}$

(d) $\begin{vmatrix} g(x) & h(x) \\ I'(x) & J'(x) \end{vmatrix} + \begin{vmatrix} g'(x) & h(x) \\ I(x) & J(x) \end{vmatrix}$

---

177 If $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = e^{\tan x}$, then for all values of $x \in \mathbb{R}$ we find that $\dots\dots$

(a) $f'(x) \geq f(x)$ (b) $f'(x) \leq f(x)$

(c) $f'(x) > f(x)$ (d) $f'(x) < f(x)$

178 If $0 < X < \frac{\pi}{2}$ and $f(\sin X) = a \cos X$ where a is a constant and $\acute{f}\left(\frac{3}{5}\right) = -6$

, then a = ...

(a) 2 (b) 4 (c) 6 (d) 8

179 If $f(X) = g(X^2)$, then $\frac{\acute{f}(2)}{\acute{g}(4)} =$ ....

(a) 1 (b) 2 (c) 3 (d) 4

180 If the function g is the inverse of the function f where f and g are differentiable functions on $\mathbb{R}$ and $\acute{f}(a) = 2$, $f(a) = b$, then $\acute{g}(b) =$ ....

(a) $\frac{1}{2}$ (b) 2 (c) $\frac{2}{3}$ (d) 1

181 If $y = \cot X$ where X is in degree , then $\frac{dy}{dX} =$ ....

(a) $-\csc^2 X$

(b) $-\csc X \cot X$

(c) $\frac{-\pi}{180} \csc^2 X$

(d) $\frac{-180}{\pi} \csc^2 X$

182 If $y = \sin 2X$ and $\frac{d^n y}{dX^n} = 2^n \sin 2X$, then ....

(a) n is an even number.

(b) n is an odd number.

(c) n is divisible by 3

(d) n is divisible by 4

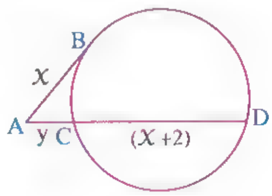
183 If $f(X) = \frac{e^{5X} + e^{4X} + e^{3X}}{e^{2X} + e^{X} + 1}$, then $\acute{f}(0) =$ ....

(a) 1 (b) 2 (c) 3 (d) 4

184 **In the opposite figure :**

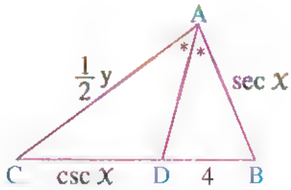
If $\overline{AB}$ is a tangent to the circle

, then $\frac{dy}{dX} =$ .... at $X = 4$



(a) 0.4 (b) 0.6 (c) 0.8 (d) 0.9

185 **In the opposite figure :**



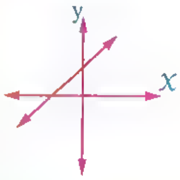
If $\overrightarrow{AD}$ bisects $\angle$ BAC

, then $\frac{dy}{dX} =$ ....

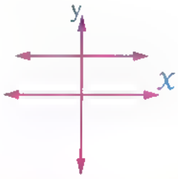
(a) $-\csc 2X \cot 2X$
(b) $-2 \csc X \cot X$
(c) $-2 \csc 2X$
(d) $-2 \csc 2X \cot 2X$

---

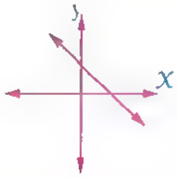
186 If $y = a X^n - b X^{n+1} + 5$ is a polynomial and a , b $\in \mathbb{R}^+$, then $\frac{d^n y}{d X^n}$ may by represented by the figure ....



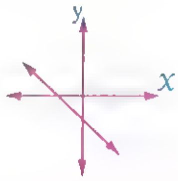
(a)



(b)



(c)



(d)

Third Questions on geometric applications (the two equations of the tangent and the normal to a curve

Choose the correct answer from the given ones :

1 If $f(X) = 3X + g(X)$, and $\grave{g}(2) = -5$, then the slope of the tangent to the curve of the function f at $X = 2$ equals ....

(a) -2
(b) 3
(c) -15
(d) $\frac{1}{2}$

---

2 If the tangent to the curve $y = f(X)$ at the point (3 , 4) makes an angle of measure $\frac{3\pi}{4}$ with the positive direction of X-axis , then $\grave{f}(3) =$ ....

(a) -1
(b) $-\frac{3}{4}$
(c) $\frac{3}{4}$
(d) 1

---

3 The tangent to the curve : $y = 3X^2 - 5$ at the point (1 , -2) passes through the point ....

(a) (5 , -2)
(b) y (3 , 1)
(c) (2 , -4)
(d) (0 , -8)

---

4 If the curves of the two functions $f(X)$ and $g(X)$ are touching at the point (2 , 4) , and $\grave{f}(2) = 3$, then $\grave{g}(2) =$ ....

(a) 2
(b) 3
(c) 4
(d) 5

5. Slope of the normal to the curve of the function $y = |x^3|$ at the point $(-2, 8)$ is ....

(a) -12 (b) 12 (c) $\frac{1}{12}$ (d) $-\frac{1}{12}$

6. The slope of the tangent to the curve $y = e^x$ at the point $(1, e)$ lies on it equals ....

(a) 1 (b) -1 (c) e (d) e^2

7. The slope of the tangent to the curve $y = \ln x$ at the point $(e^2, 2)$ lies on it equals ....

(a) 2 (b) -2 (c) e^2 (d) e^{-2}

8. If $f : f(x) = x - x \ln x$, then the slope of the tangent to the curve at $x = e$ equals ....

(a) zero (b) -1 (c) 1 (d) e

9. If the tangent drawn from the point $(2, 4)$ to the curve $y = \frac{1 - 2x}{x - 2}$ touches the curve at the point B, then B = ...

(a) $(1, 1)$ (b) $(2, 2)$ (c) $(2, 1)$ (d) $(-2, 0)$

10. The measure of the angle which the tangent to the curve $\sin 2x = \tan y$ makes with the positive direction of x-axis at the point $\left(\frac{3\pi}{4}, \frac{3\pi}{4}\right)$ equals ....

(a) zero (b) $135°$ (c) $45°$ (d) $26° 34'$

11. The tangent to the curve : $x^2 - xy + y^2 = 27$ which drawn at the point $(6, 3)$, makes an angle of measure ....with the positive direction of x-axis.

(a) $90°$ (b) zero (c) $45°$ (d) $180°$

12. The slpoe of the tangent to the curve $y = t^2 + 2t$, $x = t^2 - 2t$ equals .... at $t = 2$

(a) $\frac{1}{3}$ (b) $\frac{-1}{2}$ (c) 2 (d) 3

13. Which of the following curves has a tangent with constant slope ?

(a) $x = \sin t$, $y = \cos t$ (b) $x = t^2$, $y = 3t^2$

(c) $x = 2t - 1$, $y = t^2 - 4$ (d) $xy = 7$

14 The slope of the tangent to the curve of the circle $X^2 + y^2 = 1$ at $X = \frac{3}{5}$ equals ....

(a) $\pm \frac{4}{3}$ (b) $\pm \frac{3}{4}$ (c) $\frac{4}{5}$ (d) $\pm \frac{4}{5}$

15 If the tangent to the curve of the function $y = X^2 + a$ at the point $(1 , b)$ intersects the X-axis at $X = -1$, then $a \times b =$ ....

(a) 3 (b) 4 (c) 12 (d) -12

16 The equation of the tangent to the curve $X = y^2$ at the origin point is ....

(a) $y = 0$ (b) $X = 0$ (c) $X = y$ (d) $X + 4y = 0$

17 If the equation of the normal to the curve $y = f(X)$ at the point $(1 , 1)$ is $X + 4y = 5$, then $\acute{f}(1) =$ ....

(a) -3 (b) $-\frac{1}{4}$ (c) 4 (d) -4

18 The equation of the normal to the curve of the function : $y = X|X|$ at the point $(-2 , -4)$ is ....

(a) $y + 4X + 12 = 0$ (b) $4y + X + 18 = 0$
(c) $4y + X + 14 = 0$ (d) $y + 4X - 4 = 0$

19 The equation of the normal to the curve : $y = \sin X$ at the point $(0 , 0)$ is ....

(a) $X = 0$ (b) $y = 0$ (c) $X + y = 0$ (d) $X - y = 0$

20 The equation of the tangent to the curve of the function f where $f(X) = e^{2X+1}$ at the point $\left(\frac{-1}{2} , 1\right)$ is ....

(a) $2y = X + 1$ (b) $y = 2X + 2$ (c) $y = 2X - 3$ (d) $2y = 3X + 1$

21 The equation of the normal to the curve $y = 3e^{X}$ at point lies on it and its X-coordinate is -1 is ....

(a) $e^2 X = 0$ (b) $y - \frac{3}{e} = \frac{3}{e}(X + 1)$
(c) $y - 3 = \frac{e}{3}(X + 1)$ (d) $e^2 X + 3ey + e^2 - 9 = 0$

22 The equation of the normal to the curve $y = e^{2X} \cos X$ at $X = 0$ is ....

(a) $y + 2X = 1$ (b) $2y + X = 2$ (c) $X + y = 2$ (d) $y - 2X = 1$

23 If $y = x + c$ is a tangent to the curve $9x^2 + 16y^2 = 144$, then $c =$ ....

(a) ± 2 (b) ± 3 (c) ± 5 (d) ± 6

24 The normal to the circle $x^2 + y^2 = 12$ at any point on it , passes through the point ....

(a) $(2, 2)$ (b) $(1, 1)$ (c) $(0, 0)$ (d) $(-2, -2)$

25 The ratio between slope of the tangent to the curve : $y = \ln\left(3\sqrt{x+1}\right)$ and slope of tangent to the curve : $y = \ln\left(5\sqrt{x+1}\right)$ at $x = a$ equal the ratio ....

(a) $3 : 5$ (b) $5 : 3$ (c) $1 : 1$ (d) $\ln 3 : \ln 5$

26 The rate of change of slope of the tangent of the function $f(x) = 2x^3$ at $x = 3$ equals ....

(a) 36 (b) 54 (c) 6 (d) 12

27 If the normal to the curve $y = x \ln x$ parallel to the straight line $2x - 2y + 3 = 0$, then the equation of this normal is ....

(a) $x - y = 3e^{-2}$ (b) $x - y = 6e^{-2}$ (c) $x - y = 3e^2$ (d) $x - y = 6e^2$

28 If the tangent to the curve : $y^2 = 4ax$ is perpendicular to x-axis , then : ....

(a) $\frac{dy}{dx} = 0$ (b) $\frac{dy}{dx} = 1$ (c) $\frac{dx}{dy} = 1$ (d) $\frac{dx}{dy} = 0$

29 The tangent to the curve $x = t^2 - 1$, $y = t^2 - t$ parallel to x-axis at $t =$ ....

(a) zero (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{1}{2}$ (d) $\frac{-1}{\sqrt{3}}$

30 The tangent to the curve : $x = 3\cos\theta$, $y = 3\sin\theta$ where $(0 \le \theta \le \pi)$, parallel to x-axis if $\theta =$ ....

(a) zero (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) π

31 The curve $y = x^{\frac{1}{5}}$ at $(0, 0)$ has ....

(a) vertical tangent. (b) Horizontal tangent.

(c) inclined tangent. (d) no tangent.

32 If the curve $x = 2z^3 - 5z^2 - 4z + 12$, $y = 2z^2 + z - 4$ has Horizontal tangent , then $z = \dots\dots$

(a) $\frac{-1}{4}$ (b) $\frac{-1}{3}$ (c) 2 (d) $\frac{-2}{3}$

33 The curve $y - e^{xy} + x = 0$ has a vertical tangent at the point $\dots\dots$

(a) $(1, 1)$ (b) $(0, 0)$ (c) $(1, 0)$ (d) $(2, e^2)$

34 The tangent to the curve $x = e^{\theta}\cos\theta$, $y = e^{\theta}\sin\theta$ at the point which at $\theta = \frac{\pi}{4}$ makes with the positive direction of x-axis an angle of measure $\dots\dots$

(a) zero (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

35 The slope of the tangent to the curve $x = y^{100} + \log(100)^y$ at the point $(3, 1)$ equals $\dots\dots$

(a) 102 (b) 100 (c) 100×3^{99} (d) $\frac{1}{102}$

36 If the tangent to the curve : $y = x^3 - 3x^2$ makes obtuse angle with the positive direction of x-axis , then $x \in \dots\dots$

(a) $[0, 2]$ (b) $]0, 2[$ (c) $\mathbb{R} - [0, 2]$ (d) $\mathbb{R} -]0, 2[$

37 If the straight line : $y + x - 1 = 0$ touches the curve of the function $f : f(x) = x^2 - 3x + a$, then $a = \dots\dots$

(a) 1 (b) 2 (c) 3 (d) 4

38 The tangent to the curve of the function $y = \sqrt[3]{x}$ at $x = 0$ parallel to $\dots\dots$

(a) x-axis (b) y-axis

(c) the straight line $y = x$ (d) the straight line $x + y = 0$

39 If the curve : $y = x^2 - ax + a - 1$ touches x-axis where $a \in \mathbb{R}$, then $a = \dots\dots$

(a) 2 (b) zero (c) $\frac{-1}{2}$ (d) -1

40 The area of the triangle pounded by two coordinate axes and the tangent to the curve $xy = a^2$ at the point (x_1, y_1) lies on it equals $\dots\dots$

(a) $\frac{a^2 x_1}{y_1}$ (b) $\frac{a^2 y_1}{x_1}$ (c) $2a^2$ (d) $4a^2$

41 The curve $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$ touches the straight line $\frac{x}{a} + \frac{y}{b} = 2$ at the point (a , b) when ....

(a) $n = 3$ (b) $n = 2$ (c) for all values of n (d) false for all values of n

42 The equation of the tangent to the curve $y = be^{-\frac{x}{a}}$ at the point of intersection with y-axis is ....

(a) $\frac{x}{a} - \frac{y}{b} = 1$ (b) $a x + by = 1$ (c) $a x - by = 1$ (d) $\frac{x}{a} + \frac{y}{b} = 1$

43 The straight line : $a x + by + c = 0$ is a normal to the curve $x y = 1$, then ....

(a) $a > 0 , b > 0$ (b) $a < 0 , b < 0$

(c) $a = 0 , b \neq 0$ (d) $a > 0 , b < 0$ or $a < 0 , b > 0$

44 Length of the intercepted part from y-axis by the tangent to the curve $y = x \sin x$ at $x = \pi$ equals .... length unit.

(a) $-\pi$ (b) π (c) $-\pi^2$ (d) π^2

45 If the normal to the curve : $9 y^2 = x^3$ at the point (a , b) which lies on the curve cuts equal parts of the coordinate axes , then a = ...

(a) 2 (b) -4 (c) 4 (d) -2

46 If the tangent to the curve : $2 y^3 = a x^2 + x^3$ at the point (a , a) which lies on the curve cuts from the coordinate axes two parts of lengths L , M where $L^2 + M^2 = 61$, then a = ...

(a) ± 20 (b) ± 30 (c) ± 40 (d) ± 50

47 Slope of the tangent to the curve $y = \ln \tan t + \ln \cos t$, $x = \ln \sin t + \ln \cot t$ at $t = \frac{\pi}{4}$ is ....

(a) -1 (b) zero (c) 1 (d) 2

48 Measure of the angle that the tangent to the curve $y = e^{\tan x}$ makes with the positive direction of the x-axis at $x = \pi$ equals ....

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{3\pi}{4}$

49 If $f(X) = \frac{X + 1}{g(X)}$, $g(X) \neq 0$ and the curve of $g(X)$ has a horizontal tangent at the point (1 , 2)

, then $\acute{f}(1) = \dots\dots\dots$

ⓐ 2 ⓑ 1 ⓒ – 2 ⓓ $\frac{1}{2}$

50 If the equation of the normal to the common tangent of the two functions f and g at $X = 1$

is $y = \frac{-1}{3} X + \frac{3}{2}$, then $(f \times g)\acute{}(1) = \dots\dots\dots$

ⓐ 4 ⓑ 7 ⓒ 2 ⓓ 10

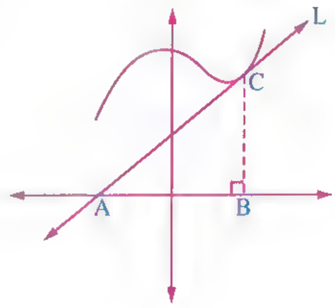
51 **In the opposite figure :**

If the straight line ℓ is a tangent of the function f

at the point C and cuts X-axis at the point A (– 4 , 0)

and if B (4 , 0) , $f(4) + \acute{f}(4) = 9$, then the area

of Δ ABC = ... square unit.



ⓐ 9 ⓑ 64

ⓒ 32 ⓓ 16

52 **In the opposite figure :**

If $f(X) = g(X) - 3\,h(X)$

, then $\acute{f}(5) = \dots\dots\dots$

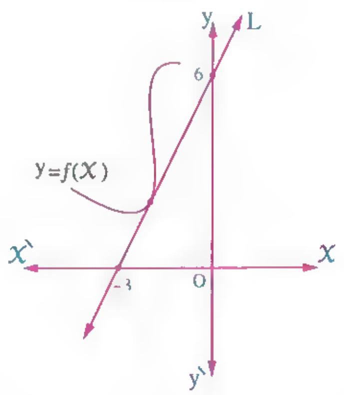
ⓐ zero ⓑ 2

ⓒ 3 ⓓ 4

53 **In the opposite figure :**

The straight line L is a tangent to the curve $y = f(X)$

at (– 2 , m) and $g(X) = f(2X)$, then $\acute{g}(-1) = \dots\dots\dots$

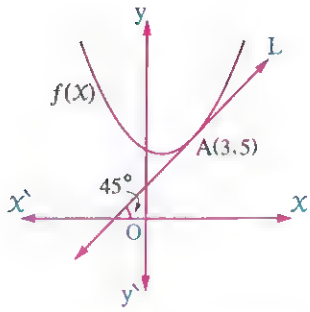


ⓐ 3 ⓑ 4

ⓒ 6 ⓓ 9

54 The opposite figure represents the function f and the straight line L touches the curve of f at the point A $(3 , 5)$ and $g(X) = X \cdot f(X)$

, then $g'(3) = \dots\dots\dots\dots$



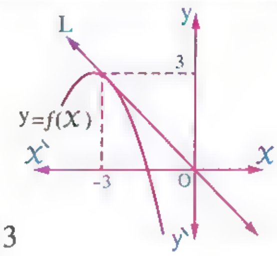
ⓐ 3 ⓑ 1

ⓒ 5 ⓓ 8

55 **In the opposite figure :**

If the straight line L touches the curve $y = f(X)$ at $(-3 , 3)$ and $h(X) = \dfrac{f(2-X)}{X-2}$

, then $h'(5) = \dots\dots\dots\dots$

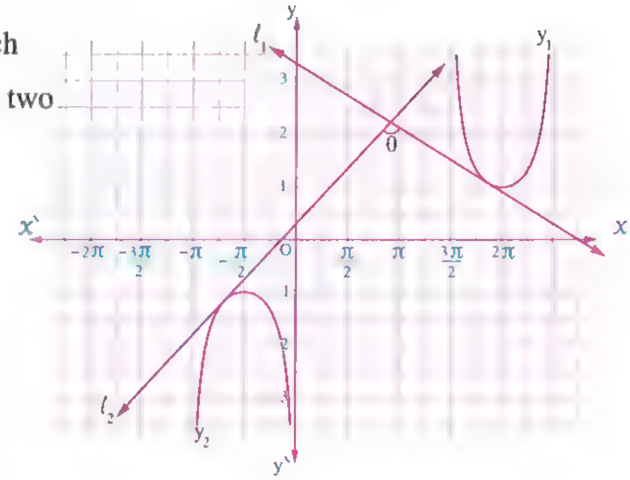


ⓐ zero ⓑ -2 ⓒ 2 ⓓ 3

56 **In the opposite figure :**

If $y_1 = \sec X$, $y_2 = \csc X$ and ℓ_1 , ℓ_2 touch the two curves y_1 , y_2 respectively at the two points $\left(\frac{11\pi}{6} , \frac{2}{\sqrt{3}}\right)$, $\left(\frac{-2\pi}{3} , \frac{-2}{\sqrt{3}}\right)$

, then $\tan \theta = \dots\dots\dots\dots$



ⓐ $-\frac{2}{3}$ ⓑ $\frac{2}{3}$

ⓒ $-\frac{12}{5}$ ⓓ $\frac{12}{5}$

Fourth Questions on related time rates

Choose the correct answer from the given ones :

1 A square with side length 5 cm. , the length of its side start to increase by rate 2 cm./sec. , then the side length of the square after t sec. given by the relation $\dots\dots\dots\dots$

ⓐ $2t$ ⓑ $5+2t$ ⓒ $2t-5$ ⓓ $5+4t^2$

2 The length of a rectangle is twice its width , if the rate of change of the length is 6 cm./sec. , then the rate of change of its width $= \dots\dots\dots\dots$ cm./sec.

ⓐ 12 ⓑ 3 ⓒ 6 ⓓ -3

3 The length of a rectangle is three times its width , if the rate of change of its width is 3 cm./sec. , then the rate of change of its diagonal length = ... cm./sec.

(a) 9 (b) $2\sqrt{10}$ (c) $3\sqrt{10}$ (d) $\sqrt{105}$

4 If the side length of an equilateral triangle increases by rate 2 cm./sec. , then the perimeter of the triangle increases by rate .... cm./sec.

(a) 2 (b) 8 (c) 4 (d) 6

5 If the height of an equilateral triangle increases at rate $\sqrt{3}$ cm./sec. , then the rate of change of its side length equals .... cm./sec.

(a) 4 (b) 2 (c) $\frac{4}{3}$ (d) $\frac{3}{4}$

6 The side length of a cube decreases by rate 3 cm./sec. , then the length of the diagonal of the cube is decreasing at a rate .... cm./sec.

(a) $3\sqrt{2}$ (b) $3\sqrt{3}$ (c) 6 (d) 9

7 A point moves on the curve $y = 2X + 1$, then the ratio between the rate of change of the X-coordinate with respect to time for a point to the rate of change of the y-coordinate with respect to time is equals ....

(a) 2 (b) -2 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

8 If the radius of a circle increases by rate $\frac{4}{\pi}$ cm./sec. , then the circumference of the circle increases at this instant by rate .... cm./sec.

(a) $\frac{4}{\pi}$ (b) $\frac{\pi}{4}$ (c) $\frac{1}{8}$ (d) 8

9 A metal disc , the rate of decreasing of its diameter by cooling is 0.5 cm./sec. , then the decreasing rate of its surface area = ... cm./sec. when its radius 14 cm. $\left(\pi = \frac{22}{7}\right)$

(a) 5.5 (b) 11 (c) 16 (d) 22

10 An empty container , its volume 45 cm.^{3} , water is poured in it at a rate 5 cm.^{3}/sec. , the container becomes full after .... sec.

(a) 9 (b) 135 (c) 45 (d) 50

11 The side length of a cube increases by rate 5 cm./sec. , then the volume of the cube increases by rate .... cm^3/sec. at the side length = 10 cm.

(a) 1500 (b) 150 (c) 45 (d) 50

12 A cubic water tank of side length 4 m. , water is poured in it by rate $\frac{1}{2}$ m^3/min. , then the rate of increasing of water height in the tank = ... m./min.

(a) $\frac{1}{96}$ (b) $\frac{1}{32}$ (c) $\frac{1}{24}$ (d) $\frac{1}{48}$

13 A cubic water tank of side length 5 m. , water is poured in it by rate $\frac{1}{4}$ m^3/min.
, then the rate of change of the upper surface area of the water = ... m^2/min.

(a) zero (b) 1 (c) $\frac{1}{8}$ (d) $\frac{1}{100}$

14 A circle touches the sides of a square internally , then the rate of change of the radius of the circle equals .... the rate of change of the side length of the square at any instant.

(a) double (b) half (c) quarter (d) four times

15 A square lamina expands regularly , the rate of increasing in surface area of the lamina 75 cm^2/sec. , then the rate of increasing of side length = ... cm./sec. when the side length is 5 cm.

(a) 2.5 (b) 5 (c) 7.5 (d) 15

16 If $y = x^2 - 3x$, then $\frac{dy}{dt} = \frac{dx}{dt}$ at $x =$ ....

(a) 1 (b) 2 (c) 3 (d) 4

17 If a particle moves on the curve : $y^2 + x^2 = 10$ such that $\frac{dy}{dt} = 4$, then $\frac{dx}{dt} =$ .... at the point $(\sqrt{5} , -\sqrt{5})$

(a) 2 (b) $2\sqrt{5}$ (c) 4 (d) $4\sqrt{5}$

18 A point moves on the curve : $x^2 + y^2 - 5x + 3y - 6 = 0$, if the rate of change of its x-coordinate respect to the time t at the point (1 , 2) equals 3 , then the rate of change of its y-coordinate respect to the time is ....

(a) $1\frac{2}{7}$ (b) $\frac{7}{12}$ (c) 3 (d) $\frac{-9}{7}$

19) A point (x , y) moves on the curve : $y = x^2 - \frac{1}{4}$, then the position of this point at the moment which the rate of change of its y-coordinate respect to the time equals three times the rate of change of x-coordinate respect to the time is ....

(a) $(6 , \frac{143}{4})$ (b) $(\frac{1}{2} , 0)$ (c) $(3 , \frac{35}{4})$ (d) $(\frac{3}{2} , 2)$

20) A point moves on the curve : $y = x - \frac{x}{x^2 + 1}$, the rate of chagne of its x-coordinate with respect to time equals 9 at $x = \sqrt{2}$, then the rate of change of its y-coordinate with respect to time at the same point = ...

(a) 4 (b) 6 (c) 8 (d) 10

21) If x is the radian measure of an angle , $x \in \left]0 , \frac{\pi}{2}\right[$, then the rate of increasing of tangent of an angle equals 8 times the rate of increasing of sine the same angle at $x =$ ....

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{5}$ (d) $\frac{\pi}{6}$

22) A stone fell in still water , then a circular waves is formed whose radius increases at rate of 3 cm./sec. , then the rate of increase of surface area of the wave after 4 sec. equals .... cm^2/sec.

(a) 8π (b) 72π (c) 12π (d) 24π

23) A square lamina , its side length changes by rate 0.2 cm./sec. , then rate of change of its surface area .... cm^2/sec. when its diagonal length = $8\sqrt{2}$ cm.

(a) $\frac{1}{25}$ (b) 3.2 (c) $\frac{4}{10}$ (d) 16

24) The radius of a circle increases by rate 2 cm./min. , and its area by rate 20π cm^2/m. , then length of its radius at this moment equals .... cm.

(a) $\frac{5}{2}$ (b) 5 (c) 10 (d) 20

25) A cube melt preserving its shape by rate 1 cm^3/sec. , then the rate of change of its edge length when its volume 8 cm^3 is .... cm./sec.

(a) $\frac{1}{12}$ (b) $\frac{1}{192}$ (c) $\frac{-1}{24}$ (d) $\frac{-1}{12}$

26 If the rate of change of area of circle equals the rate of change of its diameter , then r = ...

(a) $\frac{2}{\pi}$ (b) $\frac{1}{\pi}$ (c) $\frac{\pi}{2}$ (d) π

27 If the side length of an equilateral triangle = a , and it increases by rate k , then the rate of increasing of the area of the triangle equals ....

(a) $\frac{2}{\sqrt{3}}$ a k (b) $\sqrt{3}$ a k (c) $\frac{\sqrt{3}}{2}$ a k (d) $\frac{\sqrt{2}}{\sqrt{3}}$ a k

28 The surface area of a sphere increases by constant rate its value 6 cm^2/sec. at the moment which its radius was equal 30 cm. , then the rate of increasing of its volume = ... cm^3/sec.

(a) 180 (b) 40 (c) 90 (d) 90 π

29 If the rate of change of the volume of a sphere equals the rate of change of its radius , then r = ...

(a) $\frac{1}{2\sqrt{\pi}}$ (b) $\sqrt{2\pi}$ (c) $\frac{1}{\sqrt{2\pi}}$ (d) $\frac{1}{\sqrt{2}\,\pi}$

30 If (A) is the area of a circle of radius (r) , the radius changes at constant rate , then ....

(a) A is constant. (b) $\frac{dA}{dt}$ is constant. (c) $\frac{dA}{dt} \propto r$ (d) $\frac{dA}{dt} \propto r^2$

31 The number of sides of a regular polygon is m and its side length increases at constant rate a cm./sec. , then ....

(a) its perimeter increases at rate a cm./sec.
(b) its area increases at rate a cm^2/sec.
(c) its perimeter increases at rate m a cm./sec.
(d) its area increases at rate m a cm^2/sec.

32 The number of sides of a regular polygon is m and its side length increases at constant rate a cm./sec. , then the angle at any vertex of the polygon ....

(a) increases at constant rate a rad./sec.
(b) increases at constant rate m a rad./sec.
(c) increases at non constant rate and can not be determined.
(d) remains constant.

33 A circular segment , the radius of its circle is 10 cm. and measure of its central angle ($X°$) changes in the rate $3^{rad.}$ per minute , then the rate of increasing in the area of the circular segment at $X = 60°$ is .... cm^2/min.

(a) 125 (b) 75 (c) 150 (d) 300

34 If $X + y$ = constant , then ....

(a) each of X and y increases at the same rate.

(b) each of X and y decreases at the same rate.

(c) one of them increases and the other decreases at the same rate.

(d) nothing of the previous.

35 A spherical balloon whose radius r , its volume v , it is filled with gas but the gas leaks at constant rate , then ....

(a) $\frac{dr}{dt} > 0 , \frac{dv}{dt} > 0$ (b) $\frac{dr}{dt} > 0 , \frac{dv}{dt} < 0$

(c) $\frac{dr}{dt} < 0 , \frac{dv}{dt} > 0$ (d) $\frac{dr}{dt} < 0 , \frac{dv}{dt} < 0$

36 A 10 meter ladder is leaning against a vertical wall and its lower end on a horizontal ground , if the lower end slides 2 m./min. , then the rate of change of inclination angle with the horizontal at the moment the lower end at a distance 8 m. equals .... rad. min.

(a) 3 (b) -3 (c) $\frac{1}{3}$ (d) $-\frac{1}{3}$

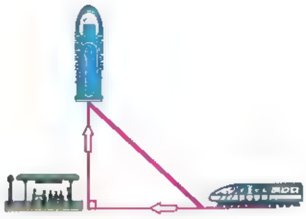
37 A 2 meter ladder leaning againest a vertical smooth wall with its top and its base on horizontal smooth ground , then the distant between the base to the wall in the moment which the rate of change of sliding of the two ends are equals is .... m.

(a) 2 (b) $2\sqrt{2}$ (c) $\sqrt{2}$ (d) $-\sqrt{2}$

38 A 4 meter ladder leaning a gainest a vertical wall with its top and its base on horizontal ground , if its base slides by velocity $\sqrt{3}$ m./sec. when the ladder inclined with the wall by angle with measure 60° , then the velocity of sliding of the other end on the wall at this moment = ... m./sec.

(a) $\frac{3}{2}$ (b) 3 (c) $\sqrt{3}$ (d) $\frac{-\sqrt{3}}{3}$

39 A train moves with velocity 30 km./h. in direction of west towards the station of the train and at the same moment another train moves from the same station with velocity 30 km./h. in direction of north , then the distance between the two trains is ....



(a) alwasy increasing.
(b) alwasy decreasing.
(c) increasing until reach a certain moment , then decreasing.
(d) decreasing until reach a certain moment , then increasing.

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40 A 1.6 meter man walks away from a lamppost at rate of 4 m./sec. , the height of the lamppost is 4.8 m. from the ground , then the rate of change of the length of the man's shadow equal .... m./sec.

(a) 1 (b) 2 (c) 3 (d) 4

---

41 A metal fine lamina in rectangular form , its length is $\frac{4}{5}$ of its diagonal , shrinking by cooling uniformly preserving its geometrical shape and same ratio between its dimensions , at a certain moment its diagonal shrinked at a rate 2.5 cm./min. and at the same moment its surface area decreases at a rate 60 cm^2/min. , then the surface area of the lamina at this moment = ... cm^2.

(a) 300 (b) 600 (c) 150 (d) 625

---

42 The radius of a cylindrical tank is 25 cm. and its height 120 cm. , oil is poured in it at a rate $\frac{5\,000}{L+40}$ m^3/sec. where L is the height of the oil at any moment then the rate of change of its height in the tank = ... cm./sec. when its half full.

(a) $\frac{4}{25}\pi$ (b) $\frac{1}{25\pi}$ (c) $\frac{2}{25\pi}$ (d) $\frac{8}{25\pi}$

---

43 A right circular cylinder expands preserving its shape , the rate of increasing of its radius is 0.5 cm./sec. and its height (h) increase at a rate of 0.25 cm./sec. then the rate of change of its volume when r = 3 cm. , h = 5 cm. equals .... cm^3/sec.

(a) $\frac{69}{4}\pi$ (b) $\frac{15}{4}\pi$ (c) $\frac{13}{2}\pi$ (d) $\frac{3}{4}\pi$

---

44 An isosceles triangle , the length of each of the two equal sides is 6 cm. , and the measure of their included angle equals (X) if X change at a rate of 3 per minute , then the rate of change of its surface area when $X = 30°$ is .... cm^2/min.

(a) $\frac{\sqrt{3}}{10}\pi$ (b) $\frac{\pi}{10}$ (c) $9\sqrt{3}$ (d) 9

The relation between the vertical displacement (s) and time (t) sec is given by $s = 49\,t - 4.9\,t^2$, then the maximum displacement can the body reach after .... sec.

(a) 9.8 (b) 10 (c) 5 (d) 50

The slope of the tangent to the curve $y = f(X)$ at a point $= \frac{1}{2}$ and the X-coordinate of this point decreases at a rate 3 unit./sec. , then the rate of change of its y-coordinate equals .... unit./sec.

(a) $-\frac{1}{6}$ (b) $\frac{-3}{2}$ (c) $\frac{1}{6}$ (d) $\frac{3}{2}$

A man observes a plane flies at 3 km. high horizontally above him and with speed 480 km./h. , then the rate of change of the distance between the man and the plane after 30 sec. later = ...

(a) $\frac{320}{3}$ km./h. (b) 384 km./sec. (c) 384 m./sec. (d) $\frac{320}{3}$ m./sec.

A right circular cone its height equals length of its base diameter if the rate of change of the radius of its base $= \frac{1}{\pi}$ cm./sec. , then rate of change of volume of the cone = ... $cm.^3$/sec. when the radius of its base = 5 cm.

(a) $50\,\pi$ (b) $\frac{250}{3}\pi$ (c) 150 (d) 50

If y is positive and is increasing then value of y at which rate of increasing in y^3 equals 4 times the rate of increasing in y is ....

(a) $\frac{2}{\sqrt{3}}$ (b) $\frac{\sqrt{3}}{2}$ (c) $2\sqrt{3}$ (d) $3\sqrt{2}$

In a right-angled triangle the length of two sides of the right angle are 12 cm. , 16 cm. if the length of the first side increases at rate 2 cm./sec. and the length of second side decreases at rate 1 cm./sec. , then the rate of change of its area after 2 seconds = ... $cm.^2$/sec.

(a) 6 (b) 3 (c) 12 (d) 18

A water pipe of length 5 m. and its ends are A and B it rests with its end A on a horizontal ground and one of its points D on a vertical fence of height 3 m. If the lower end A slide away from the fence at a rate $\frac{5}{4}$ m./min. , then the rate of descending the end B when it reaches to the fence top = ... m./min.

(a) $\frac{3}{5}$ (b) $\frac{3}{4}$ (c) $\frac{5}{4}$ (d) $\frac{4}{5}$

52. A car begins to move west at 12 pm. with speed 30 km./h. Another car begins at the same point at 2 pm. to travel North with speed 45 km./h. , then the rate of change of the distance between the two cars at 4 pm. is ………… km./h.

(a) 49 (b) 51 (c) 53 (d) 55

53. The base of metalic cuboid is a square its side length increases at a rate of 1 cm./min. and its height decreases at a rate of 2 cm./min. , then the volume of the cuboid stop increasing after ………… min. from the moment in which the length of the base side is 5 cm. and its height 20 cm.

(a) 5 (b) 3 (c) 12 (d) 6

54. If the point A moves in the positive direction of X-axis starting from the origin (O) with velocity $\frac{2}{3}$ length unit/min. and B (0 , 2) , C (0 , 4) , then the rate of change in the measure of the angle ($\angle$ BAC) when A reaches to (2 , 0) is ………… rad./min.

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{2}{15}$ (d) $\frac{1}{30}$

55. The length of rectangle 12 cm. and its width 5 cm. the length decreases at a rate 1 cm./min. while the width increases at a rate $\frac{1}{2}$ cm./min. , find the time the area stop increasing and find the area at this moment.

(a) 1 sec. , 60.5 cm^2 (b) $\frac{14}{3}$ sec. , 60.5 cm^2

(c) 1 sec. , 30 cm^2 (d) $\frac{14}{3}$ sec. , 30 cm^2

56. A metal regular quad. pyramid whose height equals its base side length. the volume of the pyramid increases at a rate of 1 cm^3/sec. when the rate of increasing of both the pyramid's height and its base side length equals 0.01 cm./sec. , then its base side length = ………… cm. at this moment.

(a) 4 (b) 8 (c) 10 (d) 12

57. 50 metre rop passes over a pulley which is 24 m. high , one of its ends tied to a heavy mass and the other end tied to a car moves on the ground with velocity 18 m./sec. then the rate of change of height of the mass at the moment when the car at a distance 32 m. from the projection of the pulley = ………… m./sec.

(a) 7.2 (b) 14.4 (c) 18.8 (d) 21.6

58 A right circular conical funnel , its height is 35 cm. , and the radius length of its base is 7 cm. fixed such that its axis is vertical while its apex is downward , oil is poured in it at a rate 5.4 cm^3/sec. while its leaking from a hole in the other end at a rate 3.2 cm^3/sec. , then the rate of increasing of the height of oil when the oil height is 25 cm. $\left(\pi = \frac{22}{7}\right)$ equal .... cm./sec.

(a) 0.007 (b) 0.014 (c) 0.028 (d) 0.056

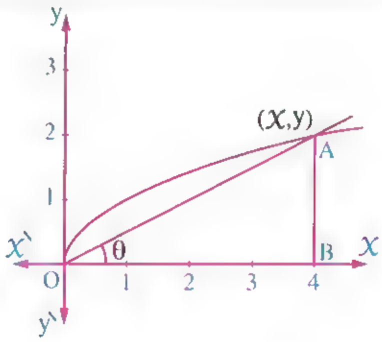
---

59 ABC is an isosceles triangle. AB = AC = 13 cm. , BC = 10 cm. If a straight line ℓ moves from point B parallel to $\overleftrightarrow{AC}$ in $\overrightarrow{BC}$ direction and intersects $\overline{AB}$, $\overline{BC}$ at D , E respectively where the rate of change in BE equals $\frac{1}{10}$ cm./min. , then the rate of change of area of Δ DEB when BE equals 1 cm. equals ....

(a) 0.12 (b) 0.24 (c) 0.48 (d) 0.96

---

60 **In the opposite figure :**



A particle (x , y) is moving along the curve of the function $y = \sqrt{x}$

When $x = 4$, the y-component of the position of the particle is increasing at rate 1 length unit/sec.

First : the rate of change of the x-component at this moment .... length unit/sec.

(a) 3 (b) 4 (c) 5 (d) 6

Second : the rate of change of the distance from the origin to the particle at the same moment = ... length unit/sec.

(a) $\frac{4\sqrt{5}}{2}$ (b) $\frac{5\sqrt{5}}{9}$ (c) $\frac{9\sqrt{5}}{5}$ (d) $9\sqrt{5}$

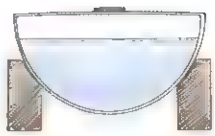
Third : What is the rate of change of the angle of inclination θ at the same moment = ... rad./sec.

(a) $-\frac{1}{5}$ (b) $\frac{-2}{5}$ (c) $\frac{-3}{5}$ (d) $\frac{-4}{5}$

Fourth : The rate of change in area of the triangle ABO at the same moment = ... square units/sec.

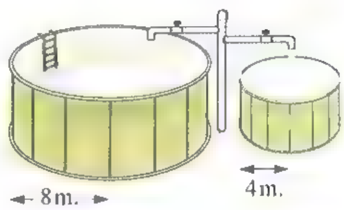
(a) 1 (b) 6 (c) 3 (d) 4

61 A hemi - sphere tank of water , radius 2 m. , water is poured into it , if the rate of change of the height of water is $\frac{1}{4}$ m./min. , then the rate of change of the area of the water surface in the tank after 2 min. from the beginning of the pouring water is .... m^2/min.



(a) $\frac{1}{4}\pi$ (b) $\frac{1}{2}\pi$ (c) $\frac{3}{4}\pi$ (d) $\frac{2}{3}\pi$

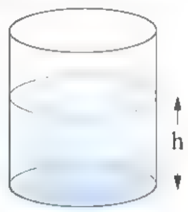
62 Two cylindrical tanks of water , the radius of the smaller is 4 m. and the radius of the bigger one is 8 m. they are being filled simultaneously at the same rate , the rate of increases of water level in the smaller tank is $\frac{1}{2}$ m./min. , then the rate of increase of water level in the bigger tank = ... m./min.

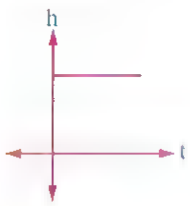


(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{8}$ (d) $\frac{1}{16}$

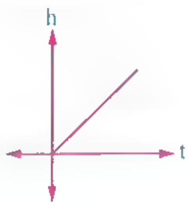
63 **In the opposite figure :**

Water is poured into a right cylindrical tank at a constant rate , which of the following figures represents the relation between the water level (h) in the tank and the time (t) ?

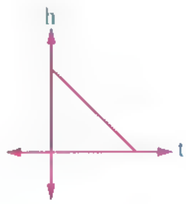




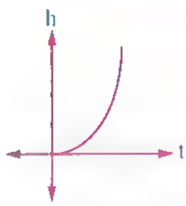
(a)



(b)



(c)



(d)

Questions on behavior of the function

1 Choose the correct answer from the given ones :

1 The 1st derivative of the decreasing functions is ....

(a) Positive (b) negative (c) Zero (d) otherwise

2 The function $f : f(X) = -X^2$ is increasing on the interval ....

(a) $]0 , \infty[$ (b) $]-\infty , 0[$ (c) $\mathbb{R} - \{0\}$ (d) $\mathbb{R}$

3. The function $f : f(X) = -|X| + 1$ is decreasing on the interval ……………

(a) $]0 , \infty[$ (b) $]-\infty , 0[$ (c) $]1 , \infty[$ (d) $]-\infty_1 , 0[$

4. The function $f : f(X) = X^3 + 4X + 2$ is increasing when $X \in$ ……………

(a) $]-4 , \infty[$ (b) $\mathbb{R}$ (c) $]-\infty , \frac{-4}{3}[$ (d) $]\frac{-4}{3} , \infty[$

5. The function $f : f(X) = \frac{X}{\ln X}$ is increasing on the interval ……………

(a) $]0 , \infty[$ (b) $]0 , e[$ (c) $]e , \infty[$ (d) otherwise.

6. If $f :]-2 , 4[\longrightarrow \mathbb{R}$, $f(X) = X^3 - 3X$, then the number of the critical points of the function f equals ……………

(a) 1 (b) 2 (c) 3 (d) 4

7. The function $f : f(X) = \ln(X^2 - 4)$, then the number of the critical points = ……………

(a) zero (b) 1 (c) 2 (d) 3

8. If $f : f(X) = a X^2 + b X + 2$ has a critical point $(1 , 4)$, then $a - b =$ ……………

(a) 2 (b) zero (c) -6 (d) -8

9. All the following functions are increasing in their domain except the function $f : f(X) =$ ……………

(a) $2X - 17$ (b) e^X (c) $(X - 3)^2$ (d) X^3

10. The function $f : f(X) = (X - 3)^2 + 2$ is increasing in the interval ……………

(a) $]2 , \infty[$ (b) $]3 , \infty[$ (c) $]-\infty , 3[$ (d) $]0 , \infty[$

11. The function $f : f(X) = X^3 - 12X$ is decreasing in the interval ……………

(a) $[-2 , 2]$ (b) $]-2 , 2[$ (c) $]0 , 12[$ (d) $]4 , \infty[$

12. If f is a continuous function on $\mathbb{R}$ and the function f has a critical point at $X = a$, so ……………

(a) $f'(a) = 0$ (b) $f'(a)$ is undefined.

(c) $f'(a^+) \neq f'(a^-)$ (d) All the previous.

13 If the function $f : f(x) = 2a x^2 + b x + 3$ has a local extrema at $(1, 2)$, then $a + b =$ ....

ⓐ -1 ⓑ $\frac{5}{2}$ ⓒ $-\frac{3}{2}$ ⓓ $\frac{3}{2}$

14 If f is an odd continuous function on $\mathbb{R}$ and the function has a local minimum value at $x = 2$, then the function has ....

ⓐ a local maximum value at $x = -2$
ⓑ a local minimum value at $x = -2$
ⓒ $f'(2) > f'(-2)$
ⓓ $f'(2) < f'(-2)$

15 If the function $f : f(x) = x + \frac{a}{x}$ has local maximum at $x = -2$, then $a =$ ....

ⓐ 4 ⓑ 2 ⓒ -2 ⓓ -4

16 Let f be a function defined by : $f(x) = \frac{x}{\ln x}$, then the local minimum value of f is ....

ⓐ e ⓑ $\frac{1}{e}$ ⓒ $\ln e$ ⓓ $-e$

17 If f is a function where $f(x) = \frac{x^4 + 1}{x^2}$, then the function is decreasing in ....

ⓐ $]-\infty, -1[$ only.
ⓑ $]0, 1[$ only.
ⓒ $]-1, 0[,]1, \infty[$
ⓓ $]-\infty, -1[,]0, 1[$

18 If f is function where $f(x) = (x^2 - 4)^{\frac{2}{3}}$, then the function is decreasing on ....

ⓐ $]-\infty, -2[,]0, 2[$
ⓑ $]-2, 0[,]2, \infty[$
ⓒ $]-\infty, -2[$ only.
ⓓ $]0, 2[$ only.

19 If $f(x) = x(a - \ln x)$ where a is constant and the curve of the function has a critical point at $x = e$, then $a =$ ....

ⓐ 1 ⓑ zero ⓒ e ⓓ 2

20 If $f(x) = x \ln x$, then the function $f : f(x)$ has a critical point at $x =$ ....

ⓐ zero ⓑ 1 ⓒ e ⓓ $\frac{1}{e}$

21 The function $f : f(x) = e^{x^2 - 2x}$ is increasing in the interval ....

ⓐ $]-\infty, 1[$ ⓑ $]-\infty, 2[$ ⓒ $]2, \infty[$ ⓓ $]1, \infty[$

22 If the function f is a polynomial function of fourth degree and its domain is $\mathbb{R}$, then the greatest number of the critical points of the function $f(X)$ is ....

(a) 1 (b) 2 (c) 3 (d) 4

23 If the curve of the function f, where f is polynomial, has a local maximum value at the point (a, b), then $f'(a) =$ ....

(a) b (b) zero (c) $\frac{-b}{a}$ (d) undefined

24 Which of the following functions has local minimum value ? $f : f(X) =$ ....

(a) $-X^2$ (b) $X^2 + 2$ (c) $-X^3$ (d) $X^3 + 2$

25 If the function $f : f(X) = X^2 + \frac{b}{X}$ has a critical point at $X = 2$, then $b =$ ....

(a) -16 (b) 16 (c) -4 (d) 2

26 If the function f is continuous on the interval $]a, b[$, if $c \in]a, b[$ where $f^1(c^+) \neq f^1(c^-)$, then $(c, f(c))$ is called ....

(a) Maximum. (b) Minimum. (c) critical. (d) in flection.

27 If $f : f(X) = \sqrt[3]{X - c}$ has critical point at $(c, 0)$, then $f'(c) =$ ....

(a) undefined. (b) zero. (c) $\frac{1}{3}$ (d) $\frac{1}{3\sqrt[3]{c^2}}$

28 If the curve of the function f has $f(5) = 7$, $f'(5) = \text{zero}$, $f''(5) = -4$, then the point $(5, 7)$ has ....

(a) local maximum. (b) local minimum.

(c) undefined. (d) inflection point.

29 If the function f is differentiable and $f'(X_1) = 0$, $f''(X_1) > 0$, then ....

(a) at $X = X_1$ the function has maximum point.

(b) at $X = X_1$ the function has minimum point.

(c) the function increases on its domain.

(d) the function decreases on its domain.

If $X \in]0, \frac{\pi}{2}[$, $f(X) = \sin\left(X + \frac{\pi}{6}\right) + \cos\left(X + \frac{\pi}{6}\right)$ has local maximum value at $X =$ ....

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{12}$ (d) $\frac{\pi}{8}$

If $f'(X) = (X - a)(X - b)$ where $a > b$, then all the following statements are true except ....

(a) at $X = a$, the function f has local minimum value.

(b) $f'(a) = f'(b)$

(c) at $X = b$, the function f has local maximum value.

(d) $f''(a) = f''(b)$

If $f''(X) = (X - 3)(X + 2)$, then curve of the function f is convex upward at the interval ....

(a) $]-\infty, -2[$ (b) $]-2, 3[$ (c) $]3, \infty[$ (d) $]-\infty, 3[$

The curve of the function f is convex upwards on a certain interval if .... on this interval.

(a) $f'(X) > 0$ (b) $f'(X) < 0$ (c) $f''(X) > 0$ (d) $f''(X) < 0$

34 If $f'(-1) = f'(3) =$ zero and $f''(X) >$ zero for all $X \in]-2, 2[$, then ....

(a) $f(-1)$ is a local maximum value. (b) $f(-1)$ is a local minimum value.

(c) $f(3)$ is a local maximum value. (d) $f(3)$ is a local minimum value.

35 The function $f : f(X) = X^4 - 4X^2$ has ....

(a) one local minimum value and two local maximum values.

(b) two different local minimum values and one local maximum value.

(c) two local minimum values and no local maximum values.

(d) two equal local minimum values and one local maximum value.

36 If $f'(X) = (X - 1)(X - 3)$, then the function f is decreasing in the interval ....

(a) $]1, 3[$ (b) $]2, \infty[$ (c) $]-\infty, 2[$ (d) $\mathbb{R} - [1, 3]$

37 If the function $f : f(X) = 2 - a X^3$ is decreasing on its domain , then ……………

(a) $a \le 0$ (b) $a > 0$ (c) $a \ge 0$ (d) $a < 0$

38 If g is increasing function on $\mathbb{R}$, h is decreasing function on $\mathbb{R}$ and $f(X) = 4\,g(X) - 3\,h(X)$, then function f is …………… on $\mathbb{R}$

(a) increasing (b) decreasing (c) constant (d) zero

39 f , g are two increasing functions on $\mathbb{R}$, which of the following is increasing on its domain ?

(a) $f + g$ (b) $f - g$ (c) $f \cdot g$ (d) $\frac{f}{g}$

40 If the function f : when $f''(X) > 0$ on an interval, then the curve of the function is ……………… on this interval

(a) increasing. (b) convex upwards.
(c) convex downwards. (d) decreasing.

41 The curve of the function f is convex downwards on $\mathbb{R}$ if $f(X)$ equals ………………

(a) $3 - X^2$ (b) $3 - X^3$ (c) $3 - X^4$ (d) $3 + X^4$

42 The curve of the function f where $f(X) = X^3 - 3X^2 + 2$ is convex upwards when $X \in$ ……………

(a) $]-\infty , 0[$ (b) $]-\infty , 1[$ (c) $]1 , 3[$ (d) $]1 , \infty[$

43 If $f(X) = (a - 2)X^2 + 3X - 5$, $X \in \mathbb{R}$, then the curve of the function f is concave downward when ………………

(a) $a > 2$ (b) $a < 2$ (c) $a = 2$ (d) $a = 0$

44 The curve of the function f where $f(X) = (X - 2)e^X$ is convex downwards on the interval …… ………

(a) $]-\infty , \infty[$ (b) $]-1 , 2[$ (c) $]0 , 2[$ (d) $]0 , \infty[$

45 The curve of the function f where $f(X) = \begin{cases} X^3 + 2 & , X > 0 \\ 3 - 2X^2 & , X < 0 \end{cases}$ is convex upwards when $X \in$ ………………

(a) $]0 , \infty[$ (b) $]-\infty , 0[$ (c) $\mathbb{R}$ (d) $\mathbb{R}^*$

46 If the curve of the function f lies above all tangents which drawn from the points of the curve , then the curve of the function is ....

(a) convex upwards.
(b) decreasing.
(c) increasing.
(d) convex downwards.

47 If the function f has $\lim\limits_{h \to 0} \frac{f'(X+h) - f'(X)}{h} > 0$ for all values of $X \in \mathbb{R}$, then the curve of the function f ....

(a) convex upwards.
(b) increasing.
(c) decreasing.
(d) convex downwards.

48 If The curve $y = (2X - c)^3 + 4$ has inflection point at $X = 5$, then $c =$ ....

(a) 2
(b) 4
(c) 5
(d) 10

49 The function $f : f(X) = X^3 - 3X - 1$ has an inflection point at ....

(a) $(0 , 1)$
(b) $(0 , -1)$
(c) $(1 , 0)$
(d) $(-1 , 0)$

50 If the function $f : f(X) = kX^3 + 9X^2$ has an inflection point at $X = -1$, then $k =$ ....

(a) -3
(b) 3
(c) -9
(d) 9

51 If the curve : $y = X^3 + aX^2 + bX$ has an inflection point at $(3 , -9)$, then $a + b =$ ....

(a) 6
(b) -9
(c) 15
(d) 24

52 The points which separate the convexity up and down parts are called ....

(a) critical.
(b) local Minimum.
(c) local Maximum.
(d) inflection.

53 If the function $f : f(X) = X^4$, then the point $(0 , 0)$ is ....

(a) local Minimum.
(b) local Maximum.
(c) inflection point.
(d) a , c together.

54 If the curve of the function has an inflection point, then the maximum number of the points which the curve intersects any straight line equals ....

(a) 1
(b) 2
(c) 3
(d) 4

55 If function f of fourth degree , then the maximum number of inflection points of it is ....

(a) 2 (b) 1 (c) 3 (d) 4

56 If $f(X) = \sqrt[3]{X - 2}$ and if $f(X)$ has an inflection point at $(2, 0)$, then $f''(2) =$ ....

(a) undefined (b) zero (c) 1 (d) $-\frac{2}{9}$

57 If the function $f : f(X) = g(X) - h(X)$ such that : $g'(2) = h'(2)$, $g''(2) > h''(2)$, then at $X = 2$ $f(X)$ has ....

(a) local Minimum value. (b) local Maximum value.

(c) inflection point. (d) Absolute Maximum value.

58 If the function f is defined on the interval $[a, b]$ and $f'(X) < 0$ on the same interval , then the absolute maximum value of $f(X)$ on this interval = ...

(a) $f(a)$ (b) $f(b)$ (c) a (d) b

59 For the function f such that $f'(X) = -2X + 6$, then all of the following statements are true except ....

(a) the curve of the function f convex upwards in the interval $]-\infty, \infty[$

(b) the function f has a local minimum value at $X = 3$

(c) the curve of the function f has no inflection points.

(d) $f(X)$ is decreasing in the interval $]3, \infty[$

60 The curve $y = X e^{X}$ has at ....

(a) $X = -1$ a minimum value. (b) $X = -1$ a maximum value.

(c) $X = 0$ a minimum value. (d) $X = 0$ a maximum value.

61 The curve $y = e^{X} - X e^{X}$ has a local ....

(a) maximum point at $X = 1$ (b) minimum point at $X = 1$

(c) maximum point at $X = 0$ (d) minimum point at $X = 0$

62 The absolute maximum value of the function f where $f(X) = 10 X e^{-X}$, $X \in [\text{zero}, 4]$ is ....

(a) $\frac{10}{e}$ (b) zero (c) 1 (d) e

63 The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = 2X + \cos X$, then ....

(a) has minimum value at $X = \pi$ (b) has maximum value at $X = 0$

(c) decreasing. (d) increasing function.

64 The function $f : f(X) = X^X$ has a stationary point at $X =$ ....

(a) e (b) $\frac{1}{e}$ (c) 1 (d) $\sqrt{e}$

If f is a continuous function in the interval $[a , b]$ and for every $X_1 , X_2 \in [a , b]$, then $f'(X_2) - f'(X_1) > 0$ when $X_2 > X_1$, then .... in the interval $]a , b[$

(a) the function f increases (b) the function f decreases

(c) the curve of f convex upward (d) the curve of f convex downward

If f , g are two differentiable twice functions on $\mathbb{R}$ and $f''(X) < g''(X)$ for all values of X and $h(X) = f(X) - g(X)$, then h ....

(a) increases on its domain. (b) decreases on its domain.

(c) has a curve convex downwards. (d) has a curve convex upwards.

If f is an even function and continuous on $\mathbb{R}$ and the function has an inflection point at $X = -a$, then the sign of $f''(a^-) \times f''(a^+)$ is the same as ....

(a) $f'(a^-) \times f'(a^+)$ (b) $f(a^-) \times f(a^+)$

(c) $f''(-a)$ (d) $f''(-a^-) \times f''(a^-)$

Let f be an increasing function on its domain , which of the following function not necessary to be increasing on its domain ?

(a) $y = \sqrt{f(X)}$ (b) $y = \sqrt[3]{f(X)}$ (c) $y = [f(X)]^2$ (d) $y = e^{f(X)}$

If $f : \mathbb{R}^* \longrightarrow \mathbb{R}$ where $f(X) = X + \frac{1}{X}$ and the function f has a local maximum value at $X = a$ and a local minimum value at $X = b$, then ....

(a) $f(a) > f(b)$ (b) $f(a) < f(b)$ (c) $a > b$ (d) $f'(a) < f'(b)$

If $y = aX + b$ is a tangent to the curve of the function f at any point on it , and $f(X) \leq aX + b$, then $f(X)$ .... for all values of X

(a) increases (b) decreases

(c) has a curve convex downwards (d) has a curve convex upwards

If $f : f(X) = \cos X$ where $X \in \left[\frac{-\pi}{2}, \frac{5\pi}{2}\right]$ has an absolute maximum value , then the number of times it reaches to the maximum values is ....

(a) 1 time (b) 2 times (c) 3 times (d) 4 times

If f is a differentiable function on $\mathbb{R}$ such that $f'(X) < 0$ for all values of $X \in \mathbb{R}$, then ....

(a) $f(X) > f(X-1)$
(b) $f(X) < f(X+1)$
(c) $f(X) < f(X-1)$
(d) $f(X) + f(X+1) = 1$

If f is a polynomial function and $f'(X) = a X^2 + b X + c$, then f is decreasing in its domain if ....

(a) $a > 0 , b^2 - 4ac \le 0$
(b) $a > 0 , b^2 - 4ac \ge 0$
(c) $a < 0 , b^2 - 4ac \le 0$
(d) $a < 0 , b^2 - 4ac \ge 0$

If f is a positive increasing functions , g is a positive decreasing functions and $z(X) = \frac{f(X)}{g(X)}$, then z is ....

(a) negative and increasing
(b) negative and decreasing
(c) positive and increasing
(d) positive and decreasing

If $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = X^3 + 3X^2 - 9X$ and a , b are the absolute minimum and maximum values of function f on the interval $[-2, 2]$, then $b - a =$ ....

(a) 17 (b) -17 (c) 27 (d) -27

If $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = \sqrt[3]{X^2}\,(3X - 7)$ is increasing for every $X \in]-\infty, a[$, $X \in]b, \infty[$, then $7a + 15b =$ ....

(a) 7 (b) 14 (c) 21 (d) 22

If function $f : f(X) = \frac{X^2 + mX + 4}{X - 1}$, where m is a constant is an increasing function , then $m \in$ ....

(a) $]-\infty, -5]$ (b) $[-5, \infty[$ (c) $]-5, 0]$ (d) $]-5, 0[$

If $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(X) = X^3 + aX^2 + 12X + 1$, then the set of values of a which makes the function f has no critical points is ....

(a) $-9 < a < 9$ (b) $-6 \le a \le 6$ (c) $-3 < a < 3$ (d) $0 < a < 9$

1 If f , g , h are differentiable functions respect X , by using the given variables in the following table :

Choose the correct answer from the given ones

x	1	2
$f(x)$	-1	4
$g(x)$	2	1
$\grave{f}(x)$	1	5
$\grave{g}(x)$	2	-3

1. $h(x) = 3f(x) - 2g(x)$, then $\grave{h}(1) =$ ....

(a) -5
(b) 2
(c) 1
(d) -1

2. $h(x) = f(x)[5 + g(x)]$, then $\grave{h}(2) =$ ....

(a) 8 (b) Zero (c) 18 (d) -18

3. $h(x) = f(x) \div [g(x) + 2]$, then $\grave{h}(1) =$ ....

(a) $\frac{-1}{4}$ (b) $\frac{1}{4}$ (c) $\frac{3}{8}$ (d) $\frac{-3}{8}$

4. $h(x) = f[g(x)]$, then $\grave{h}(1) =$ ....

(a) 2 (b) Zero (c) 10 (d) -6

5. $h(x) = g[3x - f(x)]$, then $\grave{h}(2) =$ ....

(a) 6 (b) -8 (c) -1 (d) 8

6. $h(x) = [x^3 + g(x)]^{-2}$, then $\grave{h}(1) =$ ....

(a) $\frac{-2}{27}$ (b) $\frac{-10}{27}$ (c) Zero (d) -30

2 **By using the opposite table where f is a polynomial :**

Choose the correct answer from the given ones

x	-1	1	2	3	4
$\grave{f}(x)$	24	0	-3	0	9
$\grave{\grave{f}}(x)$	-18	-6	0	6	12

1. f has local maximum value of $x =$ ....

(a) -1 (b) 1 (c) 2 (d) 3

2. f has local minimum value at $x =$ ....

(a) 1 (b) 2 (c) 3 (d) 4

3 f is decreasing when $x \in$ ....

(a) $]-\infty , 1[$ (b) $]1 , 3[$ (c) $]3 , \infty[$ (d) $\mathbb{R}$

4 The curve of $f(x)$ is convex upwards when $x \in$ ....

(a) $]-\infty , -1[$ (b) $]-\infty , 2[$ (c) $]3 , \infty[$ (d) $]1 , \infty[$

5 The curve $f(x)$ has inflection point at $x =$ ....

(a) -1 (b) 1 (c) 2 (d) 3

6 If $g(x) = f(x) + 5$, then g has local maximum value when $x =$ ....

(a) -1 (b) 1 (c) 2 (d) 3

Questions on application of maxima and minima

Choose the correct answer from the given ones :

1 The greatest value of the expression $8x - x^2$ where $x \in \mathbb{R}$ is ....

(a) 8 (b) 12 (c) 16 (d) 20

2 The function f has local maximum value if $f(x)$ equals ....

(a) $x^2 - 3$ (b) $x^3 + 1$ (c) $x^3 + 3x$ (d) $x^4 - 3x^2$

3 The curve : $y = x^2 - 6x + 7$ has local minimum value at $x =$ ....

(a) 2 (b) 3 (c) 4 (d) zero

4 If $x \in \mathbb{R}^+$ and $x + \frac{1}{x} \geq k$, then $k =$ ....

(a) 2 (b) 4 (c) 3 (d) 1

5 If $x < 0$ then the maximum value of $x + \frac{1}{x}$ equals ....

(a) -1 (b) -2 (c) -3 (d) otherwise.

6 The maximum value of $(\sin x + \sqrt{3} \cos x)$ at $x =$ ....

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) zero

7 The minimum value of the function $x \ln x$ equals ....

(a) e (b) $\frac{1}{e}$ (c) $\frac{-1}{e}$ (d) $-e$

8 The maximum value of the function $f : f(x) = 3 - \sin x$ is .... where $x \in \mathbb{R}$

(a) 1 (b) 2 (c) 3 (d) 4

9 If $x \in [0, \pi]$, then the function $f : f(x) = \sin x + \cos x$ has absolute Minimum value at $x =$ ....

(a) zero (b) 1 (c) π (d) $\frac{\pi}{2}$

10 If $x + y = 13$ where x and y are positive numbers , then the value of x which makes the expression $(2x + 3y + xy)$ has a maximum value is ....

(a) 2 (b) 4 (c) 6 (d) 8

11 If the sum of two numbers is 16 and the sum of their squares is as maximum as possible , then the two numbers are ....

(a) 8 , 8 (b) 7 , 9 (c) 6 , 10 (d) 5 , 11

12 The sum of two positive integers is 5 and the sum of cube of the smaller and twice the square of the other is the smallest value , then the two numbers are ....

(a) 4 , 1 (b) 2 , 3 (c) 5 , zero (d) $3\frac{1}{2}$, $1\frac{1}{2}$

13 A rectangle its area 50 m^2 , then its perimeter be minimum value when its dimensions are .... meters.

(a) 10 , 5 (b) $5\sqrt{2}$, $5\sqrt{2}$ (c) 15 , $\frac{10}{3}$ (d) otherwise

14 A rectangle with perimeter 14 cm. , then the maximum area of the rectangle = ... cm^2

(a) 10 (b) 12 (c) 12.25 (d) 49

15 The rectangles which inscribed in a circle its radius "r" , then the dimensions of the rectangle when its area is maximum are ....

(a) $\sqrt{2}\,r$, $\sqrt{2}\,r$ (b) r , $\sqrt{2}\,r$ (c) 2 r , $\sqrt{2}\,r$ (d) 2 r , 2 r

16 If the perimeter of the circular sector is P , then its area has maximum value at r = ...

(a) $\frac{P}{2}$ (b) $\frac{2}{\sqrt{P}}$ (c) $\sqrt{P}$ (d) $\frac{P}{4}$

17 A circular sector-piece of metal whose area 16 cm^2 , then the radius of the sector circle which makes its primeter as minimum as possible = ... cm.

(a) 2 (b) 4 (c) 6 (d) 8

18 A rectangular piece of land. bounded by a river from one side and of area 2048 m^2 , such that the length of a fence surrounding its three remaining sides is minimum , then the dimensions are .... , ... m.

(a) 16 , 128 (b) 32 , 64 (c) 8 , 256 (d) 4 , 512

19 The points on the curve $x^2 - y^2 = 8$ such that their distance from the point (0 , 2) is minimum , then the points are ....

(a) (3 , 1) , (– 3 , 1) (b) (3 , – 1) , (– 3 , – 1)
(c) (– 3 , – 1) , (– 3 , 1) (d) (3 , 1) , (3 , – 1)

20 The shortes distance between the straight line $x - 2y + 10 = 0$ and the curve $y^2 = 4x$ equal .... length unit.

(a) $6\sqrt{5}$ (b) $\frac{3\sqrt{5}}{5}$ (c) $\frac{4\sqrt{5}}{5}$ (d) $\frac{6\sqrt{5}}{5}$

21 The perimeter of an isosceles triangle is equal to 30 cm. , then the side lengths of the triangle such that its surface area is maximum.

(a) 9 , 9 , 12 (b) 10 , 10 , 10 (c) 8 , 8 , 14 (d) 4 , 13 , 13

22 If the hypotenuse length of a right-angled triangle equals 10 cm. , then the lengths of the two legs of the right angle when the area of the triangle is as maximum as possible are .... cm.

(a) 6 , 8 (b) $4\sqrt{5}$, $2\sqrt{5}$ (c) $5\sqrt{2}$, $5\sqrt{2}$ (d) $2\sqrt{10}$, $4\sqrt{15}$

23 The area of the largest rectangle that can be inscribed in a circle of radius 4 cm. equals .... cm^2.

(a) 28 (b) 32 (c) 48 (d) 64

24. A thin metalic lamina in the form of a square , then length of whose side is 20 cm. A box without a lid in the form of a rectangular parallelepiped is to be made of this lamina by cutting equal squares of its corners , and turning up the sides , then the length of the side of the removed square when the volume of the box is to be maximum equals .... cm.

(a) $2\frac{1}{2}$ (b) 3 (c) $3\frac{1}{3}$ (d) $4\frac{1}{2}$

25. A box in the form of a rectangular parallelepiped with a square base. , then its maximum volume = ... cm^3 given that its total surface area = 384 cm^2

(a) 1000 (b) 128 (c) 256 (d) 512

26. A rectangular parallelepiped with a square base , the sum of all its edges equals 240 cm. , then the dimensions of the rectangular parallelepiped when its volume is to be maximum. are .... cm.

(a) 20 , 20 , 20 (b) 15 , 15 , 30 (c) 16 , 16 , 28 (d) 18 , 18 , 24

27. A rectangular parallelepiped the length of its base is twice its width. If the sum of its three dimensions 180 cm. , then these dimensions that make the volume of the rectangular parallelepiped is to be maximum. are .... cm.

(a) 50 , 60 , 70 (b) 40 , 60 , 80 (c) 35 , 65 , 80 (d) 30 , 70 , 80

28. A rectangular parallelepiped has a volume of 576 cm^3 and the ratio between the two lengths of its base is 2 : 1 , then the dimensions of the parallelepiped that makes its total surface area minimum are .... cm.

(a) 4 , 8 , 18 (b) 8 , 16 , $\frac{9}{2}$ (c) 6 , 8 , 12 (d) 4 , $6\sqrt{2}$, $12\sqrt{2}$

29. The selling price of each unit of a product is $(100 - 0.02\,X)$ pounds. Where X is the number of units which are produced. If the cost price of X units is $40\,X + 1500$ pounds , then the number of units to be produced to satisfy the maximum profit. = ... units.

(a) 120 (b) 1 400 (c) 1 500 (d) 1 800

30. A factory is producing electric appliances profits L.E. 30 in every appliance if it produces 50 appliances monthly. When the production increased than this number , then the profit in the appliance decreases by 50 piasters for every extra appliance produced , then the number of appliances produced monthly if the profit is to be maximum = ... appliance

(a) 50 (b) 55 (c) 60 (d) 65

31 The current intensity I (Ampere) in a circuit for the alternating current at any moment t (second) is given by the relation $I = 2 \cos t + 2 \sin t.$, then is the maximum value of the current in this circuit = ... Ampere

(a) 2 (b) 3 (c) $2\sqrt{2}$ (d) $2\sqrt{3}$

32 If the sum of the surface area of a sphere and the surface area of a right circular cylinder , whose radius is equal to the radius of the sphere is $250\,\pi$ cm^2 , then the radius of the sphere if the sum of their volumes is maximum equals .... cm.

(a) 2 (b) 3 (c) 4 (d) 5

33 The height of a circular cylinder has maximum volume inside a sphere its radius is "r" equals ....

(a) $\frac{2\,r}{\sqrt{5}}$ (b) $\frac{2\,r}{\sqrt{3}}$ (c) $2\,r$ (d) $2\sqrt{3}\,r$

34 Height of a cone which has maximum volume inscribed inside a sphere with radius "r" equals ....

(a) $\frac{4\,r}{5}$ (b) $\frac{4\,r}{3}$ (c) $\frac{2\,r}{3}$ (d) $\frac{8\,r}{3}$

35 The height of a right cone , which we can put it inside sphere with radius length 9 cm. , such that its volume is maximum equals .... cm.

(a) 7 (b) 12 (c) 8 (d) 10

36 In a perpendicular coordinate plane , $\overleftrightarrow{AB}$ is drawn to pass through the point C (3 , 2) and intersect the positive parts of the coordinate axes at point A and point B , then the minimum area of triangle AOB where O is the origin point (0 , 0) equals .... square units.

(a) 10 (b) 11 (c) 12 (d) 13

37 If (a , b) is a fixed point in lattice plane where $a > 0$, $b > 0$, a straight line is draw passes through (a , b) and cuts the positive part of each X-axis and y-axis at X , y respectively , if "0" is the origin point , then the smallest area of the triangle OXY , equals .... square unit.

(a) ab (b) 2 ab (c) 4 ab (d) $\frac{1}{2}$ ab

38 Range of the function $f : f(X) = \sin X + \cos X$ equals ....

(a) $[-\sqrt{2}, \sqrt{2}]$ (b) $[-1, 1]$ (c) $[0, 1]$ (d) $[0, \sqrt{2}]$

39 If ℓ , m are the two roots of the function $x^2 - (k-1)x + k + 2 = 0$, then the value of k that makes the expression $\ell^2 + m^2$ as small as possible equals ....

(a) 2 (b) 3 (c) 1.5 (d) – 1.5

40 The minimum distance between the origin point and the curve $x = 2\sin t - \sin 2t$, $y = 2\cos t - \cos 2t$, equals .... length unit.

(a) 1 (b) 2 (c) 3 (d) 4

41 ABCD is a square whose side length 10 cm. and $M \in \overline{BC}$ where $BM = x$ cm. and $N \in \overline{CD}$ where $CN = \frac{3}{2}x$, then the value of x which makes the area of Δ AMN as minimum as possible equals ....

(a) $\frac{7}{3}$ (b) $\frac{10}{3}$ (c) $\frac{11}{3}$ (d) $\frac{13}{3}$

42 The maximum area of the trapezium ABCD in which $\overline{AB} \parallel \overline{CD}$, AB = AD = BC = 6 cm. equals .... cm^2.

(a) 27 (b) $27\sqrt{3}$ (c) $27\sqrt{6}$ (d) 81

43 A rectangle has one of its sides on the x-axis , the upper two vertices of the rectangle lie on the curve $y = 4 - x^2$, then the dimensions of the rectangle such that its area is maximum are ....

(a) $\frac{4\sqrt{3}}{3}$, $\frac{8}{3}$ (b) $\frac{2\sqrt{7}}{3}$, $\frac{8}{3}$ (c) $\frac{4\sqrt{3}}{3}$, $\frac{2}{3}$ (d) $\frac{4\sqrt{3}}{3}$, $\frac{2\sqrt{7}}{3}$

44 ABCD is a trapezoid where $\overline{AD} \parallel \overline{BC}$, $\overline{AB} \perp \overline{BC}$, AB = 20 cm. , AD = 10 cm. , BC = 30 cm. , then the dimensions of the rectangle with the largest area which can be drawn inside the trapezoid are ....

(a) 12 , 12 (b) 12 , 15 (c) 15 , 15 (d) 15 , 16

45 If a trapezoid is drawn in a semi-circle such that is base is the diameter of the semi-circle , then the measure of the base angle of the trapezoid such that its area is as maximum as possible = ...°

(a) 30 (b) 40 (c) 50 (d) 60

46 A wire of length 68 cm. is cut into two pieces. The first is bent to form a rectangle of width x cm. and of length twice its width , while the second is bent to form a square. , then the value of x such that the sum of the rectangle and square areas is minimum = ... cm.

(a) 5 (b) 6 (c) 7 (d) 8

47 If the point (a , b) $\in$ the curve of the function $y = -x^2 + 2x + 3$, then the greatest value of the expression a + b = ...

(a) $\frac{21}{4}$ (b) $\frac{21}{2}$ (c) $\frac{23}{4}$ (d) $\frac{23}{2}$

48 Let A (0 , 9) , B (0 , 4) , C $\in \overrightarrow{OX}$, then the coordinates of C which make the measure of $\angle$ ACB is as great as possible are ....

(a) (3 , 0) (b) (4 , 0) (c) (5 , 0) (d) (6 , 0)

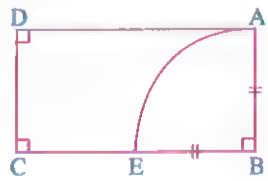
49 The area of the largest isosceles triangle that can be inscribed in a circle of radius 15 cm. approximately equals .... cm^2

(a) 248.04 (b) 284.28 (c) 292.28 (d) 312.24

50 A rectangle is drawn such that two adjacent vertices of the rectangle lie on the curve $y = x^2 - 12$ and the other two vertices lie on the curve $y = 12 - x^2$, then the maximum area of this rectangle equals .... square units.

(a) 32 (b) 48 (c) 64 (d) 96

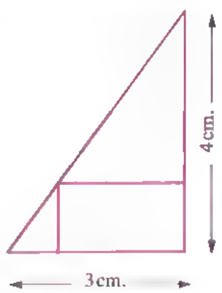
51 **In the opposite figure :**



ABCD is a rectangle of perimeter = 28 cm.
a circle of centre B is drawn and passes through the two points A and E , then the length of $\overline{AB}$ that makes the area of the shaded part is as maximum as possible equals ....

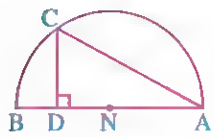
(a) $\frac{14}{4+\pi}$ (b) $\frac{14}{2+\pi}$ (c) $\frac{28}{4+\pi}$ (d) $\frac{28}{2+\pi}$

52 The dimensions of the rectangle of largest area that can be inscribed in the right-angled triangle shown in the figure are .... cm.



(a) 1 , 1.5 (b) 1.5 , 2
(c) 2 , 2.5 (d) 2.5 , 3

53 **In the opposite figure :**

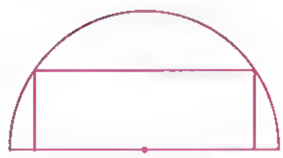


$\overline{AB}$ is a diameter in a semi-circle , AB = 16 cm.

, then the greatest area of ΔADC = ... cm^2

(a) $12\sqrt{3}$ (b) 24 (c) $24\sqrt{3}$ (d) 48

54 **In the opposite figure :**

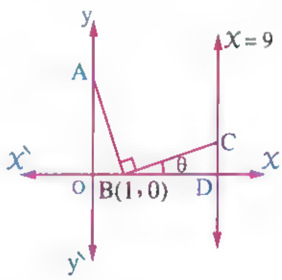


A rectangle is drawn inside the surface of semi-circle with radius 4 cm.

, then the dimensions of this rectangle when its area is maximum are .... cm.

(a) 4 , 4 (b) $4\sqrt{2}$, $2\sqrt{2}$ (c) $4\sqrt{2}$, $4\sqrt{2}$ (d) $2\sqrt{3}$, $2\sqrt{2}$

55 **In the opposite figure :**

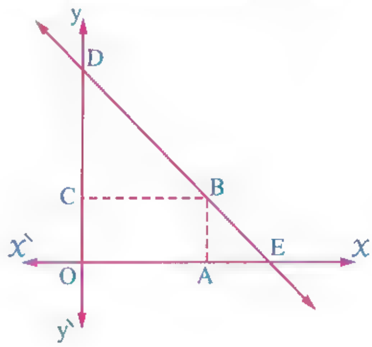


The value of tan θ which makes (AB + BC) as small as possible is ....

(a) 2 (b) 1

(c) $\frac{1}{3}$ (d) $\frac{1}{2}$

56 **In the opposite figure :**



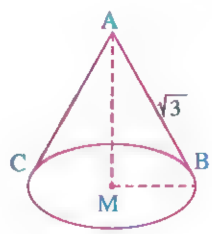
If the equation of the straight line $\overleftrightarrow{DE}$ is $y = 3 - x$

, then the greatest area of the rectangle ABCO = ... square unit.

(a) $\frac{9}{8}$ (b) $\frac{9}{2}$

(c) $\frac{9}{4}$ (d) $\frac{3}{2}$

57 **In the opposite figure :**

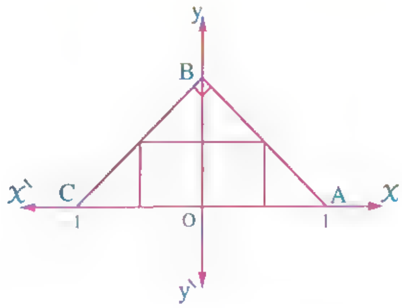


The length of $\overline{AM}$ that makes the volume of cone is as maximum as possible equals ....

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) 1 (d) $1\frac{1}{2}$

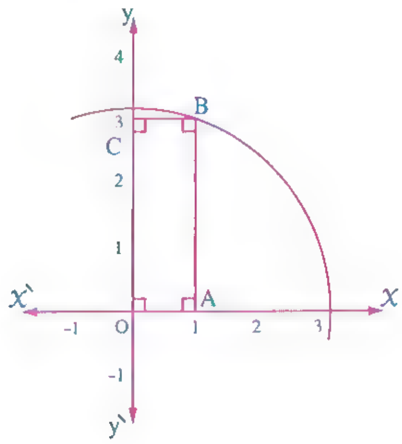
58 The opposite figure represents a rectangle inscribed in an isosceles right-angled triangle AC = 2 length units then the greatest area of the rectangle equals .... square unit(s)



(a) $\frac{1}{2}$
(b) 1
(c) $1\frac{1}{2}$
(d) 2

59 **In the opposite figure :**

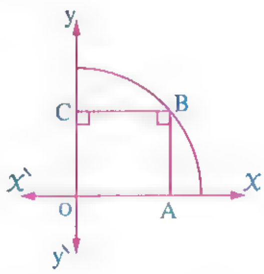
A quarter of a circle of centre (O)
, B $\in$ the curve of the circle $X^2 + y^2 = 9$
, then the greatest area of the rectangle ABCO = ... square unit.



(a) 4.5
(b) 9
(c) $\frac{3\sqrt{2}}{2}$
(d) 27

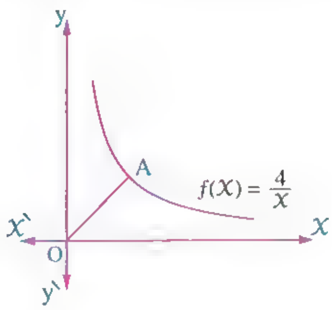
60 **In the opposite figure :**

The part is in the first quadrant from the circle $X^2 + y^2 = r^2$, then the greatest perimeter of the rectangle ABCO equals .... length unit.



(a) r
(b) $\sqrt{2}\,r$
(c) 2 r
(d) $2\sqrt{2}\,r$

61 The least length of the line segment $\overline{OA}$ = ... length unit.



(a) $\sqrt{2}$
(b) 2
(c) $2\sqrt{2}$
(d) 4

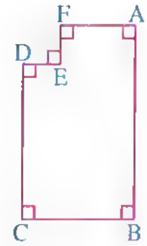
62 **In the opposite figure :**

If CD = 2 AF and FE = ED

and the perimeter of the figure ABCDEF = 40 cm.

, then the maximum area of the figure ABCDEF

equals .... cm^2.

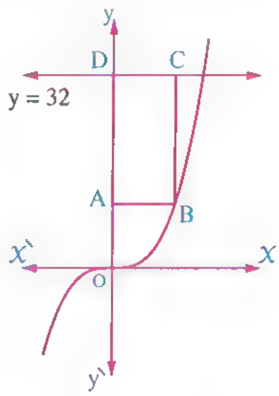


(a) 90 (b) 95 (c) 100 (d) 105

63 **In the opposite figure :**

$f(x) = x^3$, then the maximum area of

the rectangle ABCD equals .... square units.



(a) 2 (b) 8

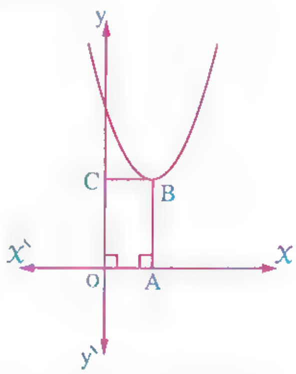
(c) 48 (d) 24

64 **In the opposite figure :**

B ∈ the curve of the function $y = x^2 - 3x + 5$

, then the least perimeter of the rectangle

OABC equals .... length unit.



(a) 3 (b) 8

(c) 12 (d) 16

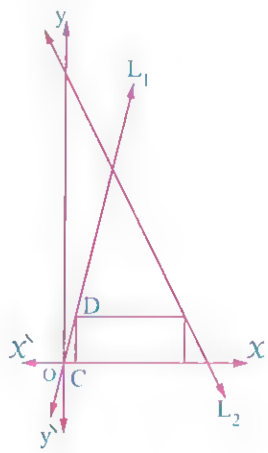
65 **In the opposite figure :**

Two straight lines

$L_1 : y = 4x$, $L_2 : y = 18 - 2x$

, then the greatest area of

the shaded rectangle = ... square unit.

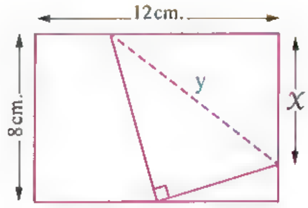


(a) 25 (b) 27

(c) 30 (d) 32

66 **In the opposite figure :**

The top right corner of a rectangular piece of paper whose dimension 8 cm. , 12 cm. is folded to align the lower edge as shown in the figure , then the value of X which makes y as minimum as possible equals .... cm.



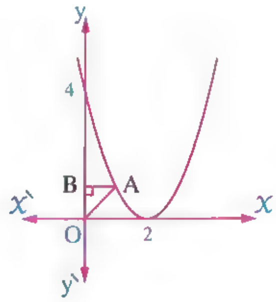
(a) 5 (b) $5\frac{1}{2}$ (c) 6 (d) $6\frac{1}{2}$

---

67 **In the opposite figure :**

If the point A ∈ the curve of the quadratic function $y = (X-2)^2$, $\overline{AB}$ // X-axis.

, then the coordinates of A which makes the area of triangle OAB as large as possible is ....

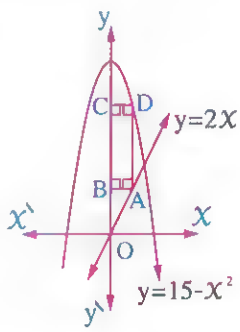


(a) $\left(1\frac{2}{3} , \frac{1}{9}\right)$ (b) $\left(\frac{2}{3} , \frac{16}{9}\right)$ (c) $\left(1\frac{1}{3} , \frac{4}{9}\right)$ (d) $(1 , 1)$

---

68 **In the opposite figure :**

The greatest area of the rectangle ABCD equals .... square units.



(a) $\frac{400}{27}$ (b) $\frac{401}{27}$

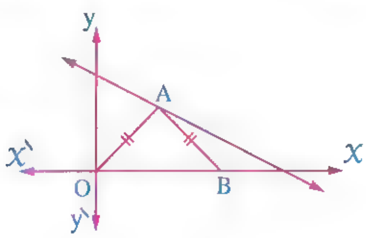
(c) $\frac{403}{27}$ (d) $\frac{404}{27}$

---

69 **In the opposite figure :**

A ∈ the straight line $X + 3y = 6$

, the greatest area of the isosceles triangle ABO = ... square units.



(a) 2 (b) 3

(c) 4 (d) 6

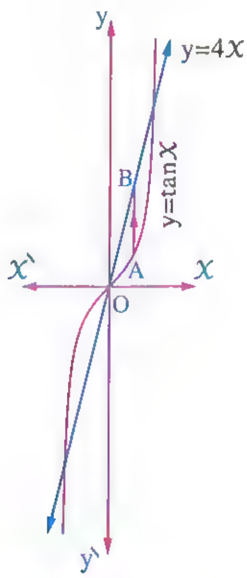
70 If $\overleftrightarrow{AB}$ // the y-axis

, then the greatest value

of length of $\overline{AB}$ = ... cm.

(a) $\frac{4\pi - 3\sqrt{3}}{3}$ (b) $\frac{4\pi + 3\sqrt{3}}{3}$

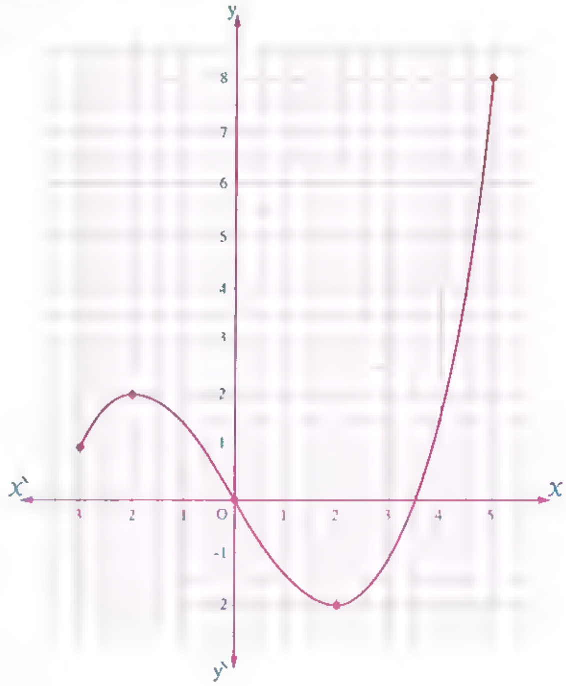
(c) $\frac{2\pi - 3\sqrt{6}}{3}$ (d) $\frac{2\pi + 3\sqrt{3}}{3}$



Seventh Questions on behavior of the curves represented graphically

Choose the correct answer from the given ones :

1 **The opposite figure represents the curve of the function f , then**



First : the function has local minimum value at X = ...

(a) – 3 (b) zero

(c) 2 (d) – 2

Second : The Absolute maximum values equals ....

(a) 5 (b) 8

(c) – 2 (d) 2

Third : The curve of the function is convex upwards at $X \in$ ....

(a)]– 3 , 0[(b)]– 2 , 2[

(c)]0 , 5[(d)]– 3 , 5[

Fourth : The curve of the function f has an inflection point is ....

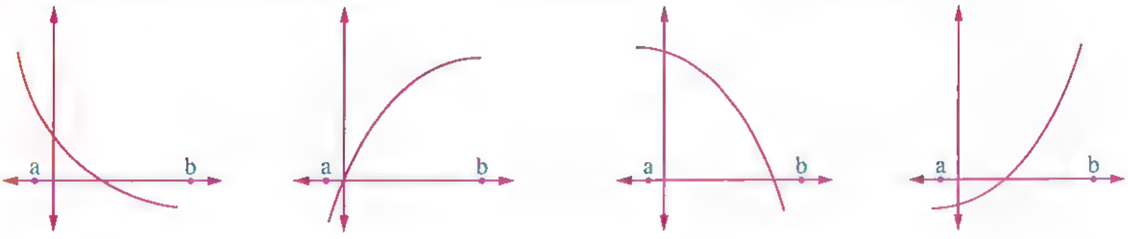
(a) (– 2 , 2) (b) (0 , 0) (c) (2 , – 2) (d) $(3\frac{1}{2}, 0)$

Fifth : The function is decreasing on the interval ....

(a)]– 3 , 0[(b)]– 2 , 2[(c)]0 , 5[(d)]– 3 , 5[

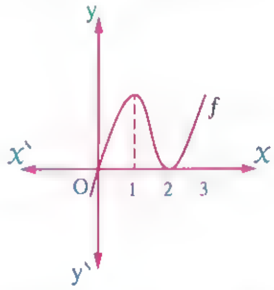
2 If $f'(X) < 0$, $f''(X) > 0$, $\forall X \in [a , b]$

, then which of the following curves represent the curve of the function f in $[a , b]$



(a) (b) (c) (d)

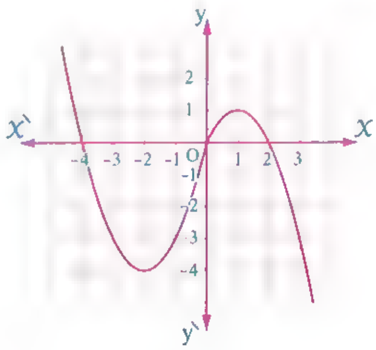
3 The opposite figure represents the curve of the function f , then f' is negative in the interval ....



(a) $]1 , 2[$ (b) $]0 , 3[$

(c) $\mathbb{R} - [1 , 2]$ (d) $]0 , 2[$

4 The opposite figure represents the curve of the function $y = f(X)$, then all the following statements are true except ....

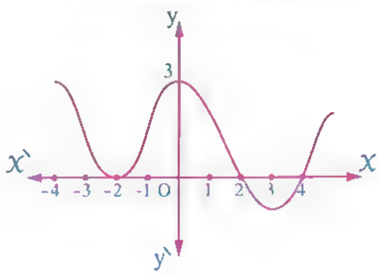


(a) $f'(-2) = \text{zero}$ (b) $f'(-1) > 0$

(c) $f(3) > f(5)$ (d) $f'(-4) < f'(-5)$

5 The opposite figure represents the curve of the function $y = f(X)$

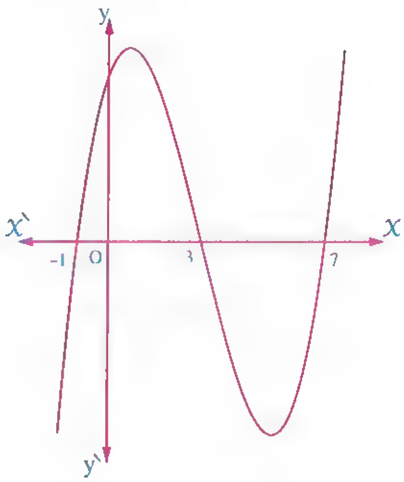
All the following statements are true except ....



(a) $f'(-3) + f'(3) < 0$ (b) $f'(-1) > f'(1)$

(c) $f'(-2) + f'(0) = 0$ (d) $f''(-2) < f''(0)$

6. If the curve of the function f has two inflection points , then the opposite figure represents the curve of y = ...

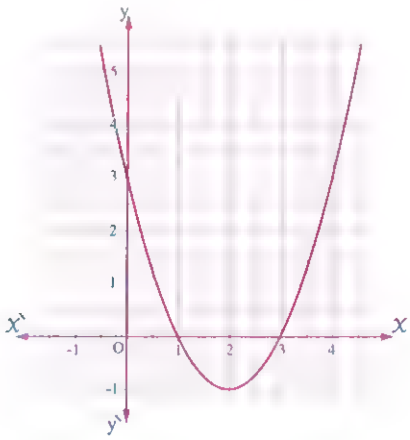


ⓐ $f(X)$ ⓑ $f'(X)$

ⓒ $f''(X)$ ⓓ otherwise

---

7. **By using the opposite figure which represents the curve of the function $f'(X)$, choose the correct answer from the given ones :**



First : $f(X)$ has local maximum value at X = ...

ⓐ zero ⓑ 1

ⓒ 2 ⓓ 3

Second : $f(X)$ has local minimum value at X = ...

ⓐ zero ⓑ 1 ⓒ 2 ⓓ 3

Third : $f(X)$ has an inflection point at X = ...

ⓐ zero ⓑ 1 ⓒ 2 ⓓ 3

Fourth : The curve of $f(X)$ is convex upwards at $X \in$ ....

ⓐ $]-\infty , 2[$ ⓑ $]2 , \infty[$ ⓒ $]1 , 3[$ ⓓ $\mathbb{R}$

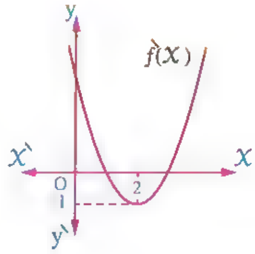
Fifth : $f(X)$ is decreasing when $X \in$ ....

ⓐ $]-\infty , 2[$ ⓑ $]2 , \infty[$ ⓒ $]1 , 3[$ ⓓ $\mathbb{R}$

Sixth : The solution set of the inequality $f'(X) \geq 0$ is ....

ⓐ $]-\infty , 2]$ ⓑ $[2 , \infty[$ ⓒ $[1 , 3]$ ⓓ $\mathbb{R}$

8 The opposite figure represents the curve of the derivative of the function f : $f(x) = x^3 + 2\,a\,x^2 + b\,x + 1$, then $f(1) = \cdots\cdots\cdots\cdots$



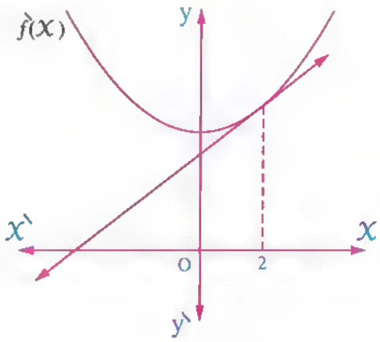
ⓐ -1 ⓑ -3

ⓒ 7 ⓓ 11

---

9 **In the opposite figure :**

If the straight line L : $4\,y = 3\,x + 12$ touches the curve $\grave{f}(x)$ at $x = 2$, then :



First : $\grave{f}(2) = \cdots\cdots\cdots\cdots$

ⓐ $\frac{1}{2}$ ⓑ $\frac{3}{4}$

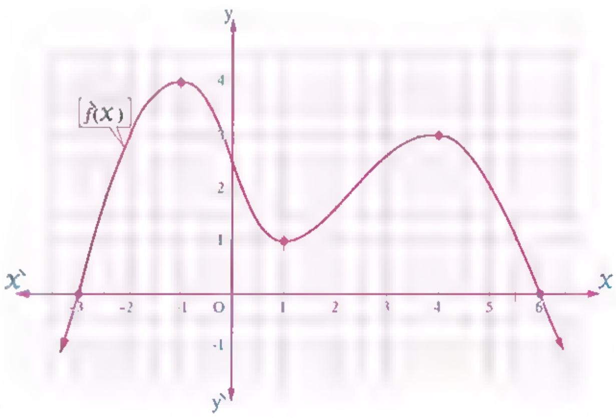
ⓒ 4.5 ⓓ 6

Second : $\grave{\grave{f}}(2) = \cdots\cdots\cdots\cdots$

ⓐ $\frac{1}{2}$ ⓑ $\frac{3}{4}$ ⓒ 4.5 ⓓ 6

---

10 If the opposite figure represents the curve $\grave{f}(x)$, then :



First : The curve $f(x)$ has local Maximum value at $x = \cdots\cdots\cdots\cdots$

ⓐ -3 ⓑ -1

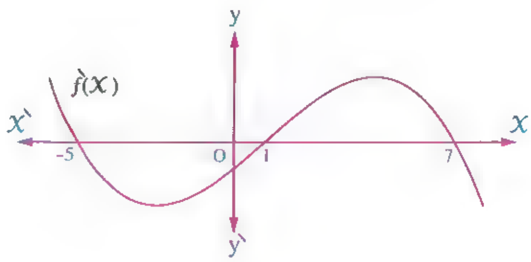
ⓒ 4 ⓓ 6

Second : The curve $f(x)$ has local Minimum value at $x = \cdots\cdots\cdots\cdots$

ⓐ -3 ⓑ 1 ⓒ 4 ⓓ 6

---

11 The opposite figure represents the curve of $\grave{f}$, then all the following statements are correct except $\cdots\cdots\cdots\cdots$



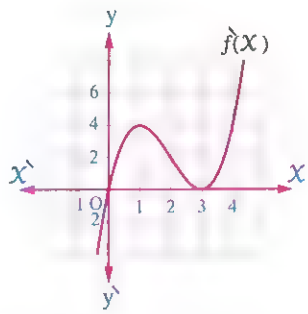
ⓐ At $x = -5$ there is a maximum value of f

ⓑ $\grave{\grave{f}}(1) >$ zero

ⓒ At $x = 7$ there is a minimum value of f

ⓓ The function f is decreasing on $]-5\,,\,1[$

12 The opposite figure represents the curve $f'(X)$, then the function f ....



(a) has a local maximum value but not a local minimum value.

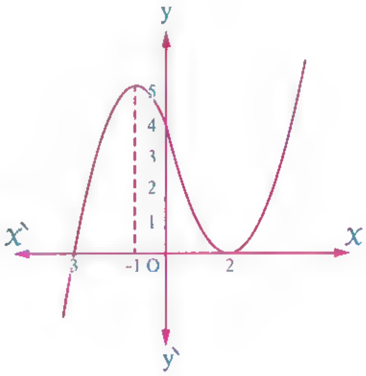
(b) has a local minimum value but not a local maximum value.

(c) has a local minimum value and a local maximum value.

(d) has neither local maximum value nor local minimum value.

---

13 The opposite figure represents the curve of the first derivative of the function $y = f(X)$ All the following statements are true except ....



(a) $f'(-4) >$ zero

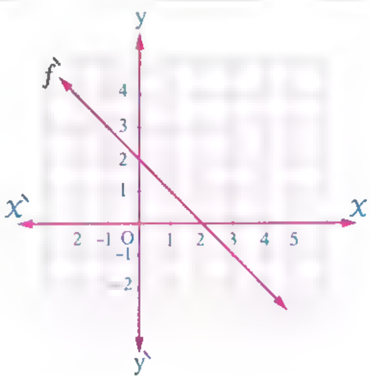
(b) $f''(-1) =$ zero

(c) $f''(1) <$ zero

(d) $f'(-2) . f'(3) >$ zero

---

14 The opposite figure represents f' , then the function f is increasing in the interval ....



(a) $]0 , \infty[$

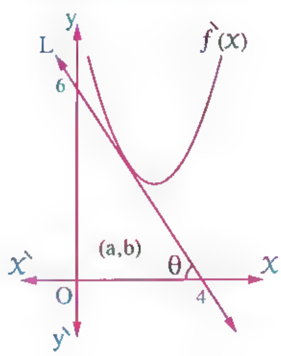
(b) $]-\infty , 0[$

(c) $]2 , \infty[$

(d) $]-\infty , 2[$

---

15 The opposite figure represents the curve $f'(X)$, the straight line L touches the curve at (a , b) , then $f''(a) =$ ....



(a) $\tan\theta$

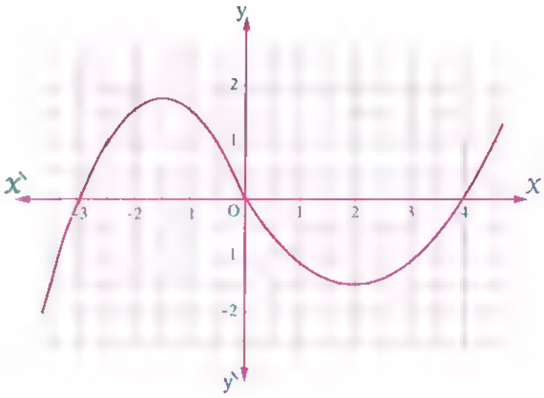
(b) $-\tan\theta$

(c) zero

(d) $\frac{-a}{b}$

16 The opposite figure represents the function $f''(x)$, then :

First : The curve of the function f is convex upwards in the interval ....



(a) $]-\infty , 0[$ (b) $]-\infty , -3[$

(c) $]0 , 4[$ (d) b , c together

Second : The curve of the function is convex downwards in the interval ....

(a) $]-3 , 0[$ (b) $]4 , \infty[$ (c) $]0 , \infty[$ (d) a , b together

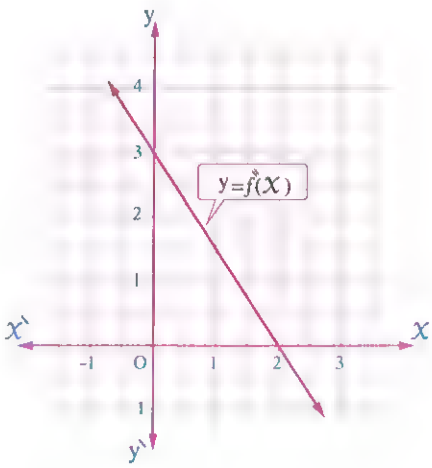
Third : The inflection point at $x =$ ....

(a) -3 (b) zero

(c) 4 (d) all the previous are true.

17 The opposite figure represents the curve of the function $f'(x)$, then :

First : The curve of the function is convex upwards when $x \in$ ....



(a) $]-\infty , 2[$ (b) $]2 , \infty[$

(c) $]0 , 2[$ (d) a , c together

Second : The curve of the function has inflection point at $x =$ ....

(a) 1 (b) 2

(c) 3 (d) zero

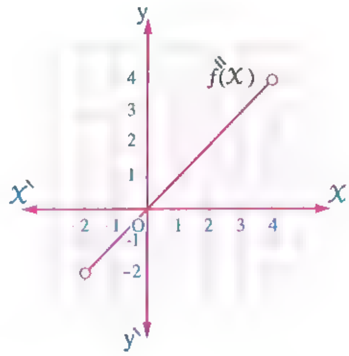
Third : If $f'(-1) = f'(5) =$ zero , then the function has local Maximum value at $x =$ ....

(a) -1 (b) 2 (c) 3 (d) 5

Fourth : The function f is decreasing for all $x \in$ ....

(a) $]-\infty , -1[$ (b) $]5 , \infty[$ (c) $\mathbb{R}$ (d) a , b together

18 If the opposite figure represents a continuous curve of $f'(x)$ in the interval $]-2\ ,4[$, then $f'(x)$ is increasing in the interval ....



(a) $]-2\ ,4[$

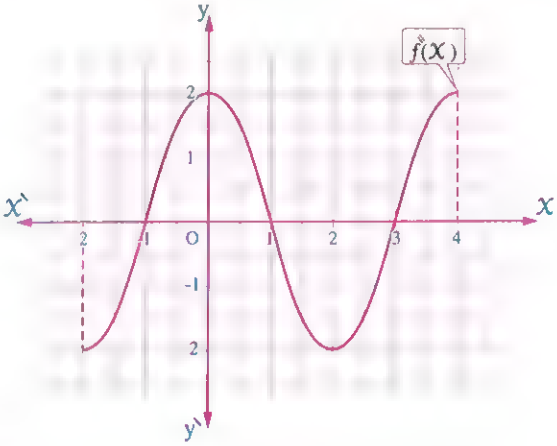
(b) $]2\ ,4[$

(c) $]0\ ,4[$

(d) $]-2\ ,0[$

---

19 If the opposite figure represents the curve of $f''(x)$ of the function f where $-2 \le x \le 4$, then the curve of the function f is convex upwards in ....



(a) $-1 < x < 1$

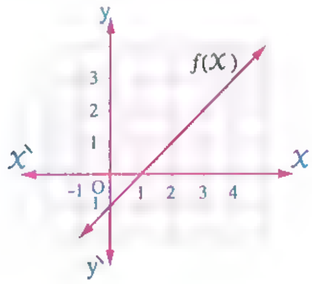
(b) $0 < x < 2$

(c) $-2 < x < -1$ only

(d) $-2 < x < -1$ and $1 < x < 3$

---

20 The opposite figure represents the curve of the function f , then $f(x) > f'(x)$ at $x \in$ ....

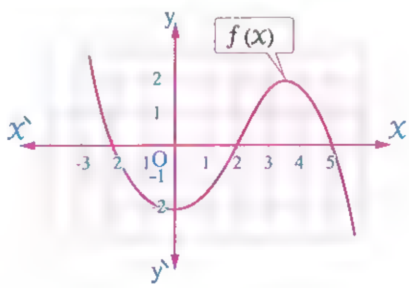


(a) $\mathbb{R}$ (b) $]-\infty\ ,1[$

(c) $]1\ ,\infty[$ (d) $]2\ ,\infty[$

---

21 The opposite figure represents the curve of the function f which is polynomial has an inflection point at $x = 2$, then $f(x)$, $f'(x)$, $f''(x)$ have the same sign at $x \in$ ....



(a) $\mathbb{R}$ (b) $]-\infty\ ,-2[$ (c) $]2\ ,5[$ (d) $]5\ ,\infty[$

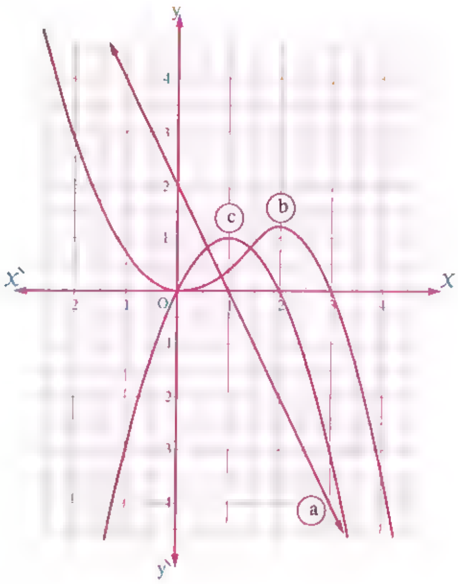
22 The opposite figure shows a graphical representation to the curves of the functions $f(x)$, $f'(x)$, $f''(x)$ where $f(x)$ is polynomial , then the curves a , b , c respectively

(a) $f(x)$, $f'(x)$, $f''(x)$

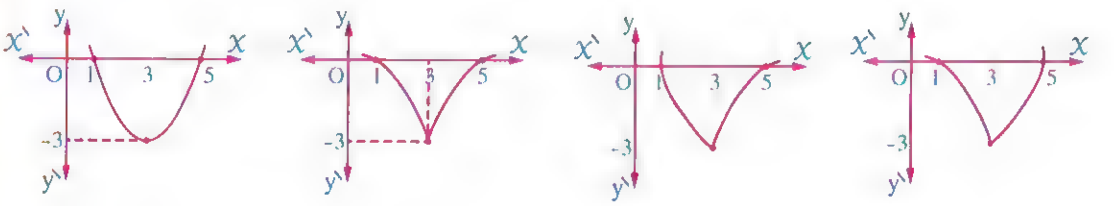
(b) $f'(x)$, $f''(x)$, $f(x)$

(c) $f''(x)$, $f(x)$, $f'(x)$

(d) $f''(x)$, $f'(x)$, $f(x)$

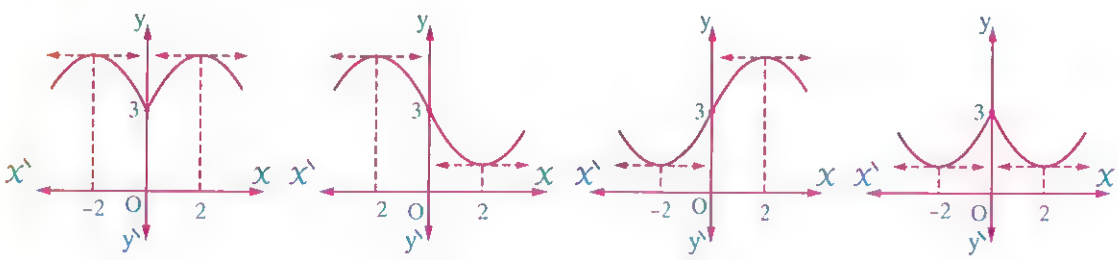


23 Which of the following figures represents a general curve of the continuous function f in which $f(1) = f(5) = 0$, $f(3) = -3$ and $f'(x) < 0$ for each $x < 3$, $f'(x) > 0$ for each $x > 3$ and $f''(x) < 0$ for each $x \neq 3$?



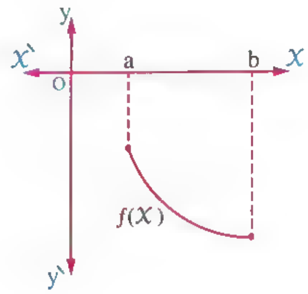
(a) (b) (c) (d)

24 Which of the following figures represents a general curve of the continuous function f in which $f(0) = 3$, $f'(2) = f'(-2) = 0$ and $f'(x) > 0$ when $-2 < x < 2$ and $f''(x) < 0$ when $x > 0$, $f''(x) > 0$ when $x < 0$



(a) (b) (c) (d)

25 The opposite figure represents the curve of function f, its domain is $[a, b]$, then the function $g : g(x) = x \cdot f(x)$ is ....
in the interval $]a, b[$



ⓐ decreasing

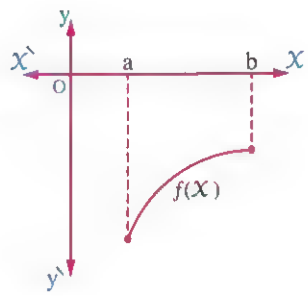
ⓑ increasing

ⓒ constant

ⓓ not possible to determine its monotony

---

26 The opposite figure represents the curve of the function f where $f : [a, b] \longrightarrow \mathbb{R}^-$, then the function $g : g(x) =$ .... is increasing in the interval $]a, b[$

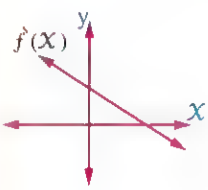


ⓐ $[f(x)]^2$ ⓑ $x \times f(x)$

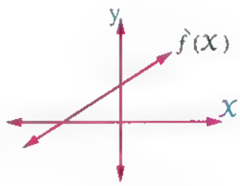
ⓒ $[f(x)]^3$ ⓓ $-2x - f(x)$

---

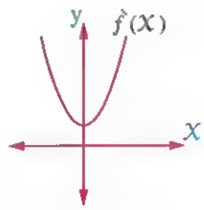
27 If $f : \mathbb{R} \longrightarrow \mathbb{R}$ and for all values of $x \in \mathbb{R}$, f is increasing function, then the figure which represents $\acute{f}(x)$ is ....



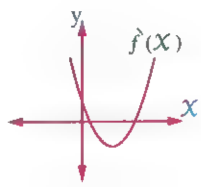
ⓐ



ⓑ

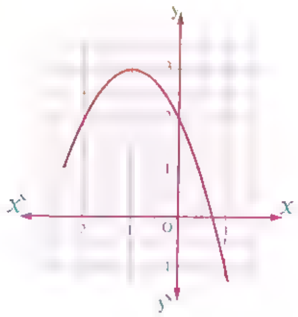


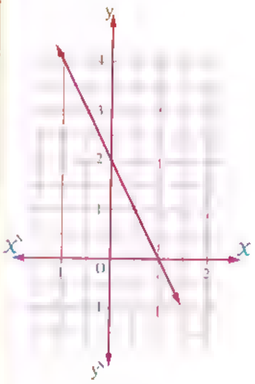
ⓒ



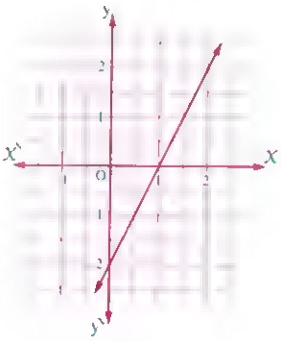
ⓓ

28 The opposite figure represents the curve of the function $y = f(X)$, which of the following represents the curve $\acute{f}(X)$?





ⓐ



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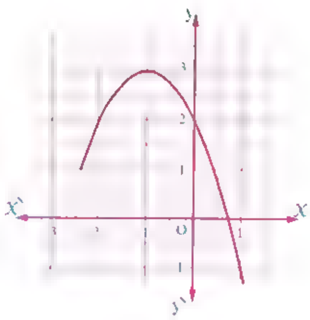

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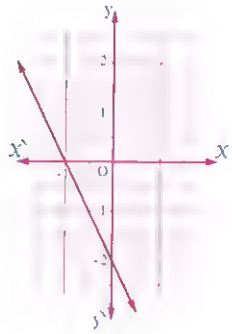
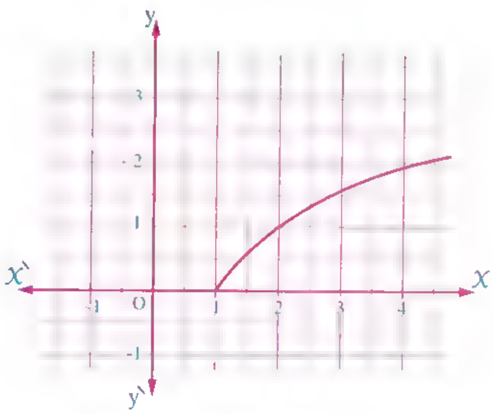


ⓓ

29 The opposite figure represents the curve of the function $y = f(X)$

If the equation of the tangent to the curve at any point (X , y) is $y = g(X)$ which of the following statements is right ....



ⓐ $g(X) = f(X)$

ⓑ $g(X) \leq f(X)$

ⓒ $g(X) \geq f(X)$

ⓓ $f(X) + g(X) < 0$

Eighth Questions on the Indefinite Integration

Choose the correct answer from the given ones :

1. $\int x(x^2+3)^5 \, dx = \dots\dots + c$

(a) $\frac{1}{6}(x^3+3)^6$ (b) $\frac{1}{12}(x^2+3)^6$ (c) $\frac{1}{4}(x^2+3)^4$ (d) $\frac{1}{8}(x^2+3)^4$

2. $\int \frac{x-\frac{1}{2}}{\sqrt{2x-1}} \, dx = \dots\dots + c$

(a) $\frac{1}{6}\sqrt{2x-1}$ (b) $\frac{1}{6}\sqrt{(2x-1)^3}$ (c) $\frac{1}{6}\sqrt{2x-1}$ (d) $\frac{1}{2}\sqrt{(2x-1)^3}$

3. $\int \frac{2x+1}{(x^2+x)^2} \, dx = \dots\dots + c$

(a) $\frac{-1}{x^2+x}$ (b) $\frac{1}{x^2+x}$ (c) $\frac{-2}{(x^2+x)^2}$ (d) $\frac{2}{(x^2+x)^3}$

4. If $\int \frac{x^2 \, dx}{\sqrt{2x^3+1}} = n\sqrt{2x^3+1} + c$, then $n = \dots\dots$

(a) 3 (b) $\frac{1}{3}$ (c) 6 (d) $\frac{1}{6}$

5. If $\int 3x^2(x^n+1)^5 \, dx = \frac{(x^n+1)^6}{6} + c$, then $n = \dots\dots$

(a) 2 (b) 3 (c) 5 (d) 6

6. $\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$, for every $n \neq \dots\dots$

(a) zero (b) 1 (c) 2 (d) -1

7. $\int \frac{(4x^2-4x+1)^7}{(2x-1)^2} \, dx = \dots\dots$

(a) $\frac{1}{13}(2x-1)^{13}+c$ (b) $\frac{1}{26}(2x-1)^{13}+c$

(c) $\frac{2}{3}(2x-1)^{13}+c$ (d) $\frac{1}{16}(2x-1)^8+c$

8. $\int \frac{(2x+1)(x-2)}{\sqrt{x}} \, dx = \dots\dots + c$

(a) $2x^{1\frac{1}{2}} - 3x^{\frac{1}{2}}$ (b) $\frac{(2x+1)^2(x-1)^2}{x^{1\frac{1}{2}}}$

(c) $2x^{1\frac{1}{2}} - 3x^{\frac{1}{2}} - 2x^{-\frac{1}{2}}$ (d) $\frac{4}{5}x^{2\frac{1}{2}} - 2x^{1\frac{1}{2}} - 4x^{\frac{1}{2}}$

9. $\int \left(x+\frac{1}{x}\right)^2 dx = \cdots\cdots + c$

(a) $\frac{1}{3}\left(x+\frac{1}{x}\right)^3$
(b) $\frac{1}{3}x^3+2x-\frac{1}{x}$
(c) $\frac{1}{3}x^3+2x+\frac{1}{x}$
(d) $\left(x+\frac{1}{x}\right)^3$

10. $\int \left(x-\frac{1}{x}\right)\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}\right) dx = \cdots\cdots + c$

(a) $x^4-\frac{1}{x^4}$
(b) $\frac{1}{5}x^5-\frac{5}{x^5}$
(c) $\frac{1}{5}x^5-x^{-3}$
(d) $\frac{1}{5}x^5+\frac{1}{3}x^{-3}$

11. $\int 6x^5\left(1-\frac{1}{x}\right)^5 dx = \cdots\cdots + c$

(a) $6(x-1)^6$
(b) $(x-1)^6$
(c) $\frac{1}{6}(x-1)^6$
(d) $(6x-1)^6$

12. $\int x\sqrt{x-2}\, dx = \cdots\cdots + c$

(a) $(x-2)^{\frac{3}{2}}+2\sqrt{x-2}$
(b) $\frac{2}{3}(x^2-2x)^{\frac{3}{2}}$
(c) $\frac{2}{5}(x-2)^{\frac{5}{2}}+\frac{2}{3}(x-2)^{\frac{3}{2}}$
(d) $\frac{2}{5}(x-2)^{\frac{5}{2}}+\frac{4}{3}(x-2)^{\frac{3}{2}}$

13. $\int \frac{x+3}{\sqrt{x-1}} dx = \cdots\cdots + c$

(a) $(x-2)^{\frac{3}{2}}-(x-1)^{\frac{1}{2}}$
(b) $\frac{3}{2}(x-1)^{\frac{3}{2}}+\frac{1}{2}(x-1)^{\frac{1}{2}}$
(c) $\frac{2}{3}(x-1)^{\frac{3}{2}}+2(x-1)^{\frac{1}{2}}$
(d) $\frac{2}{3}(x-1)^{\frac{3}{2}}+8(x-1)^{\frac{1}{2}}$

14. $\int x(x+2)^8 dx = \cdots\cdots + c$

(a) $\frac{1}{9}(x+2)^9$
(b) $\frac{1}{9}(x^2+2x)^9$
(c) $\frac{1}{10}(x+2)^{10}-\frac{2}{9}(x+2)^9$
(d) $\frac{1}{10}(x+2)^{10}+\frac{1}{9}(x+2)^9$

15. $\int x f'(1-x^2)\, dx = \cdots\cdots + c$

(a) $-f(1-x^2)$
(b) $-2f(1-x^2)$
(c) $-\frac{1}{2}f(1-x^2)$
(d) $x f(1-x^2)$

16. $\int x \cdot f'(x^2) \cdot f(x^2)\, dx = \cdots\cdots + c$

(a) $\frac{1}{4}[f(x^2)]^2$
(b) $\frac{1}{2}[f(x^2)]^2$
(c) $[f(x^2)]^2$
(d) $2[f(x^2)]^2$

17 $\int \sqrt{x^3 - 3x^2 + 3x - 4}\,(x-1)^2\,dx = \cdots\cdots\cdots + c$

(a) $\sqrt{\frac{1}{4}x^4 - x^3 + \frac{3}{2}x^2 - 4x}$
(b) $\frac{2}{9}\sqrt{(x^3 - 3x^2 + 3x - 3)^3}$
(c) $\frac{2}{9}\sqrt{x^3 - 3x^2 + 3x - 4}$
(d) $\frac{1}{2}(x^3 - 3x^2 + 3x - 4)^{\frac{2}{3}}$

18 $\int x^3 (x^2 - 1)^5\,dx = \cdots\cdots\cdots + c$

(a) $\frac{1}{6}(x^2 - 1)^6$
(b) $\frac{1}{6}x^6 - \frac{1}{4}x^4$
(c) $\frac{1}{7}(x^2 - 1)^7 + \frac{1}{6}(x^2 - 1)^6$
(d) $\frac{1}{14}(x^2 - 1)^7 + \frac{1}{12}(x^2 - 1)^6$

19 $\int \frac{dx}{\sqrt{x}\left(\sqrt{x} + 2\right)^4} = \cdots\cdots\cdots + c$

(a) $\frac{-2}{3}\left(\sqrt{x} + 2\right)^{-3}$
(b) $\frac{-1}{3}\left(\sqrt{x} + 2\right)^{-3}$
(c) $\left(\sqrt{x} + 2\right)^{-3}$
(d) $\frac{-1}{2}\left(\sqrt{x} + 2\right)^{-3}$

20 $\frac{d}{dx}\int (\sin x + 3)^4\,dx = \cdots\cdots\cdots$

(a) $\frac{1}{5}(\sin x + 3)^5 + c$
(b) $(\sin x + 3)^4$
(c) $(\sin x + 3)^4 + c$
(d) $4(\sin x + 3)^3$

21 $\int \frac{d}{dx}\left(x^5 + \sqrt{x}\right)dx = \cdots\cdots\cdots$

(a) $\frac{1}{6}x^6 + \frac{1}{2\sqrt{x}} + c$
(b) $x^4 + \frac{1}{2}x^{-\frac{1}{2}}$
(c) $x^5 + \sqrt{x}$
(d) $x^5 + \sqrt{x} + c$

22 $\int \cos(3x - 1)\,dx = \cdots\cdots\cdots + c$

(a) $\sin(3x - 1)$
(b) $\frac{1}{3}\sin(3x - 1)$
(c) $3\sin(3x - 1)$
(d) $-\frac{1}{3}\sin(3x - 1)$

23 $\int \csc^2(3x)\,dx = \cdots\cdots\cdots + c$

(a) $-\cot(3x)$
(b) $\frac{1}{3}\cot(3x)$
(c) $-3\cot 3x$
(d) $-\frac{1}{3}\cot(3x)$

24 $\int \sec y \tan y\,dy = \cdots\cdots\cdots + c$

(a) $\sec y$
(b) $\csc y$
(c) $\tan y$
(d) $\cot y$

25 $\int (\sin^2 5x + \cos^2 5x)^{2021}\, dx = \cdots\cdots\cdots + c$

ⓐ $\sin^{-1} x + \cos^{-1} x$ ⓑ 1

ⓒ x ⓓ $\frac{1}{5}(\sin^3 5x + \cos^3 5x)$

26 $\int \left(\csc^2 \frac{\pi}{4} + \cos \frac{\pi}{3}\right) dx = \cdots\cdots\cdots + c$

ⓐ $-\cot \frac{\pi}{4} + \sin \frac{\pi}{3}$ ⓑ $-4\cot \frac{\pi}{4} + 3\sin \frac{\pi}{3}$

ⓒ $2.5\, x$ ⓓ $\cot \frac{\pi}{4} - \sin \frac{\pi}{3}$

27 $\int \frac{3}{\sin^2 3x}\, dx = \cdots\cdots\cdots + c$

ⓐ $\frac{-3}{\cos^2 3x}$ ⓑ $3\csc^2 3x$ ⓒ $-\cot 3x$ ⓓ $-3\cot$

28 $\int (\sin^2 x + \cos^2 x + \tan^2 x)\, dx = \cdots\cdots\cdots$

ⓐ $\sin x + \cos x + \tan x + c$ ⓑ $\tan x + c$

ⓒ $d\,x + c$ ⓓ $\sec^2 x + c + \cos x \sec x$

29 $\int (\sin x \csc x + \cos x \sec x + \tan x \cot x)\, dx = \cdots\cdots\cdots$

ⓐ 3 ⓑ $3x$ ⓒ zero ⓓ $-\cot x$

30 $\int \left(\cos \frac{x}{2} + \sin \frac{x}{2}\right)^2 dx = \cdots\cdots\cdots + c$

ⓐ $1 + \sin x$ ⓑ x ⓒ $x - \cos x$ ⓓ $x + \cos x$

31 $\int (2\cos^2 x - 1)\, dx = \cdots\cdots\cdots + c$

ⓐ $\frac{1}{2}\sin 2x$ ⓑ $\frac{1}{2}\cos 2x$ ⓒ $\sin 2x$ ⓓ $\cos 2x$

32 $\int (1 + \cot^2 x)\, dx = \cdots\cdots\cdots + c$

ⓐ $\cot x$ ⓑ $-\cot x$ ⓒ $\tan^2 x$ ⓓ $-\cot^2 x$

33 $\int (4 - \csc x \cot x)\, dx = \cdots\cdots\cdots$

ⓐ $4x - \csc x + c$ ⓑ $4x + \csc x + c$

ⓒ $4x - \cot x + c$ ⓓ $4x + \cot x + c$

34) $\int (\sin 3x \cos x - \cos 3x \sin x)\, dx = \cdots\cdots + c$

(a) $\sin 2x$ (b) $-\frac{1}{2}\cos 2x$ (c) $\cos 2x$ (d) $-\frac{1}{4}\cos 4x$

35) $\int \cos x \cos \frac{\pi}{4} - \sin x \sin \frac{\pi}{4}\, dx = \cdots\cdots + c$

(a) $\cos\left(x + \frac{\pi}{4}\right)$ (b) $\sin\left(x + \frac{\pi}{4}\right)$

(c) $\frac{1}{4}\cos\left(x + \frac{\pi}{4}\right)$ (d) $\frac{1}{4}\sin\left(x + \frac{\pi}{4}\right)$

36) $\int \sin x \cos x \cos 2x \cos 4x\, dx = \cdots\cdots + c$

(a) $-\cos 8x$ (b) $-\frac{1}{64}\cos 8x$ (c) $\cos 8x$ (d) $\frac{1}{64}\cos 8x$

37) $\int \tan^2 x \csc^2 x\, dx = \cdots\cdots + c$

(a) $\tan x$ (b) $\text{sce}^2 x$ (c) $\csc^2 x$ (d) $-\cot x$

38) $\int (\sin^2 x + \sin^2 x \tan^2 x)\, dx = \cdots\cdots + c$

(a) $\sin^2 x + \csc^2 x$ (b) $\tan x - x$ (c) $\tan^2 x$ (d) $\sec x$

39) $\int (\cot^4 x + \cot^2 x)\, dx = \cdots\cdots + c$

(a) $\frac{1}{3}\cot^3 x$ (b) $\tan x$ (c) $\log|\sin^2 x|$ (d) $-\frac{1}{3}\cot^3 x$

40) $\int (1 + \tan^2 x)\cos^2 x\, dx = \cdots\cdots + c$

(a) x (b) $\frac{1}{3}\cos^3 x$ (c) $\frac{1}{3}\sec^3 x$ (d) $\frac{1}{3}\tan^3 x$

41) $\int (1 + \csc x)(1 - \csc)\, dx = \cdots\cdots + c$

(a) $x - \cot x$ (b) $x + \cot x$

(c) $\frac{1}{2}\cot^2 x$ (d) $\frac{1}{2}x^2 + \cot x$

42) $\int \sec x (\sec x + \tan x)\, dx = \cdots\cdots + c$

(a) $\csc x + \cot x$ (b) $\sec^2 x + \tan x$ (c) $\sec x + \tan x$ (d) $\sec x - \tan x$

43 $\int \frac{1}{1-\cos^2 x} \, dx = \cdots\cdots + c$

(a) $\cot x$ (b) $-\cot x$ (c) $\tan x$ (d) $-\tan x$

44 $\int \frac{\cos x + \cos^3 x}{\sin^2 x + 2\cos^2 x} \, dx = \cdots\cdots + c$

(a) $\sin x$ (b) $\cos x$ (c) $\tan x$ (d) $\csc x$

45 $\int \cos^5 x \sin x \, dx = \cdots\cdots + c$

(a) $\frac{1}{6}\sin^6 x$ (b) $-\frac{1}{6}\sin^6 x$ (c) $\frac{1}{6}\cos^6 x$ (d) $-\frac{1}{6}\cos^6 x$

46 $\int x^2 \sec^2 (x^3+5) \, dx = \cdots\cdots + c$

(a) $\frac{1}{3} x^3 \sec^3 (x^3+5)$ (b) $\frac{1}{3}\sec^3 (x^3+5)$

(c) $\frac{1}{3}\tan (x^3+5)$ (d) $3\tan (x^3+5)$

47 $\int 10\, x \sec (5x^2+2021) \tan (5x^2+2021) = \cdots\cdots + c$

(a) $\sec^2 (5x^2+2021)$ (b) $\cos (5x^2+2021)$

(c) $\sin (5x^2+2021)$ (d) $\sec (5x^2+2021)$

48 $\int (\sin x + \cot x)^8 (\cos x - \csc^2 x) \, dx = \cdots\cdots$

(a) $\frac{1}{9}(\sin x + \cot x)^9$ (b) $8(\sin x + \cot x)^7$

(c) $\frac{1}{9}\sin^9 x + \frac{1}{9}\cot^9 x$ (d) $\frac{1}{2}(\cos x - \cot^2 x)^2$

49 $\int \cos (\tan x + 1) \sec^2 x \, dx = \cdots\cdots + c$

(a) $\cos^2 (\tan x + 1)$ (b) $\sin (\tan x + 1)$

(c) $\frac{1}{2}\sec^3 x \times \sin (\tan x + 1)$ (d) $\sec (\tan x + 1) \tan (\tan x + 1)$

50 $\int \frac{\sec x}{\sec x + \tan x} \, dx = \cdots\cdots + c$

(a) $\tan x \sec x$ (b) $\frac{\sec x + \tan x}{\sec x}$ (c) $\tan x - \sec x$ (d) $\tan x + \sec x$

51 $\int \cos x\, f'(\sin x)\, dx = \dots\dots + c$

ⓐ $f(\cos x)$ ⓑ $f(\sin x)$

ⓒ $\frac{1}{2}(f(x))^2$ ⓓ $\frac{1}{2}(f(\cos x))^2$

52 $\int \left[(1-\cot x)^2 + 2\cot x\right] dx = \dots\dots + c$

ⓐ $-\cot x$ ⓑ $\csc^2 x$ ⓒ $x + \frac{\cot^2 x}{3}$ ⓓ $\cot x$

53 $\int \left(\frac{\sin^2 x + 1}{\sin x}\right)^2 dx = \dots\dots + c$

ⓐ $2\frac{1}{2}x - \frac{1}{2}\sin 2x + \cot x$ ⓑ $\frac{1}{2} - \frac{1}{2}\cos 2x - \frac{1}{4}\sin 2x + \cot x$

ⓒ $2\frac{1}{2}x - \frac{1}{4}\sin 2x - \cot x$ ⓓ $2\frac{1}{2}x - \frac{1}{4}\sin x - \cot x$

54 $\int \frac{1+\sin^2 x}{1-\sin^2 x}\, dx = \dots\dots + c$

ⓐ $\sec^2 x + \tan^2 x$ ⓑ $\tan x + x$ ⓒ $\tan x - x$ ⓓ $2\tan x - x$

55 $\int \frac{\sin^3 x + \cos^3 x}{\sin x + \cos x}\, dx = \dots\dots + c$

ⓐ $1 - \frac{1}{2}\sin 2x$ ⓑ $x + \frac{1}{4}\cos 2x$ ⓒ $1 + \cos x \sin x$ ⓓ $1 - \frac{1}{4}\cos x$

56 $\int \frac{\cos 2x}{\cos x + \sin x}\, dx = \dots\dots + c$

ⓐ $\sin x - \cos x$ ⓑ $\cos x - \sin x$

ⓒ $\sin x + \cos x$ ⓓ $\cos 2x + \sin 2x$

57 $\int 4\sin^4 x\, dx = \dots\dots + c$

ⓐ $\frac{-4}{5}\cos^5 x$ ⓑ $\frac{4}{5}\cos^5 x$

ⓒ $x - \sin 2x + \cos 4x$ ⓓ $\frac{3}{2}x - \sin 2x + \frac{1}{8}\sin 4x$

58 $3\int \sin 2x \sin^4 x\, dx = \dots\dots + c$

ⓐ $\sin^4 x \cos^2 x$ ⓑ $\cos^4 x \sin^2 x$ ⓒ $\sin^6 x$ ⓓ $\cos^6 x$

59 $\int \frac{\sin^6 x}{\cos^8 x}\, d\,x = \cdots\cdots\cdots + c$

(a) $\tan^7 x$ (b) $\frac{1}{7}\tan^7 x$ (c) $\frac{1}{7}\tan 7x$ (d) $\sec^7 x$

60 $\int (\sec x + \cos x)^2\, d\,x = \cdots\cdots\cdots + c$

(a) $\frac{1}{3}(\sec x + \cos x)^3$ (b) $\cot x - \sin^3 x$

(c) $\frac{5}{2}x - \sin 2x - \tan x$ (d) $\frac{5}{2}x + \frac{1}{4}\sin 2x + \tan x$

61 $\int \frac{\tan x}{\cos x}\, d\,x = \cdots\cdots\cdots + c$

(a) $\sec^2 x$ (b) $\sec x$ (c) $\tan x \sec x$ (d) $\ln \cos x$

62 $\int \frac{\cos x}{\csc x}\, d\,x = \cdots\cdots\cdots$

(a) $\frac{1}{2}\sin 2x$ (b) $\frac{1}{4}\cos 2x$ (c) $\frac{-1}{4}\cos 2x$ (d) $-4\cos 2x$

63 $\int \tan^2 x\, d\,x = \cdots\cdots\cdots$

(a) $\tan x - x + c$ (b) $\tan x + x + c$ (c) $\sec^4 x + c$ (d) $\frac{1}{3}\tan^3 x + c$

64 $\int \sec^4 x \tan x\, d\,x$ equals $\cdots\cdots\cdots$

(a) $\frac{1}{5}\sec^5 x + c$ (b) $\frac{1}{4}\sec^4 x + c$ (c) $\frac{1}{3}\tan^3 x + c$ (d) $\frac{-1}{3}\tan^3 x + c$

65 $\int \frac{\sec^2 x}{1 + \tan^2 x}\, d\,x = \cdots\cdots\cdots + c$

(a) $2\sec^2 x$ (b) $\sec^2 x \tan^2 x$ (c) $2\sec^2 x$ (d) x

66 $\int \left(\frac{\tan x}{\cot x} + 1\right) d\,x = \cdots\cdots\cdots$

(a) $\tan^2 x + c$ (b) $\tan x + c$

(c) $\tan x \sec x + c$ (d) $\cot x \csc x + c$

67 $\int e^2\, d\,x = \cdots\cdots\cdots + c$

(a) $e^2 x$ (b) $\frac{1}{3}e^3$ (c) e^2 (d) $\frac{1}{3}e^{3x}$

68 $\int e^{\ln x}\, d\,x = \cdots\cdots\cdots + c$

(a) $\frac{x^2}{2}\ln x$ (b) $\ln x$ (c) $\frac{1}{2}x^2$ (d) e^x

69 $\int \left(\sum_{n=0}^{\infty} \frac{x^n}{\underline{|n}} \right) dx =$ .... $+ c$

(a) e^x (b) $e x$ (c) x^{2e} (d) e^{x^2}

70 $\int e^{6x} \, dx =$ .... $+ c$

(a) e^{6x} (b) $\frac{1}{6} e^{6x}$ (c) $\frac{1}{7} e^{7x}$ (d) $\frac{1}{7} e^{6x+1}$

71 $\int a^{3 \log_a x} \, dx =$ .... $+ c$

(a) a^x (b) e^{x^3} (c) x^4 (d) $\frac{1}{4} x^4$

72 $\int \frac{3}{x} \, dx =$ .... $+ c$

(a) $\frac{-3}{2} x^{-2}$ (b) $3 \ln |x|$ (c) $\ln |3x|$ (d) $\ln |x|$

73 $\int \frac{\log_4 e}{x} \, dx =$ .... $+ c$

(a) $\ln |4 - x|$ (b) $\ln |4 + x|$ (c) $\log_4 |x|$ (d) $\ln |x|$

74 $\int \left(2 \sin x + \frac{1}{x} \right) dx =$ ....

(a) $-2 \cos x + \ln |x| + c$ (b) $2 \cos x + \ln |x| + c$

(c) $-\sin x - \frac{1}{x^2} + c$ (d) $-2 \cos x + \frac{1}{x^2} + c$

75 $\int \frac{e^x + e^{-x}}{e^x} \, dx =$ .... $+ c$

(a) $x + e^{-2x}$ (b) $1 + e^{-2x}$ (c) $x - e^{-2x}$ (d) $x - \frac{1}{2} e^{-2x}$

76 $\int (x^{2e} + e^{3x}) \, dx =$ .... $+ c$

(a) $2 x^{2e} + 3 x^{3e}$ (b) $\frac{1}{3} x^{3e} + e^{3x}$

(c) $\frac{1}{3e} x^{3e} + \frac{1}{3} e^{3x}$ (d) $\frac{x^{2e+1}}{2e+1} + \frac{1}{3} e^{3x}$

77 $\int 4 x e^{x^2} \, dx =$ ....

(a) $\frac{1}{2} e^{x^2} + c$ (b) $e^{x^2} + c$ (c) $2 e^{x^2} + c$ (d) $4 e^{x^2} + c$

78 $\int x^2 e^{x^3+1}\, dx = \cdots\cdots\cdots\cdots + c$

ⓐ e^{x^3+1} ⓑ $3\,e^{x^3+1}$ ⓒ $\frac{1}{3}\,e^{x^3+1}$ ⓓ $3\,e^{x^4+x}$

79 $\int (13)^x\, dx = \cdots\cdots\cdots\cdots$

ⓐ $\frac{13^x}{\ln 13} + c$ ⓑ $(14)^{x+1} + c$ ⓒ $(13)^{x+1} + c$ ⓓ $14\,x + c$

80 $\int e^x . \tan (e^x)\, dx = \cdots\cdots\cdots\cdots + c$

ⓐ $\ln |\sec e^x|$ ⓑ $\ln |\sin e^x|$ ⓒ $\ln |\cos e^x|$ ⓓ $\tan e^x$

81 $\int e^{\ln (\sin x)}\, dx = \cdots\cdots\cdots\cdots$

ⓐ $-\cos x + c$ ⓑ $\sin x + c$ ⓒ $\cos x + c$ ⓓ $-\sin x + c$

82 $\int \sin x\, e^{\cos x}\, dx = \cdots\cdots\cdots\cdots + c$

ⓐ $-e^{\sin x}$ ⓑ $-e^{\cos x}$ ⓒ $e^{\sin x}$ ⓓ $e^{\cos x}$

83 $\int \frac{x^3\, dx}{x^4 + 3} = \cdots\cdots\cdots\cdots + c$

ⓐ $\frac{1}{4}\,(x^4 + 3)$ ⓑ $\frac{1}{4}\,\ln |x^4 + 3|$ ⓒ $\ln |x^4 + 3|$ ⓓ $\frac{1}{4}\,(x^4 + 3)^{-1}$

84 $\int \frac{\ln x}{x}\, dx = \cdots\cdots\cdots\cdots + c$

ⓐ $\ln x^2$ ⓑ $(\ln x)^2$ ⓒ $\frac{1}{2}\,\ln x^2$ ⓓ $\frac{1}{2}\,(\ln x)^2$

85 $\int \frac{\ln x^2}{\ln x}\, dx = \cdots\cdots\cdots\cdots$

ⓐ $\frac{x}{2} + c$ ⓑ $\frac{1}{x} + c$ ⓒ $2\,x + c$ ⓓ $\ln |x| + c$

86 $\int \frac{1}{x \ln x^3}\, dx = \cdots\cdots\cdots\cdots$

ⓐ $3\,\ln |x| + c$ ⓑ $3\,\ln |\ln x| + c$

ⓒ $\frac{1}{3}\,\ln |x| + c$ ⓓ $\frac{1}{3}\,\ln |\ln x| + c$

87 $\int \frac{\ln x^5}{x \ln x^3}\, dx = \cdots\cdots\cdots\cdots + c$

ⓐ $\frac{5}{3}\,\ln |x|$ ⓑ $\frac{3}{5}\,\ln |x|$ ⓒ $\ln (\ln |x|)$ ⓓ $\ln \left(\frac{5}{3}\,|x|\right)$

88 $\int \frac{x+3}{x-1}\,dx = \dots\dots + c$

(a) $1 + \ln|x+3|$ (b) $x + \ln|x-1|$ (c) $1 + \ln|x-1|$ (d) $x + 4\ln|x-1|$

89 $\int \frac{x^2-25}{x^2-5x}\,dx = \dots\dots + c$

(a) $\ln|x+5|$ (b) $x + 5\ln|x|$ (c) $5x + \ln|x|$ (d) $x + \ln|x+5|$

90 $\int \tan\theta\,d\theta$ equals $\dots\dots$

(a) $-\ln|\cos\theta| + c$ (b) $-\ln\cos\theta + c$ (c) $\ln\cos\theta + c$ (d) $|\ln\cos\theta| + c$

91 $\int \frac{2\tan x}{1-\tan^2 x}\,dx = \dots\dots + c$

(a) $\frac{1}{2}\tan 2x$ (b) $2\sec^2 2x$

(c) $-\ln|\cos x|$ (d) $-\frac{1}{2}\ln|\cos 2x|$

92 $\int \frac{\sin 2x}{1+\sin^2 x}\,dx = \dots\dots + c$

(a) $(1+\sin^2 x)^{-2}$ (b) $x + \frac{1}{3}\sin^3 x$

(c) $\ln|1+\sin^2 x|$ (d) $\frac{-1}{2}\cos 2x + \frac{1}{3}\sin^3 x$

93 $\int \frac{\cos x - \sin x}{\cos x + \sin x}\,dx = \dots\dots + c$

(a) $\ln\cos x - \ln\sin x$ (b) $\ln\cos x + \ln\sin x$

(c) $\ln|\cos x - \sin x|$ (d) $\ln|\cos x + \sin x|$

94 $\int \csc x\,dx = \dots\dots + c$

(a) $-\csc x\cot x$ (b) $-\ln|\csc x + \cot x|$

(c) $\sin^{-1} x$ (d) $\frac{1}{2}\csc^2 x$

95 $\int \frac{x^2}{x+1}\,dx = \dots\dots + c$

(a) $\frac{1}{2}x^2 - x + \ln|x+1|$ (b) $\ln|x+1|$

(c) $\frac{1}{2}x^2 + \frac{1}{3}x^3$ (d) $(x-1)^2 + \ln|x+1|$

96 $\int \frac{6}{x}(\ln x)^5\,dx = \dots\dots + c$

(a) $(\ln x)^6$ (b) $\frac{1}{6}(\ln x)^6$ (c) $\ln x^6$ (d) $\frac{1}{6}\ln x^6$

97 $\int \frac{\sqrt{5+\ln x}}{x}\, dx = \cdots\cdots\cdots + c$

(a) $(5+\ln x)^{\frac{3}{2}}$ (b) $\frac{2}{3}(5+\ln x)$ (c) $\frac{1}{2}\sqrt{5+\ln x}$ (d) $\frac{2}{3}(5+\ln x)^{\frac{3}{2}}$

98 $\int \sqrt{1-\cos^2 x}\, dx = \cdots\cdots\cdots + c$ where $x \in]0, \pi[$

(a) $\csc x$ (b) $-\cos x$ (c) $\cos^2 x$ (d) $\sin^2 x$

99 $\int \cos^{10} x \sec^9 x\, dx = \cdots\cdots\cdots + c$

(a) $\frac{1}{\pi}\cos^{11} x$ (b) $\frac{6}{10}\sec^{10} x$ (c) $\cos x$ (d) $\sin x$

100 $\int \cos^9 x \sec^{10} x\, dx = \cdots\cdots\cdots + c$

(a) $\sec x$ (b) $\frac{1}{2}\sec^2 x$

(c) $\ln|\sec x + \tan x|$ (d) $\sec x \tan x$

101 $\int (1+\tan^2 x)\, e^{1+\tan x}\, dx = \cdots\cdots\cdots + c$

(a) $e^{\cot x}$ (b) $e^{1+\tan x}$ (c) $e^{1+\cot x}$ (d) $e^{\sec^2 x}$

102 $\int \ln x\, dx = \cdots\cdots\cdots + c$

(a) $\frac{1}{2}(\ln x)^2$ (b) $x \ln x$ (c) $x \ln x - x$ (d) $x \ln x - 1$

103 $\int x \ln x\, dx = \cdots\cdots\cdots + c$

(a) $x^2 \ln x - x^2$ (b) $x^2 - \ln x$

(c) $x - \frac{1}{2}x^2 \ln x^2$ (d) $\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2$

104 $\int (3x+2)\sin x\, dx = \cdots\cdots\cdots$

(a) $(3x+2)\cos x + 3\sin x + c$ (b) $-(3x+2)\cos x + 3\sin x + c$

(c) $(3x+1)\cos x + 2\sin x + c$ (d) $-(x+1)\cos x - 3\sin x + c$

105 If $\int (2x+3)\ln x\, dx = yz - \int z\, dy$, then yz equals $\cdots\cdots\cdots$

(a) $3x \ln x$ (b) $(2x+3)$

(c) $\frac{1}{2}(2x+3)\ln x$ (d) $x(x+3)\ln x$

106 If $\int (2x-1)\, e^{2x+3}\, dx = yz - \int z\, dy$, then $\int z\, dy =$ ....

(a) $e^{2x+3} + c$
(b) $\frac{1}{2} e^{2x+3} + c$
(c) $-e^{2x+3} + c$
(d) $\frac{-1}{2} e^{2x+3} + c$

107 If each of y , z is a function of x , then $\int y\, dz + \int z\, dy =$ ....

(a) $d(yz)$
(b) $\int yz\, dx$
(c) $yz + c$
(d) $y + z + c$

108 $\int x e^{x}\, dx =$ .... $+ c$

(a) $\frac{1}{2} x^2 e^{x}$
(b) $e^{x}(x-1)$
(c) $\frac{1}{2} x^2 e^{x+1}$
(d) $e^{x}(x+1)$

109 $\int x^3 e^{x}\, dx = e^{x} \times (\text{..............}) + c$

(a) $\frac{1}{4} x^4$
(b) $x^3 + 3x^2$
(c) $x^3 - 3x^2 + 6x - 6$
(d) $\frac{1}{4} x^4 + x^3 + 3x^2$

110 $\int \frac{dx}{e^{x} + e^{-x} + 2} =$ ....

(a) $\frac{1}{e^{x}+1} + c$
(b) $-\frac{1}{e^{x}+1} + c$
(c) $\frac{2}{e^{x}+1} + c$
(d) $-\frac{2}{e^{x}+1} + c$

111 $\int x \cos x^2\, dx =$ ....

(a) $\frac{-1}{2} \sin^2 x + c$
(b) $\frac{1}{2} \sin^2 x + c$
(c) $-\frac{1}{2} \sin x^2 + c$
(d) $\frac{1}{2} \sin x^2 + c$

112 $\int \frac{dx}{1 - \sin x} =$ ....

(a) $\tan x - \sec x + c$
(b) $\tan x + \sec x + c$
(c) $\sec x - \tan x + c$
(d) $\csc x + c$

113 $\int x \cos x\, dx =$ ....

(a) $x \sin x + \cos x + c$
(b) $x \sin x - \cos x + c$
(c) $\sin x\, (x^{-1}) + c$
(d) $x \sin x + \sin x + c$

114 $\int \cos^3 x \sin^5 x\, dx =$ ....

(a) $\frac{1}{6} \sin^6 x - \frac{1}{8} \sin^8 x + c$
(b) $\sin^6 x - \sin^8 x + c$
(c) $\frac{1}{3} \sin^3 x - \frac{1}{6} \sin^6 x + c$
(d) $\frac{1}{4} \cos^4 x + \frac{1}{6} \sin^6 x + c$

115 $\int \frac{\sin(\ln x)}{x}\, dx = \dots\dots\dots$

(a) $-\cos x(\ln x) + c$ (b) $\cos(\ln x) + c$ (c) $-\cos(\ln x) + c$ (d) $\frac{1}{2}[\sin(\ln x)]^2 + c$

116 $\int e^x (\sin x + \cos x)\, dx = \dots\dots\dots + c$

(a) $e^x \sin x$ (b) $e^x \cos x$ (c) $-e^x \sin x$ (d) $e^{2x} \sin x$

117 $\int e^x (1 + \tan x + \tan^2 x)\, dx = \dots\dots\dots + c$

(a) $e^x \tan x + c$ (b) $e^x \sec x + c$ (c) $e^x \sin x + c$ (d) $e^x \cos x + c$

118 $\int e^x (1 - \cot x + \cot^2 x)\, dx = \dots\dots\dots + c$

(a) $e^x \cot x$ (b) $-e^x \cot x$ (c) $e^x \csc x$ (d) $-e^x \cos x$

119 $\int e^x \left(\frac{1 + x(\ln x)}{x}\right) dx = \dots\dots\dots$

(a) $\frac{e^x}{x} + c$ (b) $e^x \ln x + c$ (c) $x e^x \ln x + c$ (d) $\frac{1}{x} + \ln x + c$

120 $\int e^x \left(f'(x) + f''(x)\right) dx = e^x \times \dots\dots\dots$

(a) x (b) $f(x)$ (c) $f'(x)$ (d) $f''(x)$

121 If $\sin^2 x\, dy = y\, dx$, then $\dots\dots\dots$

(a) $|y| = e^{-\cot x + c}$ (b) $|y| = e^{\cos x + c}$ (c) $|y| = e^{\cot x + c}$ (d) $|y| = e^{\csc x + c}$

122 If $I_2 = \int x^2 e^x\, dx$, $I_1 = \int x e^x\, dx$, then $\dots\dots\dots$

(a) $I_2 = I_1$ (b) $I_2 - I_1 = x e^x$

(c) $I_2 + I_1 = x e^x$ (d) $I_2 + 2 I_1 = x^2 e^x + c$

123 $\int (1 + 4x^4) e^{x^4}\, dx = \dots\dots\dots + c$

(a) $x e^{x^4}$ (b) $\frac{4}{5} x e^{x^4}$ (c) $4x e^{x^4}$ (d) $\left(x + \frac{4}{5} x^5\right) e^{x^4}$

124 $\int e^{2x} \cos x\, dx = \dots\dots\dots + c$

(a) $\frac{1}{2} e^{2x} \sin x$ (b) $2 e^{2x} (\sin x + \cos x)$

(c) $\frac{1}{4} e^{2x} (\sin x + \cos x)$ (d) $\frac{1}{5} e^{2x} (\sin x + 2\cos x)$

125 $\int \tan^4 x \, d(\tan x) = \cdots\cdots\cdots + c$

(a) $\frac{1}{5}\tan^5 x$
(b) $\frac{1}{3}\tan^3 x - \tan x$
(c) $\frac{1}{5}\tan^5 x + \tan x$
(d) $\frac{1}{3}\tan^3 x - \tan x + x$

126 $\int \tan^4 x \, dx = \cdots\cdots\cdots + c$

(a) $\frac{1}{5}\tan^5 x$
(b) $\frac{1}{3}\tan^3 x - \tan x$
(c) $\frac{1}{5}\tan^5 x + \tan x$
(d) $\frac{1}{3}\tan^3 x - \tan x + x$

127 $\int \sin\left(\frac{\pi}{4} + 2x\right) \sin\left(\frac{\pi}{4} - 2x\right) dx = \cdots\cdots\cdots + c$

(a) $\frac{1}{8}\sin 4x$
(b) $\frac{1}{2}\cos 2x$
(c) $\frac{1}{4}\sin 2x$
(d) $-\frac{1}{4}\cos 4x$

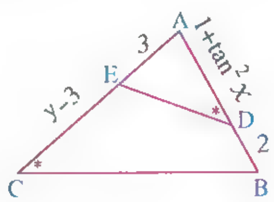
128 $\int \frac{x^{e-1} + e^{x-1}}{x^e + e^x} dx = \cdots\cdots\cdots + c$

(a) $\frac{1}{e}\ln|x^e + e^x|$
(b) $\ln|x^e + e^x|$
(c) $\ln|x^{e-1} + e^{x-1}|$
(d) $\ln|x + x^e|$

129 $\int \frac{1}{x^3}(\ln x^x)^2 dx = \cdots\cdots\cdots + c$

(a) $\frac{1}{3}x^3(\ln x) + x$
(b) $\frac{1}{3}(\ln x)^3$
(c) $3\ln|\ln x|$
(d) $\frac{1}{x}(\ln x)^2$

130 **In the opposite figure :**



If $m(\angle ADE) = m(\angle C)$

, then $\int y \, dx = \cdots\cdots\cdots + c$

(a) $\tan x + \frac{1}{3}\tan^3 x$
(b) $\tan x + \frac{1}{2}\tan^2 x$
(c) $\tan x + \frac{1}{9}\tan^3 x$
(d) $\frac{1}{3}\sec^3 x + \frac{1}{3}\tan^3 x$

Ninth Questions on the definite integration

Choose the correct answer from the given ones :

1. $\int_a^b f(x)\,dx = \ldots\ldots\ldots\ldots$
   (a) $f(b) - f(a)$ (b) $\int_b^a f(x)\,dx$ (c) $-\int_b^a f(x)\,dx$ (d) $b - a$

2. If f' is continuous on the interval $[a, b]$, then $f(a) + \int_a^b f'(a)\,dx = \ldots\ldots\ldots\ldots$
   (a) $f'(b)$ (b) $f(b)$ (c) $f(a)$ (d) $f'(a)$

3. $\int_a^b x^2\,dx + \int_b^a y^2\,dy = \ldots\ldots\ldots\ldots$ (where $a \neq b$)
   (a) $2\int_a^b x^2\,dx$ (b) zero (c) $2\int_b^a y^2\,dx$ (d) $b - a$

4. If $\int_2^4 f(x)\,dx = 7$, $\int_4^2 g(x)\,dx = 2$, then $\int_2^4 [2f(x) - 3g(x) - 5]\,dx$ equals $\ldots\ldots\ldots\ldots$
   (a) -18 (b) -8 (c) 10 (d) 14

5. If $\int_2^5 f(x)\,dx = 4$, then $\int_2^5 [3f(x) - 1]\,dx$ equals $\ldots\ldots\ldots\ldots$
   (a) 9 (b) 11 (c) 12 (d) -8

6. If f is continuous function on the interval $[2, 7]$, then $\int_2^7 f(x)\,dx + \int_7^4 f(x)\,dx = \ldots\ldots\ldots\ldots$
   (a) $\int_2^4 f(x)\,dx$ (b) zero
   (c) $\int_4^2 f(x)\,dx$ (d) $2\int_2^4 f(x)\,dx$

7. If f is continuous function on R, $\int_{-1}^3 f(x)\,dx = 7$, $\int_5^3 f(x)\,dx = -11$, then $\int_{-1}^5 f(x)\,dx$ equals $\ldots\ldots\ldots\ldots$
   (a) -4 (b) 18 (c) -18 (d) 77

8. If $\int_1^4 f(x)\,dx + \int_{2b}^8 f(x)\,dx = \int_1^8 f(x)\,dx$, then value of $b = \ldots\ldots\ldots\ldots$
   (a) 2 (b) 4 (c) 1 (d) 8

9 If f is an even function , and is continuous on the intervel $[-4, 4]$

$\int_{-4}^{4} f(x)\, dx = 20$, $\int_{0}^{2} f(x)\, dx = 6$, then : $\int_{-4}^{2} f(x)\, dx =$ ....

(a) 8 (b) 14 (c) 16 (d) 26

10 If $f(x)$ is an odd function and continuous of $\mathbb{R}$ and $\int_{-6}^{4} f(x)\, dx = -3$

, $\int_{0}^{6} f(x)\, dx = 11$, then $\int_{0}^{4} f(x)\, dx =$ ....

(a) 5 (b) 6 (c) 8 (d) 10

11 If $\int_{1}^{3} f(x)\, dx = 5$, $\int_{3}^{4} f(x)\, dx = 2$, $\int_{2}^{4} f(x)\, dx = 6$

, then $\int_{2}^{1} f(x)\, dx =$ ....

(a) 1 (b) 13 (c) -3 (d) -1

12 If $\int_{2}^{10} f(x)\, dx + \int_{8}^{11} f(x)\, dx + \int_{10}^{8} f(x)\, dx = 9$

, then $\int_{2}^{11} f(x)\, dx =$ ....

(a) 4.5 (b) 9 (c) 12 (d) 18

13 $\int_{-1}^{1} \frac{x^3}{x^4 + \cos x}\, dx =$ ....

(a) -1 (b) zero (c) 1 (d) 4

14 If $\int_{-2}^{2} (a x^3 + b x + c)\, dx$ depends on ....

(a) value of b (b) value of c (c) value of a (d) value of a , b

15 If $\int_{-2}^{2} f(x)\, dx =$ zero , then $f(x)$ may be ....

(a) $x^2 + 1$ (b) x (c) $x + 1$ (d) $x - 1$

16 $\frac{d}{dx}\left(\int_{2}^{3} x^2 \sqrt{x^2+1}\, dx\right) =$ ....

(a) -1 (b) zero (c) 1 (d) 2

17 $\int_{0}^{2} (2 - |x|)\, dx$ equals ....

(a) 4 (b) 2 (c) 1 (d) zero

18 $\int_{-8}^{8} \frac{1}{\sqrt{1+|x|}} \, dx = \dots\dots$

(a) 6 (b) -8 (c) -6 (d) 8

19 $\int_{-\pi}^{\pi} (4 + \pi \cos 2x) \, dx = \dots\dots$

(a) π (b) 2π (c) 4π (d) 8π

20 $\int_{0}^{1} 2 \sin \pi z \, dz = \dots\dots$

(a) 4π (b) $\frac{2}{\pi}$ (c) $\frac{4}{\pi}$ (d) 8π

21 $\int_{0}^{\frac{\pi}{2}} \cos^2 x \, dx = \dots\dots$

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) zero

22 $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos x}{\sin^2 x} \, dx = \dots\dots$

(a) 0 (b) 1 (c) 2 (d) -1

23 $\int_{0}^{1} x e^{x^2} \, dx = \dots\dots$

(a) $\frac{e+1}{2}$ (b) $\frac{e-1}{2}$ (c) $\frac{e}{2}$ (d) $\frac{1}{2}$

24 $\int_{0}^{\frac{\pi}{6}} \sin^4 x \, dx + \int_{\frac{\pi}{6}}^{0} \cos^4 x \, dx = \dots\dots$

(a) $\frac{\sqrt{3}}{4}$ (b) $\frac{-\sqrt{3}}{4}$ (c) $\frac{1}{4}$ (d) $\frac{-1}{4}$

25 $\int_{e}^{e^2} \frac{dx}{x} = \dots\dots$

(a) 1 (b) 2 (c) e (d) e^2

26 $\int_{2}^{3} x (x-3)^5 \, dx = \dots\dots$

(a) $\frac{-1}{6}$ (b) $\frac{-5}{14}$ (c) $\frac{-2403}{308}$ (d) $\frac{-729}{14}$

27 $\int_{0}^{6} |3 - x| \, dx = \dots\dots$

(a) 9 (b) -9 (c) $\frac{9}{2}$ (d) $\frac{-9}{2}$

28 $\int_{-3}^{3} |x^2 - 4| \, dx = \dots\dots$

(a) $7\frac{2}{3}$ (b) zero (c) $15\frac{1}{3}$ (d) -6

29 $\int_{1}^{3} (x-1)\,|x-2|\,dx =$ ....

(a) $\frac{2}{3}$ (b) 1 (c) -1 (d) $\frac{-2}{3}$

30 If $\int_{2}^{a} (4x-1)\,dx = 9$, $a \in \mathbb{Z}^+$, then $a =$ ....

(a) 3 (b) 4 (c) 1 (d) -3

31 If $\int_{k}^{3} 2x\,dx = 5$, then $k =$ ....

(a) 13 (b) 5 (c) 1 (d) ± 2

32 If $a < 2 < b$, and $\int_{a}^{b} |x-2|\,dx = 4$, then $\frac{a^2+b^2}{a+b} =$ ....

(a) $\frac{1}{2}$ (b) 4 (c) 2 (d) -4

33 If $\int_{\ln b}^{\ln a} e^{x}\,dx = 2$, $a^2 - b^2 = 12$, then $a =$ ....

(a) 8 (b) 12 (c) 4 (d) 6

34 $\int_{0}^{\ln 3} (e^{2x} + e^{x})\,dx =$ ....

(a) 12 (b) 6 (c) $7\frac{1}{2}$ (d) $5\frac{1}{2}$

35 $\int_{1}^{e} (\ln x)\,dx =$ ....

(a) $\frac{1}{e}$ (b) e (c) 1 (d) -1

36 $\int_{0}^{20\pi} |\sin x|\,dx =$ ....

(a) 20 (b) 20π (c) 40 (d) 40π

37 $\int_{0}^{2} \sqrt{4-x^2}\,dx =$ ....

(a) zero (b) 2 (c) π (d) $\frac{\pi}{2}$

38 If $f(x) = \begin{cases} 2x-1 & -1 \le x \le 2 \\ 3 & 2 < x < 5 \end{cases}$, then $\int_{-1}^{4} f(x)\,dx =$ ....

(a) 4 (b) 5 (c) 6 (d) 7

39 If $f(x) = \begin{cases} |x-1| & x \le 1 \\ x^2-1 & x > 1 \end{cases}$, then $\int_{-2}^{2} f(x)\,dx =$ ....

(a) $\frac{3}{2}$ (b) $\frac{-35}{6}$ (c) $\frac{35}{6}$ (d) $\frac{-3}{2}$

40 If $\int_{0}^{a} |x| \, dx = 32$, then a = ...

(a) 8 (b) -8 (c) ± 8 (d) zero

41 If $\int_{-1}^{1} \sqrt{16 - 16x^2} \, dx =$ .... square units

(a) 16π (b) π (c) 2π (d) 4π

42 $\int_{0}^{4} 4^{\log_2 x} \, dx =$ ....

(a) $\frac{64}{3}$ (b) $\frac{16}{3}$ (c) 4 (d) $\frac{11}{3}$

43 If $\int_{k}^{k+1} (9)^{\log_3 \sqrt{x}} \, dx = \frac{5}{2}$, then k = ...

(a) zero (b) 1 (c) 2 (d) 3

44 $\int_{\frac{1}{\pi}}^{\frac{2}{\pi}} \frac{\sin(\frac{1}{x})}{x^2} \, dx =$ ....

(a) -2 (b) -1 (c) zero (d) 1

45 If $\int_{0}^{\pi} \frac{\cos x}{1 + x^{10}} \, dx = m$, then $\int_{-\pi}^{\pi} \frac{3\cos x}{1 + x^{10}} \, dx =$ ....

(a) 2 m (b) 3 m (c) 6 m (d) 12 m

46 $\int_{e}^{e3} \frac{(\ln x)^3}{x} \, dx =$ ....

(a) 20 (b) 16 (c) 12 (d) 10

47 If $\int_{-2}^{2} (x^7 + k) = 16$, then k = ...

(a) -12 (b) -4 (c) zero (d) 4

48 If $f(x)$, $f'(x)$ two continuous functions and if $f(6) = 4$, $f'(6) = 3$, $f(7) = 14$, $f'(7) = 5$, then $\int_{6}^{7} f'(x) \, dx =$ ....

(a) 10 (b) 2 (c) 18 (d) 8

49 $\int_{-1}^{0} e^{-x} \, dx =$ ....

(a) $\frac{1}{e} - 1$ (b) $1 - e$ (c) $e - 1$ (d) $1 - \frac{1}{e}$

50 If the function f is continuous , $f(5) = 9$, $f(1) = 4$, then $\int_1^5 3\sqrt{f(x)}\, f'(x)\, dx =$ ....

(a) 19 (b) 38 (c) $10\sqrt{3} - 2$ (d) 50

51 If $\int_0^{\pi} x^2 \cos x\, dx =$ ....

(a) $-\pi$ (b) -2π (c) π (d) 2π

52 If $\int_0^{\theta} \frac{\sec^2 x}{1 + \tan x}\, dx = \ln 2$ where $0 < \theta < \frac{\pi}{2}$, then $\theta =$ ....

(a) $\frac{\pi}{12}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

53 If $\left(\int_0^a x\, dx\right)^3 = \int_0^a x^3\, dx$, $a \in \mathbb{R}^+$, then $a =$ ....

(a) 2 (b) $\sqrt{2}$ (c) 3 (d) $\sqrt{3}$

54 If $\int_0^k (3x^2 - 1)\, dx = k^3 - 2$, then $k =$ ....

(a) 6 (b) 4 (c) 2 (d) 1

55 If $\int_{-\frac{\pi}{2}}^{a} \sin^3 x\, dx =$ zero , then $a =$ ....

(a) zero (b) π (c) $-\pi$ (d) $\frac{\pi}{2}$

56 If $\int_{2k-3}^{k+2} (x^6 + x^4 - 2)\, dx =$ zero , then $k =$ ....

(a) 2 (b) 3 (c) 4 (d) 5

57 If $\int_{-1}^{a} f(z)\, dz = 4a^3 - 2a + c$, then $c =$ ....

(a) -1 (b) 1 (c) 2 (d) -2

58 If $m , n \in \mathbb{R}$ and $\int_0^1 (m x + n x)\, dx = 15$, then $\int_0^1 (m x^2 + n x^2)\, dx =$ ....

(a) 10 (b) 15 (c) 20 (d) 30

59 If f is a differentiable function on $\mathbb{R}$ and $f(1) = 4$, $f(3) = 10$

, then $\int_1^3 [f(x) + x f'(x)]\, dx =$ ....

(a) 22 (b) 24 (c) 26 (d) 28

60 If f is an even function , $\int_{0}^{1} f(x)\,dx = 7$, then $\int_{1}^{1} [f'(x) - f(x)]\,dx = \dots\dots$

(a) zero (b) -7 (c) -14 (d) 7

61 $\int_{\pi}^{2\pi} \sin^2 x\,dx + \int_{2\pi}^{\pi} (3 - \cos^2 x)\,dx = \dots\dots$

(a) 2π (b) π (c) $-\pi$ (d) -2π

62 If f is an even function and $\int_{-5}^{3} f(x)\,dx = 10$, $\int_{0}^{3} f(x)\,dx = 8$, then $\dots\dots$

(a) $\int_{0}^{5} f(x)\,dx = 2$ (b) $\int_{0}^{5} f(x)\,dx = -2$

(c) $\int_{-5}^{3} f(x)\,dx = 26$ (d) $\int_{-5}^{5} f(x)\,dx = 18$

63 $\int_{2}^{5} \frac{1}{x+2}\,dx = \dots\dots$

(a) $\int_{0}^{3} \frac{1}{x+4}\,dx$ (b) $\int_{-1}^{2} \frac{1}{x+5}\,dx$

(c) $\int_{3}^{6} \frac{1}{x+1}\,dx$ (d) All the previous

64 $\int_{0}^{2} (x+6)^2 e^{x^2}\,dx - \int_{0}^{2} (x-6)^2 e^{x^2}\,dx = \dots\dots$

(a) $12\,e^2$ (b) $12\,e^4$ (c) $12\,(e^4 - 1)$ (d) $24\,(e^4 - 1)$

65 $\int_{\sin\theta}^{\cos\theta} \frac{x}{2}\,dx = \frac{-\sqrt{3}}{8}$, then one of the values of θ equal $\dots\dots$

(a) 30° (b) 45° (c) 60° (d) 75°

66 $\int_{2019}^{2021} (x - 2020)^2\,dx = \dots\dots$

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) 1 (d) $\frac{4}{3}$

67 If $\int_{3}^{9} f(x)\,dx = 20$, $\int_{5}^{11} f(x)\,dx = 25$, then $\int_{3}^{5} f(x)\,dx - \int_{9}^{11} f(x)\,dx = \dots\dots$

(a) -10 (b) -5 (c) zero (d) 5

68 If $\int_{a}^{b} (3x^2 + 2)\,dx = 96$, $\int_{a}^{b} dx = 4$, then $a \times b = \dots\dots$

(a) 1 (b) 1.5 (c) 2 (d) 2.5

69 If $f(x) = \frac{3x^{2021} + 1}{x^{2023} + 1}$, then $\int_{0}^{1} \acute{f}(x)\,dx = \dots\dots$

(a) zero (b) 1 (c) 2 (d) 3

70 $\lim\limits_{h \to 0} \left[\frac{1}{h} \int_{3}^{3+h} \sqrt{x^2 + 16}\, d\,x\right] = \dots\dots\dots\dots$

(a) 3 (b) 5 (c) 9 (d) 16

71 If $\int_{0}^{a} (3\,x^2 - 2\,x)\, d\,x \le 2\,a$ where $a \in \mathbb{R}^+$, then $a \in \dots\dots\dots\dots$

(a) $[0\ ,2]$ (b) $]0\ ,2]$ (c) $]2\ ,\infty[$ (d) $]1\ ,\infty[$

72 If f is a function where $\int_{6}^{12} f\,(2\,x)\, d\,x = 10$

, then which of the following is correct ?

(a) $\int_{12}^{24} f\,(t)\, d\,t = 5$ (b) $\int_{12}^{24} f\,(t)\, d\,t = 20$

(c) $\int_{6}^{12} f\,(t)\, d\,t = 5$ (d) $\int_{6}^{12} f\,(t)\, d\,t = 20$

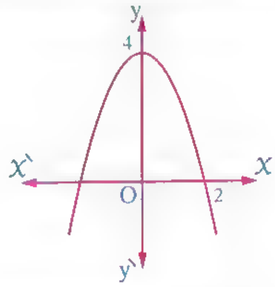
73 **In the opposite figure :**

the curve of the function $y = f\,(x)$

, then $\int_{0}^{2} f'\,(x)\, d\,x = \dots\dots\dots\dots$

(a) 12 (b) -2

(c) -4 (d) 4

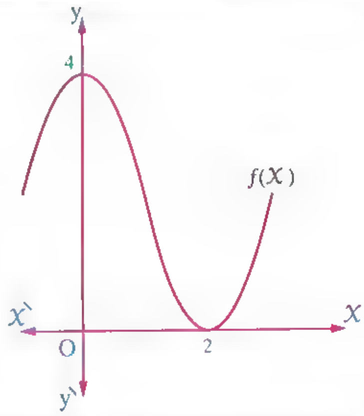


74 **In the opposite figure :**

$\int_{0}^{2} [f\,(x)]^2 f'\,(x)\, d\,x = \dots\dots\dots\dots$

(a) $-\frac{64}{3}$ (b) $\frac{64}{3}$

(c) $\frac{8}{3}$ (d) -64



75 **In the opposite figure :**

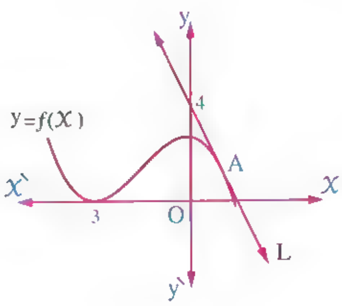
The straight line L is a tangent to the curve

$y = f\,(x)$ at the point A (1 , 2)

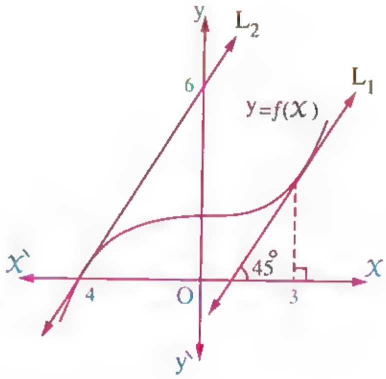
, then $\int_{-3}^{1} f'\,(x)\, d\,x = \dots\dots\dots\dots$

(a) -3 (b) 2

(c) -1 (d) 1



76 **In the opposite figure :**



$\int_{-4}^{3} \frac{f'(X)}{f(X)} \, dX = \dots\dots\dots$

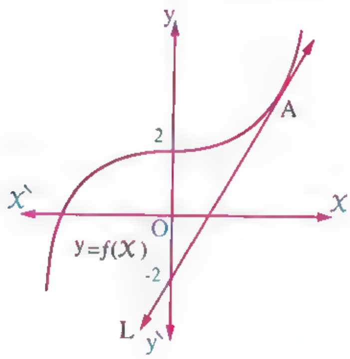
(a) 1

(b) $\ln \frac{3}{2}$

(c) $\ln \frac{2}{3}$

(d) $\log_3 2$

77 **In the opposite figure :**



If L is the tangent to the curve $y = f(X)$ at the point A (3 , 3) , then

$\int_{0}^{3} X f''(X) \, dX = \dots\dots\dots$

(a) -2

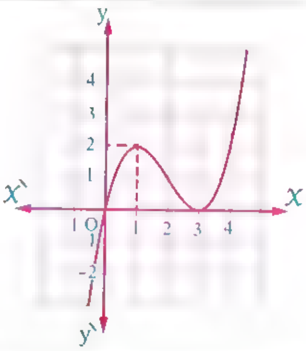
(b) 2

(c) 4

(d) 8

78 The opposite figure represents the curve of the function $y = f(X)$ and $g(X) = X \cdot f(X)$

, then $\int_{1}^{3} g''(X) \, dX = \dots\dots\dots$



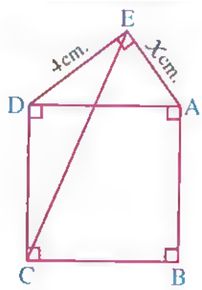
(a) -1

(b) -2

(c) -3

(d) -4

79 **In the opposite figure :**



If ABCD is a square

, m (∠ AED) = 90°

, then $\int_{0}^{1} (EC)^2 \, dX = \dots\dots\dots$

(a) $\frac{82}{3}$

(b) $\frac{91}{3}$

(c) $\frac{100}{3}$

(d) $\frac{109}{3}$

Tenth Questions on the applications of integration

Choose the correct answer from the given once :

1 If $\acute{f}(x) = 1 - \sin 2x$, $f(0) = \frac{1}{2}$, then $f\left(\frac{\pi}{2}\right) = \cdots\cdots$

(a) $\frac{\pi}{2} - \frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 1 (d) $\frac{\pi}{2}$

2 If $\frac{dy}{dx} = \csc^2 x$, $y = 2$ at $x = \frac{\pi}{4}$, then $y = \cdots\cdots$

(a) $-2 - \cot x$ (b) $2 - \cot x$ (c) $-3 - \cot x$ (d) $3 - \cot x$

3 If $\frac{dy}{dx} = x + \frac{1}{x}$, $y = \frac{1}{2}$ at $x = 1$, then $y = \cdots\cdots$ when $x = e$

(a) $\frac{1}{2} e^2 + 1$ (b) $e + \frac{1}{e}$ (c) e^2 (d) $e^2 + 1$

4 If $f(x) = \int \frac{dx}{\sqrt{2x+1}}$, $f(4) = 7$, then $f(x) = \cdots\cdots$

(a) $\sqrt{2x+1}$ (b) $2\sqrt{2x+1} + 6$ (c) $\frac{1}{2}\sqrt{2x+1}$ (d) $\sqrt{2x+1} + 4$

5 If $f(x) = \int (x+1)(2x^2 + 4x - 1)\, dx$, $f(-2) = 1$, then $f(3) = \cdots\cdots$

(a) 54 (b) 86 (c) 98 (d) 106

6 $\int f(x)\, dx = x^3 - x^2$, then $\acute{f}(1) - f(1) = \cdots\cdots$

(a) 4 (b) 3 (c) zero (d) 2

7 If $\int \frac{f(x)}{x}\, dx = \ln|x| + x^2 + c$, then $f(x) = \cdots\cdots$

(a) $x^2 + x + 1$ (b) $x^3 + x^2 + 1$ (c) $2x^2 + 1$ (d) $x^2 + 1$

8 The equation of the curve which intercpted 7 units , from negative direction of y-axis and slope of its tangent at any point on it $= 3x^2 + 2$, is $\cdots\cdots$

(a) $y = x^3$ (b) $y - x^3 - 2x = 0$

(c) $y = x^3 - 7$ (d) $y = x^3 + 2x - 7$

9 If $f(x) = \sin x + \cos x$, then $\acute{f}(x) + \int f(x)\, dx = \cdots\cdots$

(a) zero (b) constant (c) $2\sin x$ (d) $2\cos x$

10 If $f(x) = \int \frac{1}{x}\, dx$, then $f''(2) = \dots\dots$

(a) does not exist (b) $\frac{1}{x} + c$ (c) 2 (d) $\frac{1}{2}$

11 If $f''(x) = 6x - 4$, $f'(1) = 2$, $f(0) = -4$, then $f(x) = \dots\dots$

(a) $-2x^2 + 3x - 4$ (b) $x^3 - 2x^2 + 3x$

(c) $3x^2 - 4x - 4$ (d) $x^3 - 2x^2 + 3x - 4$

12 If $f''(x) = \frac{1}{2}[e^x + e^{-x}]$, $f(0) = 1$, $f'(0) = 0$, then $f(x)$ equals $\dots\dots$

(a) $-f'(x)$ (b) $f'(x)$ (c) $-f''(x)$ (d) $f''(x)$

13 If the slope of the tangent to a curve at any point on it (x , y) equals $4e^{2x}$, $f(0) = 2$, then $f(-2) = \dots\dots$

(a) 4 (b) $4e^{-4}$ (c) $2e^{-4}$ (d) $2e$

14 If the slope of the tangent to the curve : $y = f(x)$ at any point on it equals $6x + a$, where a is constant , if the equation of the tangent to the curve at the point $(1 , -1)$ is $y = 4 - 5x$, then the equation of the curve is $\dots\dots$

(a) $y = 3x^2 - 4$ (b) $y = 3x^2 - 11x + 7$

(c) $y = 3x^2 - 4x + 7$ (d) $y = 3x^2 - 11x - 4$

15 If slope of the tangent to the curve $y = f(x)$ at any point on it equals $\sec^2 x - \sin x$, the curve passes through the point $\left(\frac{\pi}{4} , \frac{1}{\sqrt{2}}\right)$, then the equation is $\dots\dots$

(a) $y = \frac{1}{3}\sec^3 x + \cos x - 1$ (b) $y = \tan x - \cos x - 1$

(c) $y = \tan x + \cos x - 1$ (d) $y = \tan x + \cos x$

16 If the slope of the tangent to the curve of the function f at any point on it (x , y) , equals $\frac{5x + 3}{x}$, and the curve passes through the point $(e , 5e + 7)$, then the equation of the curve is $\dots\dots$

(a) $y = 5x + 3\ln|x| + 4$ (b) $y = 5x + \ln|x| + 6$

(c) $y = 5x + 3\ln|x| - 3$ (d) $y = x + 7\ln|x| + 4e$

17 Find the equation of the curve passes through the point (1 , 0) and the slope of its tangent at any point on it equals $x\,e^{y}$ is ....

(a) $\frac{1}{2}x^2 + e^y = 1\frac{1}{2}$
(b) $e^{-y} + \frac{1}{3}x^3 = 1\frac{1}{3}$
(c) $e^{-y} + \frac{1}{2}x^2 = 1\frac{1}{2}$
(d) $x + e^y = 2$

18 If the slope of the tangent to the curve of the function $y = f(x)$ at any point on it equals $\frac{1}{x\sqrt{3 + \ln x}}$ where $x > 0$ and the curve passes through (e , 4) , then the relation between x , y is ....

(a) $y = \sqrt{3 + \ln x}$
(b) $y = 2\sqrt{3 + \ln x}$
(c) $y = 3 + \ln x$
(d) $y = 2\sqrt{3 + \ln x} - 5$

19 If the slope of the tangent to the curve of the function f at any point (x , y) lying on it is given by the relation $g(x) = x\,e^{-x}$ and the curve passes through the point (–1 , 3) , then the equation of the curve is ....

(a) $f(x) = -e^{-x}(x + 1) + 3$
(b) $f(x) = -x\,e^{-x} + x - e$
(c) $f(x) = -e^{x}(x^2 + 1)$
(d) $f(x) = e^{-x}(x + 1) + 3$

20 If the slope of the tangent to a curve at any point on it (x , y) equals $x\left(\sqrt{x + 1}\right)$ and the curve passes through $\left(0 , \frac{11}{15}\right)$, then the equation of the curve is y = ...

(a) $\frac{x}{\sqrt{x + 1}} + \frac{11}{15}$
(b) $2\sqrt{x + 1} - \frac{19}{15}$
(c) $\frac{2}{5}(x + 1)^{\frac{5}{2}} - \frac{2}{3}(x + 1)^{\frac{3}{2}} + 1$
(d) $\frac{2}{5}(x + 1)^{\frac{5}{2}} + \frac{1}{3}$

21 The slope of the tangent to the curve of a function f equals $\frac{1}{x - 2}$ and the curve passes through the point (3 , 0) , then $f(e^2 + 3) =$ ....

(a) $e^2 + 1$
(b) $\ln(e^2 + 1)$
(c) $\frac{1}{e^2 + 1}$
(d) 2

22 The slope of the normal to the curve at any point on it (x , y) equals $3 - 2x$, then the equation of the curve given it passes through the point (1 , 1) is ....

(a) $y = \frac{1}{2}\ln|2x - 3| + 2$
(b) $y = \frac{1}{2}\ln|2x - 3| + 1$
(c) $y = \ln|2x - 3| + 4$
(d) $y = \ln|2x - 3| + 1$

23 If the slope of the tangent to the curve of the function f at any point (x , y) lying on it is given by the relation $g(x) = \frac{x e^{x}}{(x+1)^2}$, then the equation of the curve if it passes through the point $(1 , 2e)$ is ....

(a) $y = \frac{-x e^{x}}{x+1} + \frac{1}{2} e$

(b) $y = \frac{-x e^{x}}{x+1} + e^{x} + \frac{3}{2} e$

(c) $y = \frac{x e^{x}}{x+1} + e^{x} - \frac{3}{2} e$

(d) $y = \frac{x e^{x}}{x+1} - \frac{1}{2} e$

24 If the slope of the tangent to the curve : $y = f(x)$ at any point on it (x , y) equals : $\frac{3+4x}{6y}$, then the equation of the curve , if we know that it passes through the point $(1,1)$ is ....

(a) $3y^2 = 3x + 2x^2$

(b) $y^2 = 2x^2$

(c) $3y^2 = 3x - 2$

(d) $3y^2 = 3x + 2x^2 - 2$

25 If the slope of the normal to the curve $y = f(x)$ at any point on it is $(2y + 1) \csc x$ if we know that the curve passes through the origin point , then its equation is ....

(a) $y^2 + y = \sin x - 1$

(b) $y^2 + y = \cos x - 1$

(c) $y^2 + y \csc y \cot y = 0$

(d) $y^2 + y = (\sin x)^{-2}$

26 The slope of the tangent at any point (x , y) on the curve $y = f(x)$ is equal to $3x^2 - 6x - 9$ and the local maximum value of the function f is 17 , then the local minimum value of the function f equal ....

(a) -17 (b) -15 (c) 7 (d) 15

27 If $y = f(x)$, $\frac{d^2 y}{d x^2} = a x + b$ where a and b are two constants and the curve has an inflection point at the point $(0 , 2)$ and a local minimum value at the point $(1 , 0)$, then the local maximum value to this curve = ...

(a) 3 (b) 4 (c) 5 (d) 6

28 If $y = f(x)$, and $\frac{d^2 y}{d x^2} = \frac{2}{x^3}$ and the equation of its tangent at the point $\left(2 , \frac{5}{2}\right)$ which lies on the curve is : $3x - 4y + 4 = 0$, then the equation of the curve is ....

(a) $y = x^2 + 2$ (b) $y = \frac{1}{x} + x$ (c) $y = \frac{6}{x} + x$ (d) $y = x - \frac{1}{x}$

29 If $y = f(x)$, and : $\frac{d^2 y}{d x^2} = 6(1 - x)$ and the curve has a local minimum value at the point $(0 , -6)$, then the equation of the curve is ....

(a) $y = x^3 - 3x^2$
(b) $y = 6x - 3x^2 - 6$
(c) $y = 3x^2 - x^3 - 6$
(d) $y = x^3 + 6x^2 - 6$

30 If the slope of the tangent at any point (x , y) on the curve of the function f is inversely proportional to x and the slope of the tangent equals 2 when $x = 4$ and $y = 2$, then y = ...

(a) $8 \ln|x| + 2$
(b) $8 \ln|x| + 2 - 8 \ln 4$
(c) $x^{-2} + \frac{1}{32}$
(d) $4 \ln|x| - 2$

31 If the rate of change of the slope of the tangent to a curve at any point on it is equal to $6x - 2$ and the slope of the tangent at the point $(3 , 1)$ that lies on the curve equals 2 , then the equation of this curve is ....

(a) $y = x^3 - x^2 - 18$
(b) $y = x^3 - x^2 - 2x - 12$
(c) $y = 3x^2 - 2x - 19$
(d) $y = x^3 - x^2 - 19x + 40$

32 The Capacity of an empty vessel is 1400 cm^3 , water is poured in it at a rate $(2t + 50)$ cm^3/ sec. where t is the time in seconds , then need the time to fill the vessel = ... sec.

(a) 28
(b) 20
(c) 70
(d) 700

33 A liquid is leaking from a small hole in the bottom of a vessel filled with the liquid. If the volume of the liquid changes at the rate of $(0.4t - 40)$ cm^3/sec. , where t represents the time in seconds and the volume of the liquid is 980 cm^3 after 30 seconds from the start of leaking , then the capacity of the vessel = ... cm^3

(a) 1000
(b) 2 000
(c) 3 000
(d) 4 000

34 If the rate of change of area of a lamina is a (in square centimeters) , with respect to t (in seconds) by the relation $\frac{dA}{dt} = e^{-0.2t}$, if the area of the lamina at the begining of changing is 140 cm^2 , then area of the lamina after $\frac{1}{3}$ minutes equal .... cm^2

(a) 145
(b) $145 - 5e^{-4}$
(c) $145 - e^{-\frac{1}{5}}$
(d) $145 - e^{-\frac{1}{15}}$

Eleventh Questions on the areas and the volumes of revolution solids

Choose the correct answer from the given once :

1. The area bounded by the straight line : $y = x$, $x = 1$, $y = \text{zero}$ equals ....

(a) $\frac{1}{2}$ (b) 2 (c) 1 (d) $\frac{1}{4}$

2. The area of the planar region bounded by the curve : $y = x^2$ and the two straight line $y = 0$, $x = 3$ equals ....

(a) 6 (b) 7 (c) 8 (d) 9

3. The area of planar region bounded by the curve : $y = x^3$ and the straight lines : $x = -1$, $x = 1$, $y = 0$ equals = ...

(a) zero (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) 6

4. The area of the planar region bounded by the curve : $y = x^2 + 4$ and x-axis and the two straight lines $x = -1$, $x = 2$ equals ....

(a) 15 (b) 9 (c) $14\frac{1}{3}$ (d) $12\frac{1}{3}$

5. The area of the region bounded by the curve : $y = 2x - x^2$ and x-axis equals ....

(a) $\frac{8}{3}$ (b) $\frac{4}{3}$ (c) $\frac{7}{3}$ (d) $\frac{3}{4}$

6. The area of the planar region bounded by the two curves : $y = x^2$, $y = x^3$ is .... square unit.

(a) 1 (b) $\frac{7}{12}$ (c) $\frac{1}{12}$ (d) 2

7. The area of the planar region bounded by the two curves : $y^2 = x$, $y = x^3$ equals ....

(a) $\frac{5}{12}$ (b) $\frac{5}{6}$ (c) $\frac{12}{5}$ (d) $\frac{6}{5}$

8. The area of the planar region bounded by the two curves : $y^3 = x$, $y = x$ equals ....

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{-3}{4}$ (d) $\frac{-1}{2}$

9. The area of the planar region bound by the curves : $y = x^2 - 2x + 1$, $y = x + 1$ equals ....

ⓐ $\frac{-9}{2}$ ⓑ $\frac{9}{2}$ ⓒ $\frac{3}{2}$ ⓓ $\frac{-3}{2}$

10. The area of the planar region bounded by the curve : $y = \sqrt{x - 1}$ and the straight line $y = x - 3$ and x-axis equals ....

ⓐ $3\frac{1}{3}$ ⓑ zero ⓒ $5\frac{1}{3}$ ⓓ 2

11. If f is a continuous function on the interval $[a , b]$ and A is the area bounded by the curve of the function $y = f(x)$, the x-axis and the two straight lines $x = a$, $x = b$, then A = ...

ⓐ $\int_a^b |y| \, dx$ ⓑ $\left| \int_a^b y \, dx \right|$ ⓒ $\int_a^b y \, dx$ ⓓ $\int_a^b |x| \, dy$

12. The area of the region bounded by the curve $xy = 4$ and x-axis and the two straight lines $x = 1$, $x = 3$ equals ....

ⓐ 2 ln 3 ⓑ 4 ln 3 ⓒ 3 ln 3 ⓓ 3 ln 4

13. If the area bounded by the curve $y = x^3$ and the two straight lines $y = 0$, $x = a$ where $a \in \mathbb{R}^+$ equals 4 square units , then a = ...

ⓐ 8 ⓑ 4 ⓒ 2 ⓓ 1

14. The area of the region bounded by the curve whose parametric equations $y = 3t^2$, $x = 6t$ and the x-axis and the two straight lines $x = 0$, $x = 12$ equals .... square units.

ⓐ 48 ⓑ 96 ⓒ 132 ⓓ 192

15. The area of the region bounded by the curve $y = \sqrt{4 - x^2}$ and x-axis estimated by square units equals ....

ⓐ 2 ⓑ y ⓒ 2π ⓓ 4π

16. An architect has designed an arc -like entryway of a hotel whose equation $y = -\frac{1}{2}(x - 1)(x - 7)$ where x in metres. How much does the glass cost if this entryway is covered by the glass which costs L.E. 1 500 per square metre = L.E. ....

ⓐ 9 000 ⓑ 27 000 ⓒ 54 000 ⓓ 63 000

17 An advertising company produces a poster to market an item. If the poster is shaped as an area bounded by the curve of the two functions f and g where $f(X) = 2X^2$ and $g(X) = X^4 - 2X^2$, then the area needed of adhesive paper to produce 1 000 posters for this item = ... square units.

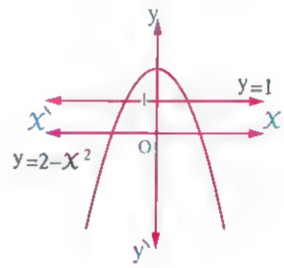
(a) $\frac{25\,600}{9}$ (b) $\frac{12\,800}{3}$ (c) $\frac{25\,600}{3}$ (d) $\frac{12\,800}{9}$

18 **In the opposite figure :**

The area of the shaded region = ... square units.

(a) $\frac{2}{3}$ (b) $\frac{4}{3}$

(c) $\frac{5}{3}$ (d) 2

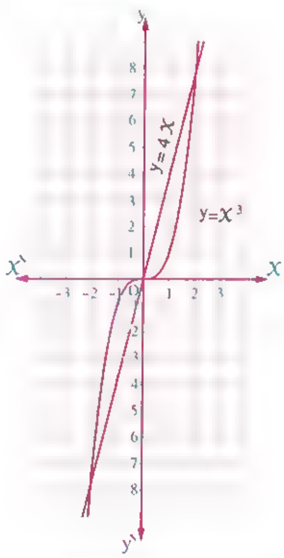


19 **In the opposite figure :**

The area of the shaded region = ... square units.

(a) 4 (b) 8

(c) 12 (d) 16

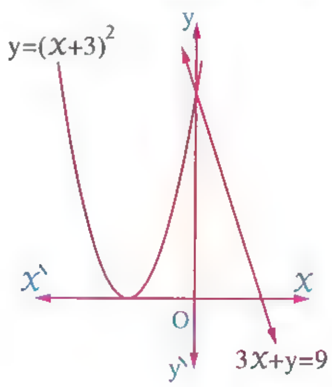


20 **In the opposite figure :**

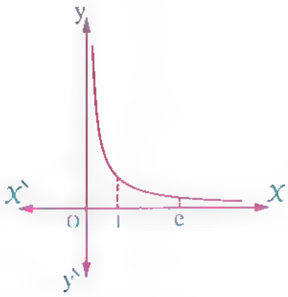
The area of the shaded region = ... square units.

(a) 20 (b) $\frac{45}{2}$

(c) 25 (d) $\frac{55}{2}$



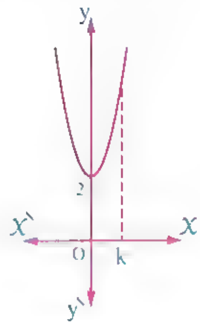
21 **In the opposite figure :**



If $f(X) = \frac{1}{X}$, then the area of the shaded region = ... square units.

(a) 1
(b) e
(c) e^2
(d) 2

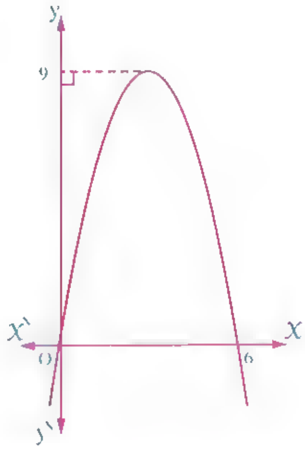
22 **In the opposite figure :**



If $f(X) = 3X^2 + 2$ and the area of the shaded region = 33 square units , then k = ...

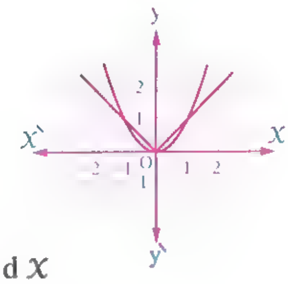
(a) 1
(b) 2
(c) 3
(d) 4

23 The opposite figure represents a quadratic function , its vertex is (k , 9) , then the area of the shaded region = ... square units.



(a) 6
(b) 9
(c) 12
(d) 18

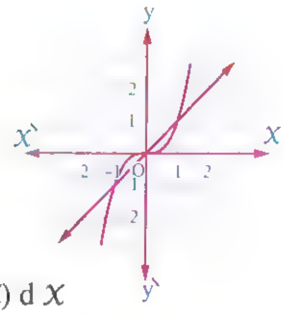
24 **In the opposite figure :**



The area of the region bounded by the two curves : $y = X^2$ and $y = |X|$ equals = ...

(a) $2 \int_{-1}^{0} (X^2 - X)\, dX$
(b) $\int_{0}^{1} (X - X^2)\, dX$
(c) $2 \int_{0}^{1} (X - X^2)\, dX$
(d) $\int_{-1}^{1} (X - X^2)\, dX$

25 **In the opposite figure :**



The area of the region bounded by

the curve $y = x^2$ and the straight line $y = x$

equals ....

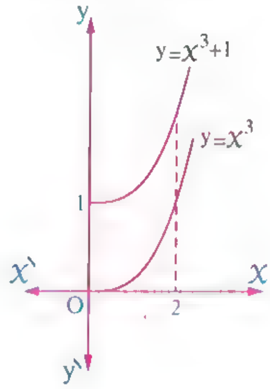
(a) $_{-1}\int^{1} (x^2 - x)\, dx$

(b) $2\,_{0}\int^{1} (x^3 - x)\, dx$

(c) $_{0}\int^{1} (x - x^3)\, dx$

(d) $2\,_{0}\int^{1} (x - x^3)\, dx$

26 **In the opposite figure :**



The area of the shaded region = ... square units.

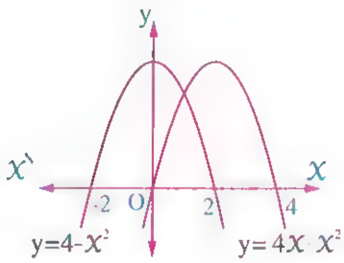
(a) 1

(b) $\frac{1}{2}$

(c) 2

(d) $\frac{3}{2}$

27 **In the opposite figure :**



The area of the shaded region = ... square units.

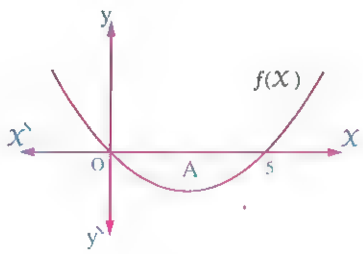
(a) 2

(b) $\frac{7}{3}$

(c) 3

(d) $\frac{10}{3}$

28 **In the opposite figure :**



If the region A between the curve $f(x)$ and the x-axis

equals 8 square units , then $_{0}\int^{5} \left(1 - f(x)\right) dx =$ ....

(a) 12

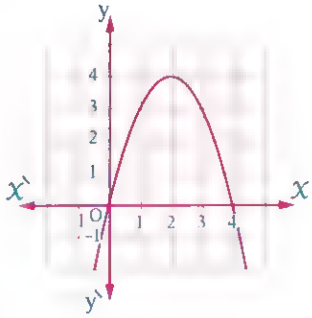
(b) 13

(c) 14

(d) 15

29 The opposite figure represents the curve of the function $f : f(x) = -x(x-4)$

, then all of the following are true except ……………



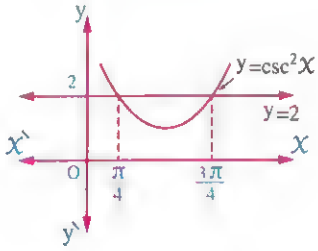
(a) $\int_0^1 f(x)\,dx = \int_3^4 f(x)\,dx$

(b) $\int_0^4 f(x)\,dx = 2\int_0^2 f(x)\,dx$

(c) $\int_{-1}^0 |f(x)|\,dx = \int_0^1 f(x)\,dx$

(d) $\int_{-1}^0 f(x)\,dx = \int_4^5 f(x)\,dx$

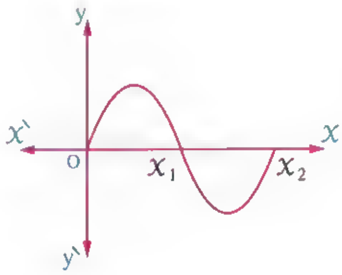
30 The shaded area in the figure is equal to ……………



(a) $\pi + 2$ (b) $\pi - 2$

(c) 2 (d) π

31 The opposite figure represents the curve of the function f , then the area included between the curve of the function f and x-axis equals ……………



(a) $\int_0^{x_2} f(x)\,dx$

(b) $\int_0^{x_1} f(x)\,dx + \int_{x_1}^{x_2} f(x)\,dx$

(c) $\int_0^{x_1} f(x)\,dx - \int_{x_1}^{x_2} f(x)\,dx$

(d) $\left|\int_0^{x_2} f(x)\,dx\right|$

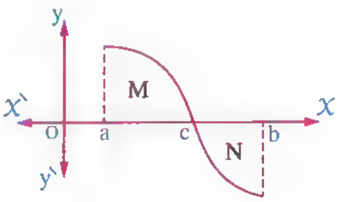
32 **In the opposite figure :**

A part of the curve $f(x)$ is drawn in interval $[a , b]$

, if the area M equals 5 square units

, and the area N equal 3 square units

, then $\int_a^b f(x)\,dx =$ ……………



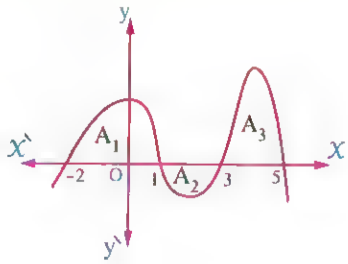
(a) -5 (b) -2 (c) 2 (d) 8

33 **In the opposite figure :**

If $A_1 = 5$ square unit , $A_2 = 2$ square unit

, $A_3 = 8$ square units

, then $\int_{-2}^{5} f(X)\, dX + \int_{-2}^{5} |f(X)|\, dX =$ ....



ⓐ 15 ⓑ 20

ⓒ 22 ⓓ 26

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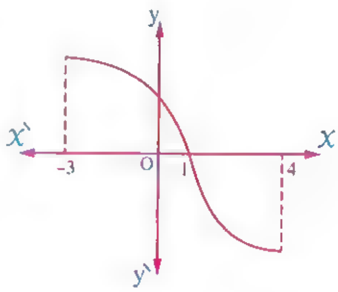
34 **In the opposite figure :**

If $\int_{-3}^{4} f(X)\, dX = 12$

and area of the shaded part = 28 square unit

, then $\int_{1}^{4} f(X)\, dX =$ ....



ⓐ – 16 ⓑ – 8

ⓒ 8 ⓓ – 20

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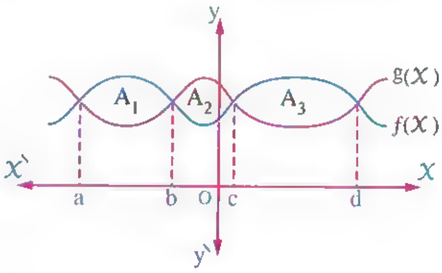
35 If each of f and g is a continuous function and the graph illustrates the curve of $f(X)$ and $g(X)$ and $A_1 = 3$ square units , $A_2 = 2$ square units

, $A_3 = 4$ square units

which of the following statements is not true ?



ⓐ $\int_{a}^{c} [f(X) - g(X)]\, dX = 1$ ⓑ $\int_{b}^{d} [g(X) - f(X)]\, dX = -2$

ⓒ $\int_{a}^{d} [f(X) - g(X)]\, dX = 5$ ⓓ $\int_{a}^{c} [f(X) - g(X)]\, dX = 4$

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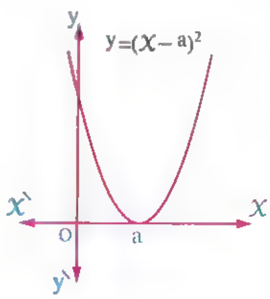
36 **In the opposite figure :**

If the area of shaded region $= \frac{8}{3}$ square units

, then a = ...



ⓐ $\frac{1}{2}$ ⓑ 1

ⓒ $\frac{3}{2}$ ⓓ 2

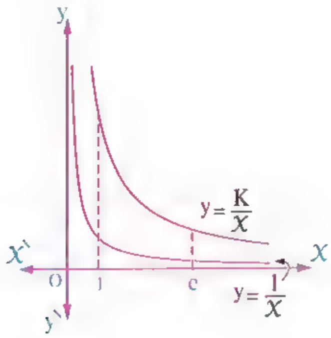
37 **In the opposite figure :**

If the area of the shaded region = 2 square unit

, then k = ...

ⓐ 2 ⓑ 3

ⓒ 4 ⓓ 5



38 **In the opposite figure :**

Find the area of the region bounded by the curve of the function f and the two straight lines y_1 and y_2 where :

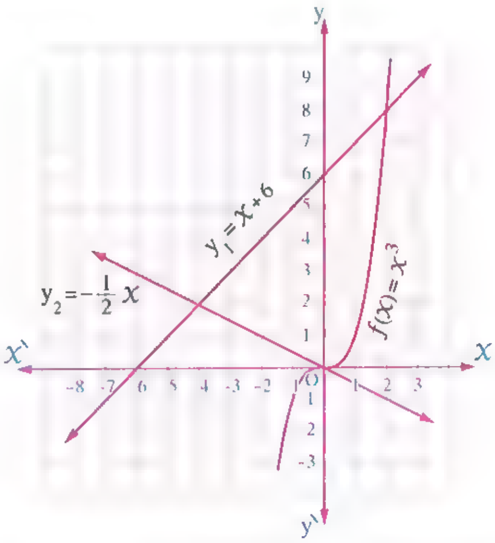
$f(X) = X^3$, $y_1 = X + 6$, $y_2 = -\frac{1}{2}X$

ⓐ 11

ⓑ 16

ⓒ 22

ⓓ 27



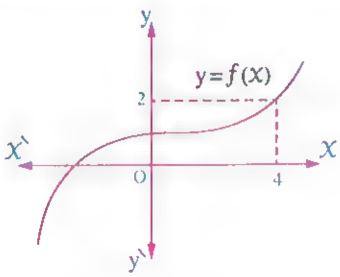
39 **In the opposite figure :**

If the area of the shaded region = 3 square units

, then $\int_0^4 f(X)\, dX =$ ....

ⓐ 3 ⓑ 4

ⓒ 5 ⓓ 6

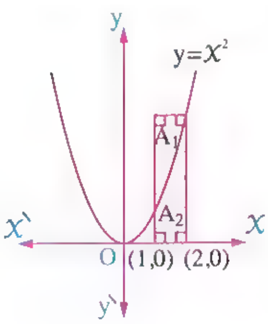


40 The opposite figure represents the curve of the function $f : f(X) = X^2$

, then $\frac{A_1}{A_2} =$ ....

ⓐ $\frac{2}{7}$ ⓑ $\frac{3}{7}$

ⓒ $\frac{4}{7}$ ⓓ $\frac{5}{7}$

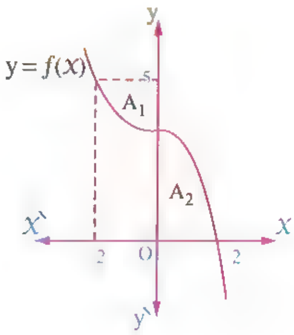


41 **In the opposite figure :**

$A_1 = 2$ square units ,

$A_2 = 7$ square units

, then $\int_{-2}^{2} f(x)\, dx = \cdots\cdots$



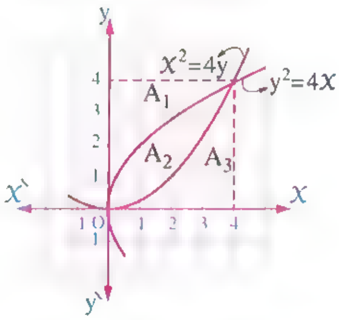
(a) 5 (b) 9

(c) 15 (d) 19

42 **In the opposite figure :**

If A_2 is the region bounded by the two curves

$y^2 = 4x$, $x^2 = 4y$, then $A_1 : A_2 : A_3 = \cdots\cdots$



(a) 2 : 1 : 2 (b) 1 : 2 : 1

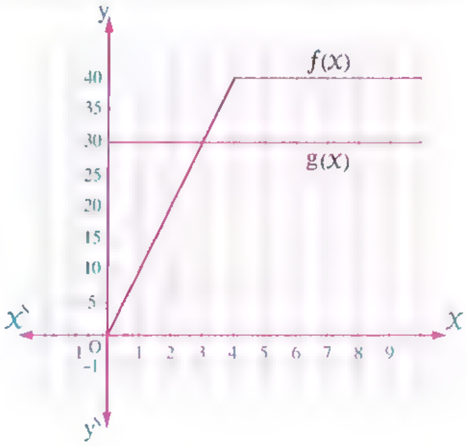
(c) 1 : 1 : 1 (d) 3 : 2 : 3

43 The opposite figure represents the curves

of two functions f , g in the interval $[0 , 9]$

If $\int_0^a f(x)\, dx = \int_0^a g(x)\, dx$

, then a = $\cdots\cdots$



(a) 3 (b) 4

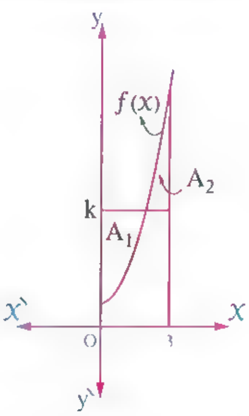
(c) 5 (d) 8

44 **In the opposite figure :**

$f(x) = x^2 + 1$

, then k which makes

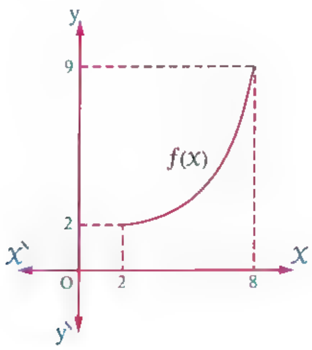
$A_1 = A_2$ equals $\cdots\cdots$



(a) 3 (b) 4

(c) 5 (d) 8

45 **In the opposite figure :**



If the curve of the function $f(X)$ is continuous and convex downward in the interval $[2, 8]$, then $\int_{2}^{8} f(X)\, dX$ can not be ……………

(a) 28 (b) 30
(c) 32 (d) 34

46 If the region bounded by the curve $y = X^2$ and the straight line $y = 2$ revolved a complete revolution about y-axis, then the volume of the solid generated by revolving equals ……………

(a) $\int_0^2 y\, dX$ (b) $\pi \int_0^2 y\, dy$ (c) $\pi \int_0^2 X\, dX$ (d) $\pi \int_0^2 X^2\, dX$

47 If the region bounded by the curve $y = X^2$ and the straight line $y = 2$ revolves a complete revolution about the X-axis, then the volume of the generated solid equals ……………

(a) $\pi \int_0^2 X^4\, dX$ (b) $\pi \int_{-\sqrt{2}}^{\sqrt{2}} (4 - X^4)\, dX$

(c) $\pi \int_{-2}^{2} (4 - X^4)\, dX$ (d) $\pi \int_{-\sqrt{2}}^{\sqrt{2}} X^4\, dX$

48 The volume of the solid generated by revolving the region bounded by the two curves $y = X^2$, $y = 1$ a complete revolution about y-axis is ……………

(a) π (b) $\frac{1}{2}\pi$ (c) $\frac{1}{4}\pi$ (d) $-\pi$

49 The volume of the solid generated by revolving the region bounded by the two curves $y = X^2$, $y = 1$ a complete revolution about X-axis equals ……………

(a) $\frac{8}{5}\pi$ (b) $\frac{-8}{5}\pi$ (c) $\frac{4}{5}\pi$ (d) $\frac{-4}{5}\pi$

50 The volume of the solid generated by revolving the region bounded by the curve $f(X) = X^2$ and X-axis, and the two straight lines $X = -2$, $X = 2$ a complete revolution about X-axis equals ……………

(a) $\frac{16\pi}{5}$ (b) $\frac{32\pi}{5}$ (c) $\frac{64\pi}{5}$ (d) 4π

51 The volume of the solid generated by revolving the region bounded by the curve $f(X) = X^2$, X-axis , $X = a$ where $a \in R^+$ a complete revolution about X-axis equal ....

(a) $\frac{1}{2} \pi a^4$ (b) πa^2 (c) $\frac{1}{5} \pi a^5$ (d) $\frac{1}{3} \pi a^3$

52 The volume of the solid generated by revolving Δ ABC such that A $(-2 , 0)$, B $(1 , 5)$, C $(4 , 0)$ a complete revolution a bout X-axis = ... cubic unit.

(a) 25π (b) 50π (c) 75π (d) 90π

53 The volume of the solid generated by revolution the region bounded by the curve $y^2 = 2 a X$ and the striagh line $X = b$ where $a , b \in R^*$ half revolution about X-axis equals .... cubic unit.

(a) $\pi a b^2$ (b) $\pi b a^2$ (c) $\pi a b$ (d) $\pi a^2 b^2$

54 $\int_0^r \pi X \, d X =$ ....

(a) Perimeter of a circle whose radius length r

(b) half the volume of a sphere whose radius length

(c) half the perimeter of a circle whose radius length r

(d) half the area of a circle whose radius length r

55 $\int_0^h \pi r^2 \, d X =$ ....

(a) volume of a circular cylinder whose height (h) and its base radius is (r)

(b) the area of a sphere whose radius length (h)

(c) the lateral area of a right circular cylinder whose height (h) and its base radius is (r)

(d) $\frac{1}{3} \pi h^3 + c$

56 volume of the solid generated by revolving the region bounded by the straight line $y = X + 1$ and the two straight lines $X = 0$, $y = 2$ a complete revolution about X-axis equals ....

(a) $\frac{5}{3} \pi$ (b) 4 (c) $\frac{7}{3} \pi$ (d) 3π

57 $\pi \int_{-2}^{2} (4 - x^2)\, dx$ is the volume of ....

(a) a sphere whose radius length 4 units

(b) a right circular cone whose height is 4 units

(c) a sphere whose radius length is 2 units

(d) A right circular cylinder whose height is 4 unit

58 The volume of a solid generated by revolving the region bounded by the curve $y = x(x-2)$ and x-axis a complete revolution about x-axis equals ....

(a) $\frac{-4}{3}\pi$ (b) $\frac{4}{3}\pi$ (c) $\frac{16}{15}\pi$ (d) $\frac{16}{15}$

59 The volume of the solid generated by revolving the region enclosed by the curve $y = 2x^2$ and the line $y = 8x$ a complete revolution about the x-axis is equal to ....

(a) $\pi \int_0^8 (8x - 2x^2)^2\, dx$ (b) $\pi \int_0^4 (8x - 2x^2)^2\, dx$

(c) $\pi \int_0^4 (64x^2 - 4x^4)\, dx$ (d) $\pi \int_0^4 (4x^4 - 64x^2)^2\, dx$

60 When the region bounded by the curve $x = \frac{1}{\sqrt{y}}$, $1 \le y \le 4$ and y-axis revolves a complete revolution about y-axis , then the volume of the solid generated measured by cubic units equals ....

(a) $\frac{2}{3}\pi$ (b) $3\sqrt{2}\,\pi$ (c) $2\pi \ln 2$ (d) $\frac{2}{3}\pi \log 3$

61 The volume of the solid generated by revolving the region bounded by the curve : $y = \sec x$ and the two straight lines : $x = 0$, $x = \frac{\pi}{3}$ a complete revolution about x-axis = ... cubic unit

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{\pi}{\sqrt{3}}$ (d) $\sqrt{3}\,\pi$

62 The solid generated by revolving the region bounded by the two curves $y = \tan x$, $y = \sec x$ and the two straight lines $x = \frac{\pi}{6}$, $x = \frac{\pi}{3}$ a complete revolution about x-axis measured by cubic units equals ....

(a) $\frac{\pi^2}{6}$ (b) $\frac{\pi^2}{3}$ (c) $\frac{2\pi^2}{5}$ (d) $2\pi^2$

63 The volume of the solid generated by revolving the region bounded by the curve $f(X) = \sqrt{25 - X^2}$ and $g(X) = 3$ a complete revolution about X-axis = ... cubic unit.

(a) $\frac{232}{3}\pi$ (b) $\frac{244}{3}\pi$ (c) $\frac{256}{3}\pi$ (d) $\frac{268}{3}\pi$

64 The volume of the solid generated by revolving the region bounded by the two curves $y = \sin X$ and $y = \cos X$ and the y-axis where $X \in \left[0 , \frac{\pi}{4}\right]$ a complete revolution about X-axis equals .... cubic unit.

(a) $\frac{1}{4}\pi$ (b) $\frac{1}{2}\pi$ (c) $\frac{3}{4}\pi$ (d) π

65 ABCD is a trapezium in which A (0 , 0) , (2 , 0) , C (2 , 5) , D (0 , 3) , then the volume of the solid generted by revolving the trapezium ABCD a complete revolution about X-axis = ... cubic unit

(a) $\frac{98}{3}\pi$ (b) $\frac{160}{3}\pi$ (c) $\frac{223}{3}\pi$ (d) $\frac{226}{3}\pi$

66 The ratio between the volume of the solid generated by revolving the curve of the function $y = f(X)$ about X-axis a complete revolution : the volume of the solid generated by revolving the same curve about X-axis two complete revolutions = ...

(a) 1 : 1 (b) 1 : 2 (c) 2 : 1 (d) 1 : 4

67 The ratio between the volume of the solid generated by revolving the curve of the function $y = f(X)$ about X-axis half revolution : the volume of the solid generated by revolving the same curve about X-axis two and half revolutions = ... : ....

(a) 1 : 1 (b) 1 : 2 (c) 5 : 1 (d) 1 : 5

68 The ratio between the volume of the solid generated by revolving circle with equation $(X - 5)^2 + y^2 = 9$ a complete revolution about X-axis = ... cubic unit

(a) 18π (b) 27π (c) 36π (d) 72π

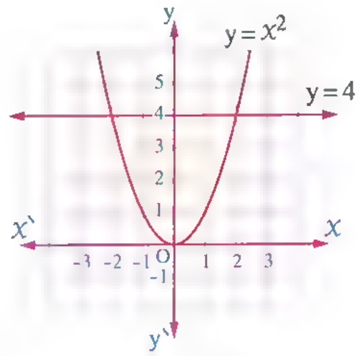
69 If $y_1 = \sqrt{X}$, A_1 is the area bounded by the curve y_1 , the X-axis and straight line $X = 2$ And $y_2^2 = X$, A_2 is the area bounded by the curve y_2 and the straight line $X = 2$, V_1 and V_2 are the volumes of two solids generated by revolving the two regions A_1 , A_2 a complete revolution about X-axis respectively , then ....

(a) $A_1 = A_2$, $V_1 = V_2$ (b) $A_1 = \frac{1}{2} A_2$, $V_1 = \frac{1}{2} V_2$

(c) $A_1 = A_2$, $V_1 = \frac{1}{2} V_2$ (d) $A_1 = \frac{1}{2} A_2$, $V_1 = V_2$

70 **In the opposite figure :**

The volume of the solid generated by revolving the shaded region a complete revolution about the X-axis = ... cube units.

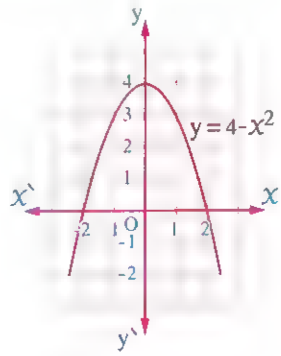


ⓐ $\frac{32}{5}\pi$ ⓑ $\frac{128}{5}\pi$

ⓒ $\frac{256}{5}\pi$ ⓓ $\frac{512}{5}\pi$

71 **In the opposite figure :**

The volume of the solid generated by revolving the shaded region a complete revolution about the y-axis equals .... cube units.

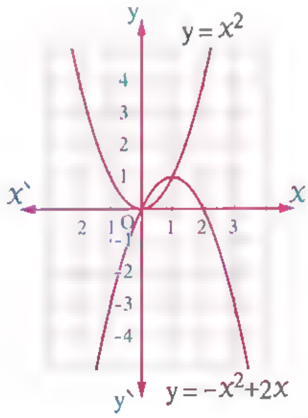


ⓐ 4π ⓑ 6π

ⓒ 8π ⓓ 10π

72 **In the opposite figure :**

The volume of the solid generated by revolving the shaded area a complete revolution about the X-axis = ... cube units.

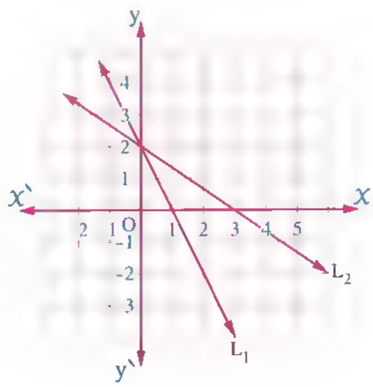


ⓐ $\frac{\pi}{3}$ ⓑ $\frac{\pi}{2}$

ⓒ $\frac{2\pi}{3}$ ⓓ π

73 **In the opposite figure :**

The volume of the solid generated by revolving the shaded area a complete revolution about the y-axis = ... cube units.

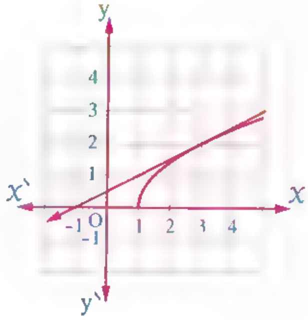


ⓐ $\frac{4}{3}\pi$ ⓑ $\frac{8}{3}\pi$

ⓒ 6π ⓓ $\frac{16}{3}\pi$

74 **In the opposite figure :**

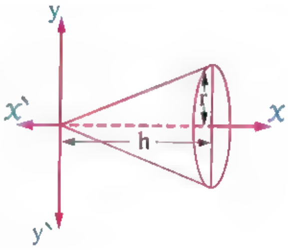
The straight line L is a tangent to the curve $y = \sqrt{2x - 2}$ at $(3, 2)$, then the volume of the solid generated by revolving the shaded region a complete revolution about the x-axis equals .... cube units.



ⓐ $\frac{4}{3}\pi$
ⓑ $\frac{3}{4}\pi$
ⓒ $\frac{1}{3}\pi$
ⓓ $\frac{2}{3}\pi$

75 **In the opposite figure :**

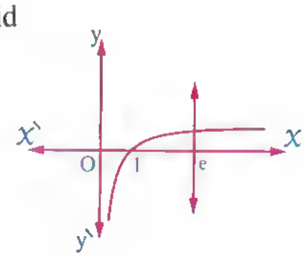
The axis of a right cone lies along the x-axis and its vertex at the origin, then its volume = ...



ⓐ $\pi \int_0^h \left(\frac{r}{h}x\right) dx$
ⓑ $\pi \int_0^r \left(\frac{h}{r}y\right) dy$
ⓒ $\pi \int_0^h \left(\frac{r^2}{h^2}x^2\right) dx$
ⓓ $\pi \int_0^r \left(\frac{h^2}{r^2}y^2\right) dy$

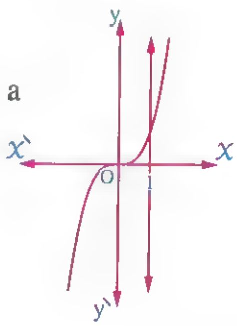
76 The opposite figure represents the curve :

$y = \frac{\ln x}{\sqrt{x}}$ and the line $x = e$, then the volume of the generated solid by revolving the shaded region a complete revolution about the x-axis = ... cube units.



ⓐ $\frac{1}{3}\pi$
ⓑ π
ⓒ 3π
ⓓ 9π

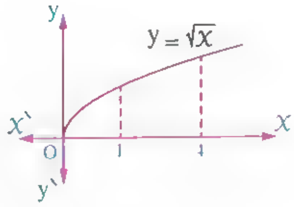
77 The opposite figure represents the curve $y = x^3$ and the line $x = 1$, then the volume of the solid generated by revolving the shaded region a complete revolution about the y-axis = ... cube units.



ⓐ $\frac{1}{7}\pi$
ⓑ $\frac{2}{5}\pi$
ⓒ $\frac{3}{5}\pi$
ⓓ $\frac{3}{4}\pi$

78 **In the opposite figure :**

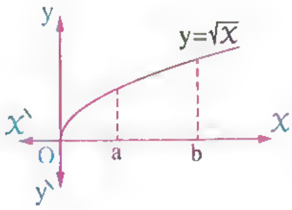
The volume of the solid generated by revolving the shaded region a complete revolution about x-axis = ... cubic units.



(a) $\frac{14}{3}\pi$ (b) $\frac{15}{2}\pi$ (c) $\frac{15}{2}$ (d) $\frac{14}{3}$

79 **In the opposite figure :**

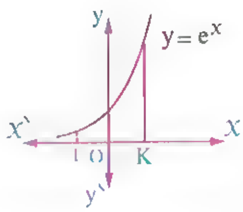
If the volume of the solid generated by revolving the shaded area a complete revolution about x-axis on the interval $[a, b]$ equals 8π, then $b^2 - a^2 =$ ....



(a) 8 (b) 12

(c) 16 (d) 20

80 If the volume of the solid generated by revolving the shaded region a complete revolution about x-axis, equals $\frac{\pi}{2}(e^{10} - e^{-2})$ cubic unit, then : k = ...



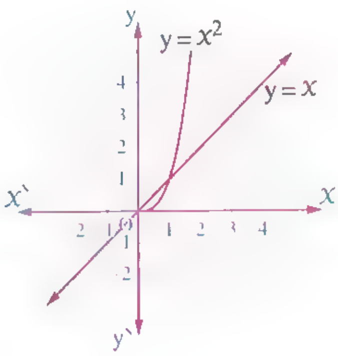
(a) 5 (b) 10

(c) 20 (d) – 5

81 **In the opposite figure :**

If the volume of the solid generated by revolving the shaded area about the x-axis = a cube units and about the y-axis = b cube units.

, then which of the following statements is true ?



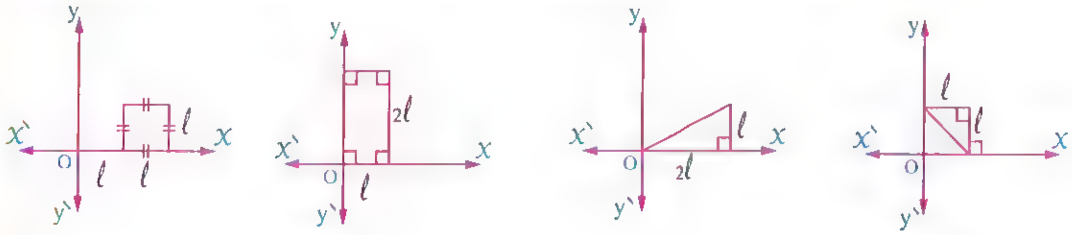
(a) a = b and the two bodies are congrunet.

(b) a = b and the two bodies are not congruent.

(c) a > b

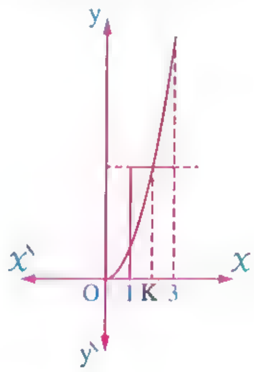
(d) a < b

82 Which of the following figures , the volume of the solid generated by revolving about the X-axis equals the volume of the solid generated by revolving about the y-axis ?



ⓐ ⓑ ⓒ ⓓ

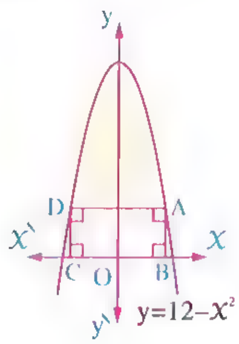
83 In the opposite figure the curve $f(X) = 6X^2$, then the value of k which makes the shaded region as small as possible equals ....



ⓐ $1\frac{1}{4}$
ⓑ $1\frac{3}{4}$
ⓒ 2
ⓓ $2\frac{1}{4}$

84 **In the opposite figure :**

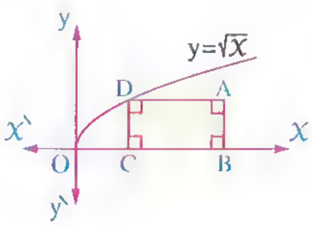
The area of the shaded region when the area of rectangle ABCD is as big as possible equals .... square unit.



ⓐ $\frac{8}{3}$
ⓑ 4
ⓒ 5
ⓓ $\frac{32}{3}$

85 **In the opposite figure :**

If B = (12 , 0) , then the greatest volume of the solid generated by revolving the shaded region a complete revolution about X-axis = ... cubic unit.



ⓐ 48π
ⓑ 36π
ⓒ 24π
ⓓ 18π

Practice Exams

on

Differential & Integral calculus

Practice Exams

Exam 1

Answer the following questions :

1. $\lim\limits_{X \to \infty} \left(1 + \frac{1}{X}\right)^{2X} = \dots\dots\dots\dots$

(a) 1 (b) 2 (c) e (d) e^2

2. The curve $y = X e^{X}$ at $\dots\dots\dots\dots$

(a) $X = -1$ has local minimum value
(b) $X = -1$ has local maximum value
(c) $X = 0$ has local minimum value
(d) $X = 0$ has local maximum value

3. The tangent to the curve $y = 3X^2 - 5$ at the point $(1, -2)$ also passes by the point $\dots\dots\dots\dots$

(a) $(5, -2)$ (b) $(3, 1)$ (c) $(2, -4)$ (d) $(0, -8)$

4. If the perimeter of a circular sector is P (where P is constant), then its surface area is maximum at r = $\dots\dots\dots\dots$

(a) $\frac{P}{2}$ (b) $\frac{1}{\sqrt{P}}$ (c) $\sqrt{P}$ (d) $\frac{P}{4}$

5. If $X = e^{2t}$, $y = t^3$, then $\frac{d^2 y}{d X^2} = \dots\dots\dots\dots$ at $t = 1$

(a) $\frac{3}{2}$ (b) $\frac{3}{2} e^{-4}$ (c) 0 (d) $3e^2$

6. If f is a continuous even function on the interval $[-4, 4]$,

$\int_{-4}^{4} f(X)\, dX = 20$, $\int_{0}^{2} f(X)\, dX = 6$, then $\int_{-4}^{2} f(X)\, dX = \dots\dots\dots\dots$

(a) 120 (b) 14 (c) 26 (d) 16

7. If $f(X) = \cot X$, then $f''\left(\frac{\pi}{4}\right) = \dots\dots\dots\dots$

(a) $\frac{-4}{9}$ (b) $\frac{4}{9}$ (c) 4 (d) $\frac{9}{2}$

8. If $f(X) = \begin{cases} 3X^2 & , \quad X \le 3 \\ 2X + 1 & , \quad X > 3 \end{cases}$, then $\int_{0}^{5} f(X)\, dX = \dots\dots\dots\dots$

(a) 125 (b) 30 (c) 110 (d) 45

9. If $f(X) = 2X^3 - 3X^2 - 12X + 12$, then the local maximum value of the function equals ....

(a) 2 (b) -8 (c) -1 (d) 19

10. The equation of the curve passes through the point (0 , 1) and the slope of its tangent at any point on it (X , y) equals $X\sqrt{X^2+1}$ is ....

(a) $y = \frac{1}{3}(X^2+1)^{\frac{2}{3}} + \frac{2}{3}$ (b) $y = \frac{1}{3}(X^2+1)^{\frac{3}{2}} + \frac{2}{3}$

(c) $y = \frac{1}{3}(X^2+1)^{\frac{3}{2}} - \frac{2}{3}$ (d) $y = \frac{4}{3}(X+1)^{\frac{2}{3}} - \frac{2}{3}$

11. The length of each side of an equilateral triangle = a , and increase at a rate k , then the rate of increasing of its surface area equals ....

(a) $\frac{2}{\sqrt{3}}$ a k (b) $\sqrt{3}$ a k (c) $\frac{\sqrt{3}}{2}$ a k (d) $\frac{2}{\sqrt{3}}$ a k

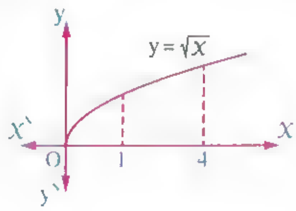
12. The curve of the function f is convex downwards in $\mathbb{R}$ if $f(X) =$ ....

(a) $3 - X^2$ (b) $3 - X^3$ (c) $3 - X^4$ (d) $3 + X^4$

13. $\int \tan\theta \, d\theta =$ ....

(a) $-\ln|\cos\theta| + c$ (b) $-\ln\cos\theta + c$ (c) $\ln\cos\theta + c$ (d) $|\ln\cos\theta| + c$

14. The volume of the solid generated by revolving the shaded region a complete revolution about X-axis equals .... cube units.



(a) $\frac{14}{3}\pi$ (b) $\frac{15}{2}\pi$

(c) $\frac{15}{2}$ (d) $\frac{14}{3}$

15. The shortest distance between the straight line : $X - 2y + 10 = 0$ and the curve $y^2 = 4X$ equals .... length unit.

(a) 4 (b) $6\sqrt{5}$ (c) $\frac{6}{5}\sqrt{5}$ (d) 2

16 The area of the region bounded by the curve of the function $y = x^3$ and the two straight lines $y = 0$, $x = 2$ equals .... square unit.

(a) 4 (b) -4 (c) $\frac{1}{2}$ (d) 8

17 $\frac{d^2}{dx^2}(\cos^4 x + \sin^4 x) =$ ....

(a) Zero (b) $-2 \sin 2x$ (c) $-4 \cos 4x$ (d) 1

18 The surface area of a sphere increases at constant rate 6 cm^2/sec. at the instant at which its raduis is 30 cm. , then the rate of increase of the volume of the sphere = ... cm^3/sec.

(a) 180 (b) 40 (c) 90 (d) 90π

19 If $f(x) = 2 \sin \frac{x}{2} \cos \frac{x}{2}$, then the 1000th derivative of $f(x)$ equals ....

(a) $\sin x$ (b) $-\sin x$ (c) $-\cos x$ (d) $\cos^{1000} x$

20 $\int e^2 \, dx =$ .... + c

(a) $e^2 x$ (b) $\frac{1}{3} e^3$ (c) e^2 (d) $\frac{1}{3} e^{3x}$

21 If $f'(x) = (x - 3)(x + 4)$, then the curve of the function f is convex upwards on ....

(a) $]-\infty , -4[$ (b) $]-4 , 3[$ (c) $]3 , \infty[$ (d) $]-\infty , 3[$

22 The slope of the normal to the curve at any point on it (x , y) equals $\frac{-2y}{x}$, then the equation of the curve , given that it intercepts 3 units from the positive part of y–axis , is ....

(a) $2y^2 = x^2 + 18$ (b) $2y^2 = x^2$

(c) $2y^2 = \frac{1}{2}x^2 + 9$ (d) $y^2 = \frac{1}{2}x^2 + 3$

23 If $f(x) = |x|$, then $f'(-6) =$ ....

(a) 6 (b) -1 (c) 0 (d) -6

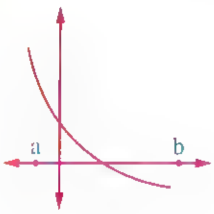
24 The length of the radius of a circle increase at a rate $\frac{4}{\pi}$ cm./sec. , then the rate of increase of its circumference at this moment is ....

(a) $\frac{4}{\pi}$ cm./sec. (b) $\frac{\pi}{4}$ cm./sec. (c) $\frac{1}{8}$ cm./sec. (d) 8 cm./sec.

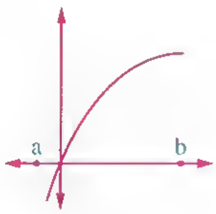
25 $\lim\limits_{x \to 0} (1 + x)^{\frac{1}{3x}} =$ ....

(a) $\frac{1}{3}$ (b) e^3 (c) $e^{\frac{1}{3}}$ (d) $\frac{e}{3}$

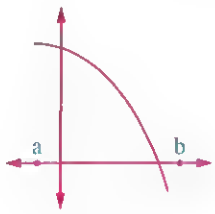
26 If $f'(x) < 0$, $f''(x) > 0$, for each $x \in [a , b]$ state which of the following represents the curve of the function f in $[a , b]$



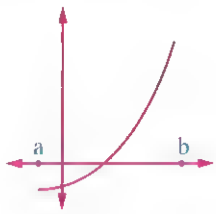
(a)



(b)



(c)



(d)

27 The greatest area of the rectangle whose perimeter equals 14 cm. equals .... cm^2.

(a) 10 (b) 12 (c) 12.25 (d) 49

28 $\int \frac{1}{1 - \cos^2 x} \, dx =$ .... + c

(a) $\csc^2 x$ (b) $\cot x$ (c) $-\sin^{-1} x$ (d) $-\cot x$

29 If $\int_{-2}^{3} f(x) \, dx = 9$, $\int_{5}^{3} f(x) \, dx = 4$, then the value of :

$\int_{-2}^{5} [3 f(x) - 6 x] \, dx =$ ....

(a) -48 (b) -58 (c) -144 (d) -147

30 $\int_{-\pi}^{\pi} \frac{4x + \sin x}{x^2 + \cos x} \, dx =$ ....

(a) $-\pi$ (b) 0 (c) π (d) 2π

Practice Exams

Exam 2

Answer the following questions :

1. The function $f : f(X) = \frac{X}{\ln X}$ is increasing in the interval ....

ⓐ $]0, \infty[$ ⓑ $]0, e[$ ⓒ $]e, \infty[$ ⓓ $]-\infty, \infty[$

2. The normal to the circle $X^2 + y^2 = 12$ at any point on it passes through the point ....

ⓐ $(2, 2)$ ⓑ $(1, 1)$ ⓒ $(0, 0)$ ⓓ $(-2, -2)$

3. $\int (4 - \csc X \cot X)\, dX =$ ....

ⓐ $4X - \csc X + c$ ⓑ $4X + \csc X + c$

ⓒ $4X - \cot X + c$ ⓓ $4X + \cot X + c$

4. $\lim\limits_{X \to \infty} \left(\frac{X+6}{X+2}\right)^{X+5} =$ ....

ⓐ e^4 ⓑ e^5 ⓒ e ⓓ e^6

5. The area of the greatest rectangle which can be drawn in a circle of radius 4 cm. equals .... cm^2.

ⓐ $4\sqrt{2}$ ⓑ $8\sqrt{2}$ ⓒ 32 ⓓ 64

6. The curve of the function f where : $f(X) = 2X^3 + 3X^2 - 12X + 5$ has inflection point at $X =$ ....

ⓐ -2 ⓑ 1 ⓒ $11\frac{1}{2}$ ⓓ $-\frac{1}{2}$

7. If $\int_{-2}^{2} f(X)\, dX = 0$, then $f(X) =$ ....

ⓐ $X^2 + 1$ ⓑ X ⓒ $X + 1$ ⓓ $X - 1$

8. If $f(X) = \sin 2X \cos 2X$, then $f'''\left(\frac{\pi}{3}\right) =$ ....

ⓐ -4 ⓑ 0 ⓒ $4\sqrt{3}$ ⓓ 8

9. If $y = (\sin x)^{\tan x}$, then $\frac{dy}{dx}$ = ...

(a) $(\sin x)^{\tan x} (\sec^2 x \ln \sin x + 1)$
(b) $(\tan x)(\sin x)$
(c) $(\cos x)^{\sec^2 x}$
(d) $\sec^2 x \ln \sin x + 1$

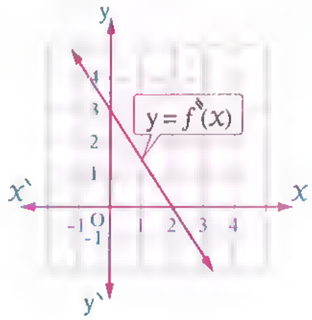
10. The height of right circular cone equals its base diameter. The rate of change of its base radius = $\frac{1}{\pi}$ cm./sec. then the rate of change of its volume = ... cm.^{3}/sec. when its base radius length = 5 cm.

(a) 50π
(b) $\frac{250}{3} \pi$
(c) 150
(d) 50

11. $\int x^2 e^x \, dx$ = ... + c

(a) $\frac{1}{3} x^3 e^x$
(b) $x^2 e^x - 2 x e^x + 2 e^x$
(c) $x^2 e^x - 2 x e^x$
(d) $2 x e^x$

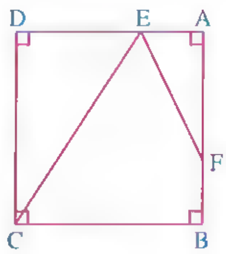
12. The opposite figure represents the curve of the function f' , then the curve of the function f has an inflection point at x = ...



(a) 0
(b) 1
(c) 2
(d) 3

13. **In the opposite figure :**

ABCD is a square of side length 24 cm. , AF = 2 AE , then the greatest area of the figure FBCE = ... cm.^{2}



(a) 324
(b) 252
(c) 6
(d) 648

14. The area of the region bounded by the curve $y = x^3$ and the straight lines $x = -1$, $x = 1$, $y = 0$ equals ....

(a) zero
(b) $\frac{1}{2}$
(c) $\frac{1}{4}$
(d) 6

15 The equation of the curve : $y = f(X)$ if $\ddot{y} = 6X - 4$ and the curve has local minimum at $(1 , 5)$ is ....

(a) $f(X) = X^3 - 2X^2 + X + 5$
(b) $f(X) = X^3 - 2X^2 + X$
(c) $f(X) = 3X^2 - 4X + 1$
(d) $f(X) = 3X^2 - 4X$

16 The curve of the function $f : f(X) = (X - 2)\, e^X$ is convex downwards in the interval ....

(a) $]-\infty , \infty[$
(b) $]-1 , 2[$
(c) $]0 , 2[$
(d) $]0 , \infty[$

17 The rate of increasing of the length of each of two sides in a triangle is 0.1 cm./sec. and the rate of increasing of angle including between them is $\frac{1}{5}$ rad/sec. , then the rate of increasing of area of the triangle at the instant when the length of each side of the triangle is 10 cm. equals .... cm.^{2}/sec.

(a) $\frac{\sqrt{3}}{4}$
(b) $\frac{\sqrt{3}}{2}$
(c) 5
(d) 5.866

18 The volume of the solid generated by revolving the region bounded by the curve $y = X^2$ and the straight line passes through the two points $(0 , 6) , (1 , 7)$ a complete revolution about X-axis equals .... cubic unit.

(a) $\frac{500}{3}$
(b) $\frac{665}{3}\pi$
(c) 55π
(d) $\frac{500}{3}\pi$

19 If $y = \sin X + \sec X , X = 3\pi z$, then $\frac{dy}{dz} =$ .... at $z = 1$

(a) 3π
(b) -3π
(c) -6π
(d) -1

20 If $f(X) = 3X - 2$, then $(f \circ f)'(1) =$ ....

(a) 6
(b) -6
(c) 9
(d) 3

21 $\int_{-1}^{3} |X - 1|\, dX =$ ....

(a) 2
(b) 0
(c) 4
(d) $\frac{3}{2}$

22 The volume of the solid generated by revolving the region bounded by the curve $y = 3 - X$, $X = 0$, $y = 0$ a complete revolution about X-axis equals .... cubic unit.

(a) 9π
(b) 9
(c) $\frac{9}{2}\pi$
(d) $\frac{9}{2}$

23 Two ships move from the same point on the same time , the first ship in direction of east with velocity 60 km./h. and the second in direction of south with velocity 80 km./h. , then the rate of change of the distance between the two ships after 2 hours from the beginning of motion = ... km./h.

(a) 100 (b) 50 (c) 200 (d) 400

24 If $y = \ln(\tan x)$, then $\frac{dy}{dx} =$ ....

(a) $2 \sec 2x$ (b) $2 \csc 2x$ (c) $\sec^2 x$ (d) $\cot x$

25 **In the opposite figure :**

If $f(x) = g(x) - 3h(x)$

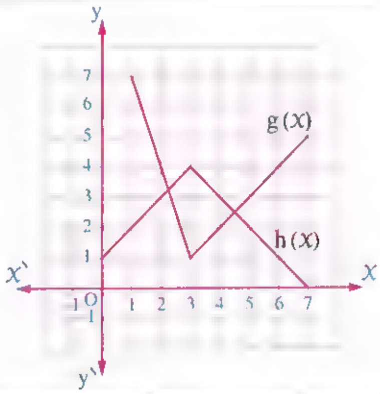
, then $\acute{f}(5) =$ ....

(a) zero

(b) 2

(c) 3

(d) 4



26 $\int \cos^{99} x \sec^{100} x \, dx =$ .... + c

(a) $\frac{1}{100} \cos^{100} x$ (b) $\frac{1}{101} \cos^{101} x$

(c) $\sec x \tan x$ (d) $\ln |\sec x + \tan x|$

27 If the point (1,12) is the inflection point to the curve of the function f where $f(x) = a x^3 + b x^2$, then $2a + b =$ ....

(a) 24 (b) – 12 (c) 12 (d) 6

28 If $y = -\sin x$, then $\frac{d^2 y}{dx^2} + y =$ ....

(a) – 4 (b) 2 (c) 4 (d) zero

29 An empty container , its volume 45 cm³ , water is poured in it at a rate 5 cm³/sec. , the container becomes full after .... second.

(a) 9 (b) 135 (c) 45 (d) 5

30 $\lim_{x \to 0} \frac{e^{5x} - 1}{x} =$ ....

(a) 5 (b) e^5 (c) ln 5 (d) 5 e

Practice Exams

Exam 3

Answer the following questions :

1. $\lim\limits_{x \to 0} \dfrac{e^x - \sin x - 1}{x} = \dots\dots\dots$

(a) zero (b) 1 (c) undefined (d) -1

2. If the curve of the function f represents a polynomial function , has a local maximum at the point (a , b) , then $f'(a) = \dots\dots\dots$

(a) b (b) zero (c) $\frac{-b}{a}$ (d) undefined

3. If the tangent to the curve $y^2 = 4ax$ is perpendicular to x-axis , then $\dots\dots\dots$

(a) $\frac{dy}{dx} = 0$ (b) $\frac{dy}{dx} = 1$ (c) $\frac{dx}{dy} = 1$ (d) $\frac{dx}{dy} = 0$

4. If $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$, then $\frac{dy}{dx} = \dots\dots\dots$

(a) $\sin\theta$ (b) $\sin 2\theta$ (c) $\cos\theta$ (d) $\tan\theta$

5. The volume of the solid generated by revolving the region bounded by the two curves $y = \tan x$, $y = \sec x$ and the two straight lines $x = \frac{\pi}{6}$, $x = \frac{\pi}{3}$ a complete revolution about x-axis is $\dots\dots\dots$ cubic unit.

(a) $\frac{\pi^2}{6}$ (b) $\frac{\pi^2}{3}$ (c) $\frac{2\pi^2}{5}$ (d) $2\pi^2$

6. If $f : \left[\frac{1}{e} , e\right] \longrightarrow \mathbb{R}$ and $f(x) = x - \ln x$

, then the function f has absolute maximum value $= \dots\dots\dots$

(a) e (b) $e - 1$ (c) 1 (d) $\frac{1}{e} + 1$

7. The rate of change for $\sqrt{x^2 + 16}$ with respect to $\frac{x}{x-1}$ at $x = 3$ equals $\dots\dots\dots$

(a) -60 (b) $\frac{-5}{12}$ (c) $\frac{-12}{5}$ (d) $\frac{-3}{5}$

8. $\int \frac{\ln x^2}{\ln x} \, dx = \dots\dots\dots$ (where : $x > 0$)

(a) $\frac{x}{2} + c$ (b) $\frac{1}{x} + c$ (c) $2x + c$ (d) $\ln|x| + c$

9. If $f(x) = 2x^2 + x - 3$, then $\int_1^2 f'(x)\,dx =$ ....

(a) 8 (b) 9 (c) 10 (d) 11

10. If the rate of change in volume of a sphere equals the rate of change of its radius, then r = ... length unit.

(a) 1 (b) $\sqrt{2\pi}$ (c) $\frac{1}{\sqrt{2\pi}}$ (d) $\frac{1}{2\sqrt{\pi}}$

11. $\int x \cos x\,dx =$ .... + c

(a) $x \sin x - \cos x$
(b) x
(c) $-\frac{1}{2} x^2 \sin x$
(d) $x \sin x + \cos x$

12. The curve $y = (2x - c)^3 + 4$ has an inflection point at $x = 5$, then c = ...

(a) 2 (b) 4 (c) 5 (d) 10

13. $\int \sqrt{x}\left(1 + \sqrt{x}\right) dx =$ .... + c

(a) $x^{\frac{1}{2}} + x$
(b) $\frac{2}{3} x^{\frac{3}{2}} + 2x^2$
(c) $\frac{3}{2} x^{\frac{2}{3}} + \frac{1}{2} x^2$
(d) $\frac{2}{3} x^{\frac{3}{2}} + \frac{1}{2} x^2$

14. If $y = e^x$, $z = \sin x$, then $\frac{dy}{dz} =$ ....

(a) $\frac{e^x}{\sin x}$ (b) $e^x \tan x$ (c) $e^x \cos x$ (d) $\frac{e^x}{\cos x}$

15. If $f(x) = 2x^3 - 3x^2 - 36x + 14$, then the curve of the function is convex downwards on the interval ....

(a) $]\frac{1}{2}, \infty[$ (b) $]-\infty, \frac{1}{2}[$ (c) $]-2, 3[$ (d) $\mathbb{R} - [-2, 3]$

16. The area of the region bounded by the curve $y = 3x^2 + 4$, x-axis and the two straight lines $x = -1$, $x = 2$ equals .... square units.

(a) 21 (b) 11 (c) -21 (d) 16

17 The slope of the tangent to the curve at any point on it (x , y) is given by the relation $\frac{dy}{dx} = \sin x \cos x$, then the equation of the curve known that it passes through the point $\left(\frac{\pi}{6} , 1\right)$ is ....

(a) $y = \frac{1}{2} \sin^2 x$

(b) $y = \frac{1}{2} \sin^2 x + 7$

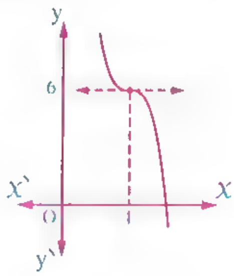
(c) $y = \frac{1}{2} \sin^2 x + \frac{7}{8}$

(d) $2y = \sin^2 x - \frac{7}{4}$

18 A trapezium is drawn in a semi-circle , and its base is the diameter of the semi-circle , then the base angle of the trapezium such that its area is as maximum as possible is of measure ....

(a) 45°
(b) 60°
(c) 30°
(d) 120°

19 The opposite figure represents the curve of function f , then all the following statments are true except ....



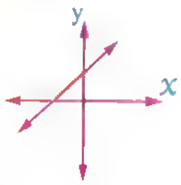
(a) $f(4) = 6$

(b) $f'(4) = 0$

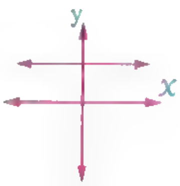
(c) $f''(x) > 0$ for $x < 4$

(d) $f''(x) < 0$ for $x < 4$

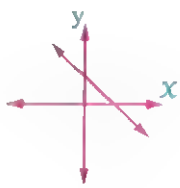
20 If $y = ax^n - bx^{n-1}$ is a polynomial function a , b $\in \mathbb{R}$, then $\frac{d^n y}{dx^n}$ could be represented by one of the following figure :



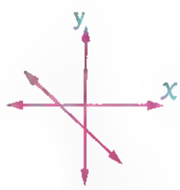
(a)



(b)

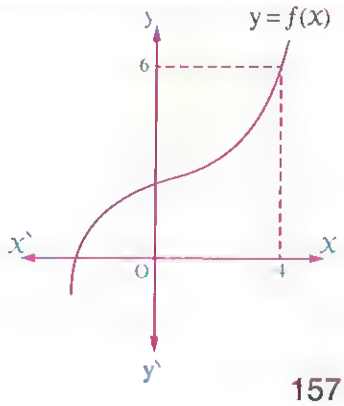


(c)



(d)

21 **In the opposite figure :**



If the area of the shaded region = 9 square units

, then $\int_0^4 f(x)\, dx =$ ....

(a) 24
(b) 4
(c) 15
(d) 33

22 $\int_{2020}^{2022} (x - 2021)^2 \, dx = \dots\dots$

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) 1 (d) $\frac{4}{3}$

23 $\int \frac{\sin^{10} x}{\cos^{12} x} \, dx = \dots\dots + c$

(a) $\tan^{11} x$ (b) $\frac{1}{11} \tan^{11} x$ (c) $\frac{1}{11} \tan 11 x$ (d) $\sec^2 x$

24 If f is a function , $f : f(x) = x^2 + a x + b$ has local minimum value = 3 at $x = 1$, then ab = ……

(a) – 48 (b) – 8 (c) 2 (d) – 12

25 ABCD is a square whose side length 10 cm. and $M \in \overline{BC}$ where $BM = x$ cm. and $N \in \overline{CD}$ where $CN = \frac{3}{2} x$, then the value of x which makes the area of Δ AMN as minimum as possible = …… cm.

(a) $\frac{10}{3}$ (b) $\frac{3}{2}$ (c) 5 (d) $\frac{15}{2}$

26 $\int \frac{2 x^3}{x^4 + 5} \, dx = \dots\dots + c$

(a) $8 \ln |x^4 + 5|$ (b) $2 \ln |x| + \frac{1}{10} x^4$

(c) $2 \ln |x^4 + 5|$ (d) $\frac{1}{2} \ln |x^4 + 5|$

27 If : $f(x) = \sin 2x$, then $\dddot{f}\left(\frac{\pi}{4}\right) = \dots\dots$

(a) zero (b) – 2 (c) – 4 (d) – 6

28 The tangent to the curve : $x^2 - x y + y^2 = 27$ drawn at the point (6 , 3) makes an angle of measure …… with the positive direction of the x–axis.

(a) 90° (b) zero (c) 45° (d) 180°

29 The side length of a square is 5 cm. The side length increases at a rate 4 cm./sec. , then the length of the side of the square after t seconds is given by the relation ……

(a) 24 t (b) 4 t + 5 (c) 4 t – 5 (d) 9

30 $\lim\limits_{n \to \infty} \left(1 + \frac{5}{n}\right)^{3n} = \dots\dots$

(a) e^5 (b) e^3 (c) e^{15} (d) 15 e

Answer the following questions :

1. The equation of the normal to the curve $y = f(x)$ at the point $(1, 1)$ is $x + 4y = 5$, then $f'(1) = \dots\dots$

ⓐ -3 ⓑ $-\frac{1}{4}$ ⓒ 4 ⓓ -4

2. $\int \tan^2 x \, dx = \dots\dots$

ⓐ $\tan x - x + c$ ⓑ $\tan x + x + c$ ⓒ $\sec^4 x + c$ ⓓ $\frac{1}{3}\tan^3 x + c$

3. If $y = \ln(\sec x + \tan x)$, then $\frac{dy}{dx} = \dots\dots$

ⓐ $\tan x$ ⓑ $\sec x$ ⓒ $\tan^2 x$ ⓓ $\csc x$

4. If $z = x + \frac{1}{x}$, then $dz = \dots\dots$

ⓐ $\left(1 + \frac{1}{x^2}\right) dx$ ⓑ $\left(1 + \frac{1}{x^2}\right) dx + c$ ⓒ $\left(1 - \frac{1}{x^2}\right) dx + c$ ⓓ $\left(1 - \frac{1}{x^2}\right) dx$

5. $\int_{-2}^{2} (a x^3 + b x + c) \, dx$ depends on $\dots\dots$

ⓐ the value of b ⓑ the value of c

ⓒ the value of a ⓓ the value of a , b

6. If $\lim\limits_{x \to 0} \frac{\ln(1 + a x)}{b x} = -1$, then $a + b = \dots\dots$

ⓐ zero ⓑ -1 ⓒ 2 ⓓ -2

7. If $y^2 = 1 - \frac{1}{x^2}$, then $y \frac{d^2 y}{d x^2} + \left(\frac{dy}{dx}\right)^2 = \dots\dots$

ⓐ $3x^4$ ⓑ $-3x^{-4}$ ⓒ $3x^{-4}$ ⓓ $-3x^{-2}$

8. If $f'(x) = x \cdot f(x)$, $f(3) = -5$, then $f''(3) = \dots\dots$

ⓐ -50 ⓑ -40 ⓒ 15 ⓓ 27

9 The absolute minimum value of the function $f : f(X) = X e^{-X}$ in the interval $[0 , 2]$ equals ....

ⓐ 1 ⓑ $\frac{1}{e}$ ⓒ zero ⓓ $\frac{2}{e^2}$

10 If $f : \mathbb{R} \longrightarrow \mathbb{R}$, $f(X) = X^3 + a X^2 + 12 X + 1$ and the function has no critical points , then $a \in$ ....

ⓐ $]-6\sqrt{2} , 6\sqrt{2}[$ ⓑ $]-3 , 3[$ ⓒ $]-12 , 12[$ ⓓ $]-6 , 6[$

11 If $X^2 y^3 = 108$, and $\frac{dX}{dt} = 2$ at $X = 2$, $y = 3$, then $\frac{dy}{dt} =$ ....

ⓐ 2 ⓑ -2 ⓒ -1 ⓓ 18

12 $\int e^{(X^3 + 2 \ln X)} \, dX =$ .... $+ c$

ⓐ $3 e^{X^3}$ ⓑ $\frac{1}{3} e^{X^3}$ ⓒ e^{X^3} ⓓ $X^2 e^{X^3}$

13 The curve of the function $f : f(X) = X^3 - 12 X$ is convex upwards in the interval ....

ⓐ $]-\infty , 0[$ ⓑ $]0 , \infty[$ ⓒ $\mathbb{R} - \{0\}$ ⓓ $\mathbb{R} -]-2 , 2[$

14 The volume of a solid generated by revolution of the region bounded by the curve $X y = 3$ and the two straight lines $y = 1$, $y = 3$ and y-axis a complete revolution about X-axis equals .... cubic unit.

ⓐ 12π ⓑ 8π ⓒ 4π ⓓ 6π

15 The dimensions of a rectangle which has the greatest area can be drawn in a triangle , the base length of the triangle equals 16 cm. and its height 12 cm. such that one of the rectangle sides coincides with the base of the triangle and its opposite vertices lie on the other two sides of the triangle are ....

ⓐ 6 cm. , 6 cm. ⓑ 8 cm. , 8 cm. ⓒ 6 cm. , 8 cm. ⓓ 4 cm. , 6 cm.

16 The area of the region bounded by the curve $y = \sqrt{4 - X^2}$ and X-axis in square unit equals .... square units.

ⓐ 2 ⓑ 4 ⓒ 2π ⓓ 4π

17 The equation of the curve passes through the point A (2 , 3) and the slope of the normal at any point on it is $3 - x$ ....

(a) $y = \ln |x - 3|$
(b) $y = (x - 3)^{-2}$
(c) $y = \ln |x - 3| + 3$
(d) $y = \ln |x - 3| - 3$

18 A regular quadrilateral pyramid of metal expands uniformaly , the height equals the side length of its base , its volume increases at a rate 1 cm^3/sec. , when the rate of increasing of each of its height and its base side equals 0.01 cm./sec. , then the base length at this moment = ...

(a) 10 cm.
(b) 100 cm.
(c) 5 cm.
(d) 125 cm.

19 If $y \in]0 , \frac{\pi}{4}[$, $x = \frac{2 \tan y}{1 - \tan^2 y}$, then $\frac{dy}{dx} =$ ....

(a) $\frac{1}{2} \cos^2 2y$
(b) $2 \sec^2 2y$
(c) $\sin^2 2y$
(d) $\cot 2y$

20 If the function $y = a x^3 + b x^2 + c x + d$ has a critical point at (1 , 5) , then $2a + b - d =$ ....

(a) -6
(b) -5
(c) 5
(d) 6

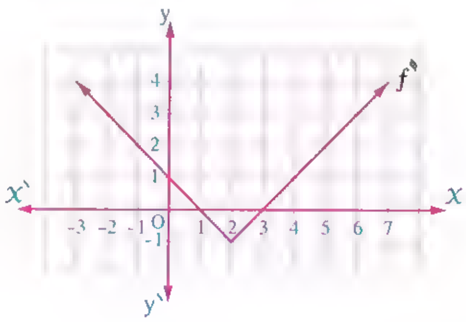
21 If $x = \frac{t}{1 + t}$, $y = \frac{t + 1}{t}$, then $\frac{d^2 y}{dx^2} =$ ....

(a) $\frac{2}{x}$
(b) $2x^{-3}$
(c) $-x^{-2}$
(d) zero

22 If $f : f(x) = 15x + 6x^2 - x^3$, then the function f has ....

(a) an inflection point (2 , 46)
(b) two inflection points at $x = -1$, $x = 5$
(c) no inflection point
(d) an inflection point is (0 , 2)

23 The opposite figure represents the curve of $\acute{f}(x)$, then the function f is convex upwards on the interval ....

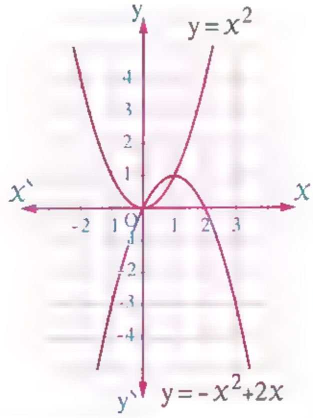


(a) $]1 , 3[$
(b) $\mathbb{R} - [1 , 3]$
(c) $]-\infty , 2[$
(d) $]2 , \infty[$

24 If $f(X)$ is a continuous function on $[-5, 8]$ and $\int_{-5}^{4} f(X)\, dX = 19$, $\int_{8}^{4} f(X)\, dX = 7$, then $\int_{-5}^{8} f(X)\, dX = \dots\dots$

(a) 26 (b) 12 (c) -12 (d) -26

25 **In the opposite figure :**



The volume of the solid generated by revolving the shaded area a complete revolution about the X-axis $= \dots\dots$ cube units.

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$

(c) $\frac{2\pi}{3}$ (d) π

26 If equation of the tangent to the curve of the function $f : f(X) = aX^3 + 2\sqrt{X}$ is $y = 4X - 2$ at $X = 1$, then $a = \dots\dots$

(a) 1 (b) 2 (c) $\sqrt{2}$ (d) 4

27 A ladder of length 10 m. rests with its upper end on a vertical wall and its lower end on a horizontal ground. If the lower end slipping away from the wall at speed 2 m./min , then the rate of change in the inclination angle of the ladder to the horizontal at the moment the lower end is 8 m. from the wall equals $\dots\dots$ rad./min.

(a) 3 (b) -3 (c) $\frac{1}{3}$ (d) $-\frac{1}{3}$

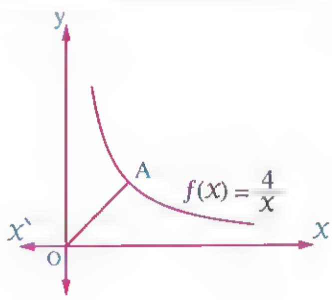
28 $\lim_{x \to 0} (e^{5X} + 2) = \dots\dots$

(a) e^5 (b) $e^5 + 2$ (c) 3 (d) 2

29 $\int \tan X\, dX = \dots\dots + c$

(a) $\ln|\cos X|$ (b) $-\ln|\sec X|$ (c) $\sec^2 X$ (d) $\ln|\sec X|$

30 **In the opposite figure :**



The least length of the line segment $\overline{OA} = \dots\dots$ length unit.

(a) $\sqrt{2}$ (b) 2

(c) $2\sqrt{2}$ (d) 4

Practice Exams

Exam 5

Answer the following questions :

1. $\lim\limits_{x \to 0} \left(\frac{\ln(1+2x)}{e^x - 1}\right) = \dots\dots$

(a) zero (b) 1 (c) 2 (d) $\frac{1}{2}$

2. The function $f : f(x) = -|x| + 1$ is decreasing in the interval $\dots\dots$

(a) $]0, \infty[$ (b) $]-\infty, 0[$ (c) $]1, \infty[$ (d) $]-\infty, 1[$

3. The straight line $y + x - 1 = 0$ touches the curve of the function $f : f(x) = x^2 - 3x + a$, then $a = \dots\dots$

(a) 1 (b) 2 (c) 3 (d) 4

4. If $\int_{-2}^{3} f(x)\,dx = 12$, $\int_{-2}^{5} f(x)\,dx = 16$, then $\int_{3}^{5} f(x)\,dx = \dots\dots$

(a) -28 (b) -4 (c) 4 (d) 28

5. The local minimum value of the function f, where : $f(x) = x^4 - 2x^2$ equals $\dots\dots$

(a) 1 (b) -1 (c) 0 (d) -4

6. $\int (2x - 1)\, e^{2x+3}\, dx = yz - \int z\, dy$, then $\int z\, dy = \dots\dots$

(a) $e^{2x+3} + c$ (b) $\frac{1}{2} e^{2x+3} + c$

(c) $-e^{2x+3} + c$ (d) $-\frac{1}{2} e^{2x+3} + c$

7. $\int \frac{x^3\, dx}{x^4 + 3} = \dots\dots + c$

(a) $\frac{1}{4}(x^4 + 3)$ (b) $\frac{1}{4} \ln|x^4 + 3|$ (c) $\ln|x^4 + 3|$ (d) $\frac{1}{4}(x^4 + 3)^{-1}$

8. In the function f : where $f(x) = \frac{x^2 + 9}{x}$ the absolute minimum value of the function f where $x \in [1, 6]$ equals $\dots\dots$

(a) 10 (b) 6 (c) 7.5 (d) zero

9. $\int \sec^5 x \tan x \, dx =$ .... + c

(a) $\frac{1}{6} \sec^6 x$ (b) $\frac{1}{5} \sec^5 x$ (c) $\sec^7 x$ (d) $\frac{1}{2} \tan^2 x$

10. The area of the region bounded by the curve $y = 6 - x^2$, and the straight line $y = -x$ equals .... square unit.

(a) 125 (b) 0 (c) $\frac{125}{6}$ (d) $\frac{55}{3}$

11. If $y = x^{n+1} + n x^{n-1} + 1$, then $\frac{d^n y}{d x^n} =$ ....

(a) $\underline{|n+1}$ (b) $x \underline{|n+1}$ (c) $x \underline{|n}$ (d) $x^{-1} \underline{|n}$

12. $\frac{d^3}{d x^3} (\sin^2 x) =$ ....

(a) $\sin 2x$ (b) $2 \cos 2x$ (c) $4 \cos 2x$ (d) $-4 \sin 2x$

13. When the region bounded by the curve $x = \frac{1}{\sqrt{y}}$, $1 \le y \le 4$ and y-axis revolves a complete revolution about y-axis , then the volume of the solid generated in cubic units equals ....

(a) $\frac{2}{3} \pi$ (b) $3\sqrt{2} \pi$ (c) $2 \pi \ln 2$ (d) $\frac{2}{3} \pi \log 3$

14. On the perpendicular coordinate system a straight line $\overleftrightarrow{AB}$ passes through the point C (3 , 2) and intersects the positive part of x-axis at the point A and the positive part of y-axis at the point B , then the smallest area of the triangle AOB equals .... square unit (where O is the origin).

(a) 12 (b) 6 (c) 3 (d) 24

15. A ladder of length two metres is leaning against a smooth vertical wall. If the top of the ladder slid down at the same rate as the lower end slid away from the wall , then the distance of the lower end from the wall equals .... m.

(a) 2 (b) $2\sqrt{2}$ (c) $\sqrt{2}$ (d) $-\sqrt{2}$

16. If $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = \sqrt[3]{x^2}(3x - 7)$ and the function is increasing for $x \in]-\infty , a[$, $x \in]b , \infty[$, then $7a + 30b =$ ....

(a) 14 (b) 28 (c) -28 (d) $\frac{98}{15}$

17 The slope of the normal to the curve : $x = \cos\theta$, $y = \sqrt{2} + \sin\theta$ at $\theta = \frac{\pi}{4}$ is ....

(a) 1 (b) -1 (c) zero (d) undefined

18 A regular hexagonal like lamina shrinks by cooling. The rate of change of its side length is 0.1 cm./sec. , then the rate of change in the area of the lamina when its side length is 10 cm. equals .... cm^2/sec.

(a) $3\sqrt{3}$ (b) $30\sqrt{3}$ (c) $-3\sqrt{3}$ (d) $-30\sqrt{3}$

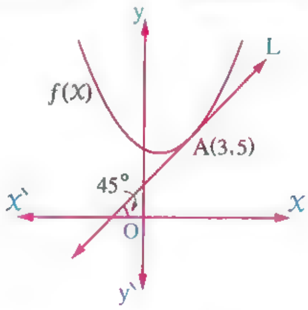
19 $\frac{d}{dx}\left(2 \sin \frac{\pi}{8} \cos \frac{\pi}{8}\right) =$ ....

(a) $\sin \frac{\pi}{4}$ (b) $\cos \frac{\pi}{4}$ (c) $\frac{1}{4} \cos \frac{\pi}{4}$ (d) zero

20 If : $f(x) = \frac{x^{65}}{\underline{|65}}$, then $f^{(65)}(x) =$ ....

(a) zero (b) 1 (c) x (d) $\frac{x}{\underline{|65}}$

21 The opposite figure represents the function f and the straight line L touches the curve of f at the point A (3 , 5) and $g(x) = x \cdot f(x)$, then $\grave{g}(3) =$ ....



(a) 3 (b) 1

(c) 5 (d) 8

22 The height of right circular cone equals to its base diameter , the rate of change of its base radius $= \frac{1}{\pi}$ cm./sec. , then the rate of change of its volume = ... cm^3/sec. when its base raduis = 5 cm.

(a) 50π (b) $\frac{320}{3}\pi$ (c) 150 (d) 50

23 $\lim_{x \to 0} \frac{6^x - 1}{3x} =$ ....

(a) 3 ln 8 (b) 3 ln 3 (c) $\frac{1}{3}$ ln 6 (d) $\frac{1}{3} e^6$

24 If $y^x = x$, then $\frac{dy}{dx} = 0$ at $x =$ ....

(a) zero (b) 2 (c) e (d) $\frac{1}{e}$

25 $\int \frac{dx}{e^{x} + e^{-x} + 2} = \cdots\cdots\cdots + c$

(a) $\frac{1}{e^{x} + 1}$ (b) $-\frac{1}{e^{x} + 1}$ (c) $\frac{2}{e^{x} + 1}$ (d) $-\frac{2}{e^{x} + 1}$

26 The function $f : f(x) = x^4 - 4x^2$ has ....

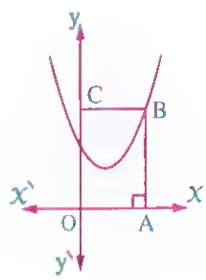
(a) local minimum value and two local maximum values.

(b) Two different local minimum values and one local maximum value.

(c) Two local minimum values and no local maximum value.

(d) Two equal local minimum values and one local maximum value.

27 **In the opposite figure :**



B $\in$ the curve $y = x^2 - 5x + 14$

, then the least perimeter of the rectangle

OABC eaquals .... length unit.

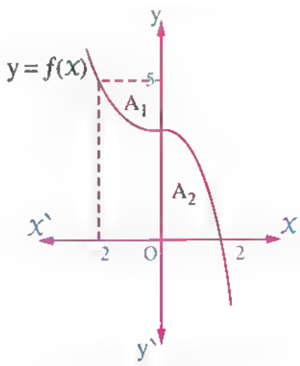
(a) 10 (b) 12

(c) 16 (d) 20

28 If $f(x) = \begin{cases} 2x - 1 & -1 \le x \le 2 \\ 3 & 2 < x < 5 \end{cases}$, then $\int_{-1}^{4} f(x)\, dx = \cdots\cdots\cdots$

(a) 4 (b) 5 (c) 6 (d) 7

29 **In the opposite figure :**



$A_1 = 2$ square units ,

$A_2 = 7$ square units

, then $\int_{-2}^{2} f(x)\, dx = \cdots\cdots\cdots$

(a) 5 (b) 9

(c) 15 (d) 19

30 $\int (\sin^2 x + \sin^2 x \tan^2 x)\, dx = \cdots\cdots\cdots + c$

(a) $\sin^2 x + \csc^2 x$ (b) $\tan x - x$ (c) $\tan^2 x$ (d) $\sec x$

Practice Exams

Exam 6

Answer the following questions :

1. The rate of change of tangent slope of the function $f : f(x) = 2x^3$ at $x = 3$ equals ....

   (a) 54 (b) 36 (c) 18 (d) 9

2. The function $f : f(x) = x^x$ has a stationary point at $x =$ ....

   (a) e (b) $\frac{1}{e}$ (c) 1 (d) $\sqrt{e}$

3. $\frac{d}{dx}\int_2^3 x\sqrt{x^2+1}\,dx =$ ....

   (a) -1 (b) zero (c) 1 (d) 2

4. The slope of the tangent to the curve of the function $y = f(x)$ at a particular point is $\frac{1}{2}$ and the x-coordinate of this point decreases at a rate 3 units/sec. , then the rate of change of its y-coordinate equals .... unit/sec.

   (a) $-\frac{1}{6}$ (b) $-\frac{3}{2}$ (c) $\frac{1}{6}$ (d) $\frac{3}{2}$

5. The shortest distance between the point (0 , 5) and the curve $y = \frac{1}{2}x^2 - 4$ equals .... length units.

   (a) 4 (b) zero (c) 17 (d) $\sqrt{17}$

6. If $y = \sin^3\theta$, $z = \cos^3\theta$, then $\frac{dy}{dz} =$ ....

   (a) $-\sin\theta$ (b) $\cos\theta$ (c) $-\tan\theta$ (d) $3\sin 2\theta$

7. The slope of the tangent to the curve at a point (x , y) which lies on it is $x\sqrt{x+1}$, then the equation of the curve given that it passes through $\left(0 , \frac{11}{15}\right)$ is ....

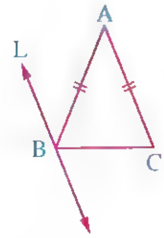
   (a) $15y = 15x + 11$
   (b) $y = \frac{2}{5}(x+1)^{\frac{5}{2}} - \frac{2}{3}(x+1)^{\frac{3}{2}}$
   (c) $y = \frac{5}{2}(x+1)^{\frac{5}{2}} - \frac{3}{2}(x+1)^{\frac{3}{2}} + 1$
   (d) $y = \frac{2}{5}(x+1)^{\frac{5}{2}} - \frac{2}{3}(x+1)^{\frac{3}{2}} + 1$

8. If $y = a(1 - \cos\theta)$, $x = a(\theta + \sin\theta)$, then $\frac{dy}{dx} =$ ....

   (a) $\tan\theta$ (b) $\cot\theta$ (c) $\tan\frac{\theta}{2}$ (d) $\cot\frac{\theta}{2}$

9 **In the opposite figure :**

ABC is isosceles triangle in which AB = AC = 13 cm. , BC = 10 cm. If the straight line L moves from point B parallel to $\overleftrightarrow{AC}$ in direction of $\overrightarrow{BC}$ to intersects $\overline{AB}$, $\overline{BC}$ at D , E respectively where the rate of change in BE = $\frac{1}{10}$ cm./min. , then the rate of change of the area of Δ DEB at BE = 1 cm. equals .... cm^2/sec.



(a) 0.48 (b) 0.12 (c) 0.96 (d) 0.24

---

10 The local maximum value of the curve : $y = \sin x(1 + \cos x)$ where $x \in]0 , \frac{\pi}{2}[$ equals ....

(a) $-3\sqrt{3}$ (b) $\frac{3}{4}\sqrt{3}$ (c) $\frac{1}{2}$ (d) $\frac{\pi}{3}$

---

11 If $y = \pi^{\sin x} + e^{\pi}$, then $\frac{dy}{dx} =$ ....

(a) $\pi^{\cos x}$ (b) $\sin x \times \pi^{\sin x - 1}$

(c) $\pi^{\sin x} \cos x$ (d) $\pi^{\sin x} \cos x \ln \pi$

---

12 $\int 2\cos^2 x \, dx =$ .... + c

(a) $1 + \frac{1}{2}\sin 2x$ (b) $x + \frac{1}{2}\sin 2x$

(c) $1 - 2\sin 2x$ (d) $x + \sin 2x$

---

13 If the function $f : f(x) = x^2 + \frac{b}{x}$ has a critical point at $x = 2$, then b = ...

(a) 16 (b) 4 (c) -1 (d) $\frac{1}{16}$

---

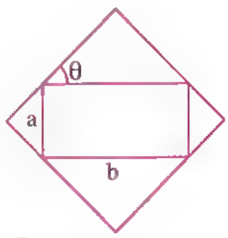
14 If $\int \frac{dx}{1 - \sin x} = a \tan x + b \sec x + c$ $\therefore a^2 + b^2 =$ ....

(a) 2 (b) 1 (c) 4 (d) 5

---

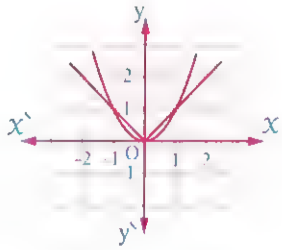
15 **In the opposite figure :**

The greatest area of a rectangle that can be drawn outside a rectangle whose dimensions are constants a and b equals .... square unit.



(a) $(a + b)^2$ (b) $a^2 + b^2 + ab$ (c) $\frac{(a + b)^2}{2}$ (d) ab

16 **In the opposite figure :**



The area of the region bounded by the two curves

$y = x^2$, $y = |x|$ equals ....

(a) $2 \int_{-1}^{0} (x^2 - x)\, dx$

(b) $\int_{0}^{1} (x - x^2)\, dx$

(c) $2 \int_{0}^{1} (x - x^2)\, dx$

(d) $\int_{-1}^{1} (x - x^2)\, dx$

---

17 If $y = \ln\sqrt{\tan x}$, then $\frac{dy}{dx} =$ .... at $x = \frac{\pi}{4}$

(a) 1 (b) zero (c) $\frac{1}{2}$ (d) ∞

---

18 If $y = \cos x$, then $\left(\frac{dy}{dx}\right)^2 - y \frac{d^2 y}{dx^2} =$ ....

(a) zero (b) 1 (c) -1 (d) $\cos 2x$

---

19 If $f(x) = 8x \sin x \cos x \cos 2x$, then $\acute{f}\left(\frac{\pi}{8}\right) =$ ....

(a) -2 (b) zero (c) 1 (d) 2

---

20 If $x = \left(1 - \sqrt[3]{y}\right)\left(1 + \sqrt[3]{y} + \sqrt[3]{y^2}\right)(1 + y + y^2)$, then $\frac{dy}{dx} =$ ....

(a) $3y^2$ (b) $-3y^2$ (c) $-\frac{1}{3y^2}$ (d) $\frac{1}{3y^2}$

---

21 The length of the intercepted part of y-axis by the tangent to the curve $y = x \sin x$ at $x = \pi$ equals .... length unit.

(a) $-\pi$ (b) π (c) $-\pi^2$ (d) π^2

---

22 A cuboid of dimensions 3 , 4 and 12 cm. If the rate of the increase of its first dimension is 2 cm./sec. and the second dimension is 1 cm./sec. but the rate of the decrease of its third dimension is 3 cm./sec. , then the rate of change of its volume at the end of two second = ... cm^3/sec.

(a) -12 (b) 468 (c) 144 (d) 252

---

23 $\lim\limits_{x \to 0} \frac{\sin 4x}{3^x - 1} =$ ....

(a) $\frac{4}{\ln 3}$ (b) $\frac{4}{e^3}$ (c) $4e^3$ (d) $\frac{1}{\ln 3}$

24 If $e^{xy} - x^2 + y^3 = 0$, then $\frac{dy}{dx}$ (at $x = 0$) equals ....

(a) -1 (b) $\frac{-1}{3}$ (c) 1 (d) $\frac{1}{3}$

25 The absolute maximum value of the function $f : f(x) = x + \frac{1}{x}$ in the interval $\left[\frac{1}{2}, 3\right]$ equals ....

(a) $2\frac{1}{2}$ (b) $4\frac{1}{4}$ (c) 3 (d) $3\frac{1}{3}$

26 ABC is a right-angled triangle at B in which : AB + BC = 20 cm. , then the greatest possible area for this triangle equals .... cm^2

(a) 50 (b) 10 (c) 150 (d) 100

27 $\int x(x-5)^3 \, dx =$ .... + c

(a) $\frac{1}{5}(x-5)^5 + \frac{5}{4}(x-5)^4$ (b) $4x(x-5)^4 + 20(x-5)^5$
(c) $\frac{1}{4}(x^2 - 5x)^4$ (d) $4(x-5)^3 + 15(x-5)^2$

28 If $f(x) = \begin{cases} x^2 & , x < 2 \\ 3x - 2 & , x \geq 2 \end{cases}$, then $\int_0^3 f(x) \, dx =$ ....

(a) $\frac{49}{6}$ (b) 9 (c) $\frac{15}{2}$ (d) $\frac{25}{3}$

29 The volume of the solid generated by revolution the region bounded by the curve : $y^2 = 4 - x$ and the two positive parts of the coordinate axes a complete revolution about y-axis equals .... volume units.

(a) $\frac{256}{15}\pi$ (b) $\frac{16}{3}\pi$ (c) $\frac{1472}{15}\pi$ (d) 8π

30 $\int \sin^2 x \, dx =$ .... + c

(a) $\frac{1}{2}x - \frac{1}{2}\sin x$ (b) $\frac{1}{3}\sin^3 x$
(c) $-\cos^2 x$ (d) $\frac{1}{2}x - \frac{1}{4}\sin 2x$

Practice Exams

Exam 7

Answer the following questions :

1. The tangent to the curve of the function $y = \sqrt[3]{x}$ at $x = 0$ is parallel to ....
   (a) x-axis. (b) y-axis.
   (c) the straight line $y = x$ (d) $x + y = 0$

2. If $\int_1^3 f(x)\, dx = 5$, $\int_3^4 f(x)\, dx = 2$, $\int_2^4 f(x)\, dx = 6$
   , then $\int_2^1 f(x)\, dx =$ ....
   (a) 1 (b) 13 (c) -2 (d) -1

3. $\lim\limits_{x \to 6} \dfrac{e^x - e^6}{x - 6} =$ ....
   (a) 1 (b) -1 (c) zero (d) e^6

4. The maximum local value of the function $y = \dfrac{\ln x}{x}$ in the interval $[2, \infty[$ is ....
   (a) 1 (b) $\frac{2}{e}$ (c) e (d) $\frac{1}{e}$

5. The function $f : f(x) = \dfrac{x^2 + 1}{x^2 + 3}$ is convex downward on the interval ....
   (a) $]-1, 1[$ (b) $]-\infty, -1[,]1, \infty[$
   (c) $]0, \infty[$ (d) $]-\infty, 0[$

6. If $f''(x) = \frac{1}{2} [e^x + e^{-x}]$, $f(0) = 1$, $f'(0) = 0$, then $f(x) =$ ....
   (a) $-f'(x)$ (b) $f'(x)$ (c) $-f''(x)$ (d) $f''(x)$

7. If $x^2 y = 2x + 5$, prove that : $x^2 \dfrac{d^2y}{dx^2} + 4x \dfrac{dy}{dx} =$ ....
   (a) $-2y$ (b) $-y$ (c) $2y$ (d) y

8. When the region bounded by the curve $y = x^2$ and the straight line $y = 2$ revolves a complete revolution about y-axis , then the volume of the generated solid equals ....
   (a) $\int_0^2 y\, dx$ (b) $\pi \int_0^2 y\, dy$ (c) $\pi \int_0^2 x\, dx$ (d) $\pi \int_0^2 x^2\, dx$

9 An equilateral triangle , its side length increase at a rate $\frac{1}{3}$ cm./sec. , then the rate of change of its perimeter at this instant equals .... cm./sec.

(a) 1 (b) 2 (c) 3 (d) 4

10 If $x = 2t^3 + 3$, $y = t^4$, then $\frac{d^2y}{dx^2} =$ .... at t = 1

(a) 9 (b) $\frac{1}{9}$ (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

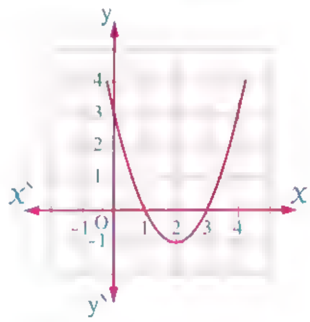
11 $\int \cot^3 x \, dx =$ .... + c

(a) $\frac{1}{4} \cot^4 x$
(b) $(\ln |\sin x|)^3$
(c) $-\frac{1}{2} \cot^2 x + \ln |\csc x|$
(d) $\frac{1}{2} \cot^3 x + \csc^2 x$

12 The function $f : f(x) = x^3 + 4x + 2$ is increasing for every $x \in$ ....

(a) $\mathbb{R}^+$ (b) $\mathbb{R}$ (c) $\mathbb{R}^-$ (d) $\mathbb{R} - \{0\}$

13 The opposite figure represents the curve $f'(x)$, then the function f has a local minimum at $x =$ ....



(a) 1
(b) 2
(c) 3
(d) 4

14 A metalic circular sector whose area is 16 cm^2 , then the radius length of the sector's circle which makes its perimeter as minimum as possible equals .... cm.

(a) 4 (b) 8 (c) 64 (d) 2

15 The area of the region bounded by the two curves : $y = 2x^2$, $y = 3 - x^2$ equals .... square unit.

(a) 2 (b) 4 (c) 0 (d) 8

16 If the slope of the normal to the curve at any point (x , y) on it equals $\frac{5 + 2y}{2 - 3x^2}$ and the curve passes through (1 , 2) , then its equation is ....

(a) $y^2 + 5y = x^3 - 3x$
(b) $y^2 + 5y = x^3 - 2x + 15$
(c) $2y + 5 = 3x^2 - 12$
(d) $y^2 + 5y = x^3 - 2x - 15$

17 The equation of the tangent to the curve : $2 + \ln y \,.\, \ln x = x^2 + y$ at the point whose x coordinate is 1 is ....

ⓐ $2x + y = 0$ ⓑ $2x + y = 3$ ⓒ $-2x + y = 3$ ⓓ $x - 2y = 3$

18 A man of tall 180 cm. moves far from the base of a 3-metre lamp post at a rate of 1.2 m./sec. , then the rate of change of the length of the man's shadow = ... m./sec.

ⓐ $\frac{5}{9}$ ⓑ 1.5 ⓒ 2.25 ⓓ 1.8

19 $\frac{d}{dx}[(\sec x - 1)(\sec x + 1)] =$ ....

ⓐ $\sec^2 x \tan^2 x$ ⓑ $2\sec^2 x \tan x$

ⓒ $\sec^2 x \tan x$ ⓓ $\sec^4 x$

20 If $\sin x \cos y - \sin y \cos x = 1$, then $\frac{dy}{dx} =$ .... where $x , y \in]0 , 2\pi[$

ⓐ $\cos(x - y)$ ⓑ $\sin(x - y)$ ⓒ 1 ⓓ -1

21 The straight line : $13x - y - 7 = 0$ touches the curve $y = a x^3 + b x^2$ at the point $(1 , 6)$, find the value of $a\,b =$ ....

ⓐ 1 ⓑ 6 ⓒ 5 ⓓ 13

22 A man walk across a bridge , 12 m. high above water surface , the speed of the man is 3 m./min. , the man observed a boat moving perpendicular to the bridge with a constant speed 6 m./min. exactly under the man , then the rate of diverge between the man and the boat after 6 minutes from the moment that they are on the same vertical line = ... m./minute

ⓐ $\frac{45}{7}$ ⓑ 540 ⓒ $\frac{2700}{7}$ ⓓ $3\sqrt{5}$

23 If $x e^{xy} = y + \sin^2 x$, then : $\frac{dy}{dx}$ (at $x = 0$) equals ....

ⓐ -1 ⓑ -2 ⓒ 2 ⓓ 1

24 The area of the largest rectangle that can be inscribed in a circle of radius 4 cm. equals .... cm.^{2}

ⓐ 32 ⓑ $4\sqrt{2}$ ⓒ 8 ⓓ 64

25 $\int x^3 e^{x^2} \, dx = \cdots\cdots$

(a) $\frac{1}{2} x^2 e^{x^2} - x e^{x^2}$
(b) $\frac{1}{2} x^2 e^{x^2} + \frac{1}{2} e^{x^2}$
(c) $2 x^2 e^{x^2} - 4 e^{x^2}$
(d) $\frac{1}{2} x^2 e^{x^2} - \frac{1}{2} e^{x^2}$

26 The derivative of $\sin^2 x$ with respect to $\cos^2 x$ is $\cdots\cdots$

(a) $-\tan^2 x$
(b) $\tan^2 x$
(c) $-(\sin^2 x + \cos^2 x)$
(d) $\cos^2 x - \sin^2 x$

27 $\int (1 + \cos x)^2 \, dx = \cdots\cdots + c$

(a) $(1 + \sin x)^2$
(b) $\frac{1}{3} (1 + \cos x)^3$
(c) $\frac{3}{2} x + 2 \sin x + \frac{1}{4} \sin 2x$
(d) $\frac{3}{2} + 2 \cos x + \frac{1}{2} \cos 2x$

28 If $f(x) = \begin{cases} 2 & , \quad x < 2 \\ x & , \quad x \geq 2 \end{cases}$, then $\int_0^6 f(x) \, dx = \cdots\cdots$

(a) 18
(b) 20
(c) 12
(d) 24

29 An architect has designed an arc -like entryway of a hotel whose equation $y = -\frac{1}{2} (x - 1)(x - 7)$ where x in metres. How much does the glass cost if this entryway is covered by the glass which costs L.E. 1500 per square metre ?

(a) L.E. 27 000
(b) L.E. 18
(c) L.E. 13 500
(d) L.E.54 000

30 If g is an increasing function on $\mathbb{R}$, k is a decreaseing function on $\mathbb{R}$ and $f(x) = 4 g(x) - 3 k(x)$, then the function f $\cdots\cdots$ on $\mathbb{R}$

(a) increases
(b) decreases
(c) constant
(d) is a zero

Practice Exams

Exam 8

Answer the following questions :

1. The function $f : f(x) = x^3 + 4x + 2$ is increasing , then $x \in$ ....

   (a) $]-4 , \infty[$ only.   (b) $\mathbb{R}$
   (c) $]-\infty , \frac{-4}{3}[$ only.   (d) $]\frac{-4}{3} , \infty[$ only.

2. The tangent equation of the curve of the function $f : f(x) = e^{2x+1}$ at the point $\left(-\frac{1}{2} , 1\right)$ is ....

   (a) $2y = x + 1$ (b) $y = 2x + 2$ (c) $y = 2x - 3$ (d) $2y = 3x + 1$

3. $\int_0^{10\pi} |\sin x| \, dx =$ ....

   (a) 10   (b) 10π (c) 20   (d) 20π

4. The absolute minimum value of the function f , where $f(x) = x + \frac{1}{x}$ in the interval $\left[\frac{1}{2} , 3\right]$ equals ....

   (a) 3   (b) 2   (c) 1   (d) $2\frac{1}{2}$

5. A regular octagon , its side length is 10 cm. the side length increase at a rate 0.2 cm./sec. , then the rate of increasing of its area at this moment = ... cm^2 /sec.

   (a) 19.3   (b) 4.825   (c) 193   (d) 38.6

6. $\lim\limits_{x \to 0} \frac{a^x + b^x + c^x - 3}{x} =$ ....

   (a) $\ln(a + b + c)$ (b) $\ln(a\,b\,c)$
   (c) $\ln a - \ln b - \ln c$ (d) $\log(a\,b\,c)$

7. The normal equation to the curve $y = f(x)$ at the point (1 , 1) is $x + 4y = 5$, then $f'(1) =$ ....

   (a) -3 (b) $-\frac{1}{4}$ (c) 4   (d) -4

8. The radius length of a circle increases at a rate 2 cm./min. and its area of a rate 20π cm^2/min. , then the radius length at this instant equals .... cm.

   (a) $\frac{5}{2}$ (b) 5   (c) 10   (d) 20

9 If $y = \ln(\sec x + \tan x)$, then $\frac{dy}{dx} = \text{.............}$

(a) $\tan x$ (b) $\sec x$ (c) $\tan^2 x$ (d) $\csc x$

10 If $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = x^3 - k x^2 + 12 x + 7$ is one to one function , then :

(a) $k \in \mathbb{R} - \{-6 , 6\}$ (b) $k \in]-\infty , -6]$

(c) $k \in [-6 , 6]$ (d) $k \in [6 , \infty[$

11 $\int (1 + 4 x^4) e^{x^4} \, dx = \text{.............} + c$

(a) $x + x e^{x^4}$ (b) $x e^{x^4}$ (c) e^{x^4} (d) $\frac{1}{4} e^{x^4}$

12 The length of the hypotenuse in a right-angled triangle equals 10 cm. , then the length of each side of the right angle when the area is as great as possible equals ....

(a) $\sqrt{10}$ cm. , $\sqrt{10}$ cm. (b) $5\sqrt{2}$ cm. , $5\sqrt{2}$ cm.

(c) $10\sqrt{2}$ cm. , 5 cm. (d) $2\sqrt{5}$ cm. , $2\sqrt{15}$ cm.

13 The slope of the tangent at any point (x , y) on the curve $y = f(x)$ is : $6 x^2 - 30 x + 36$, then given that the curve has a local maximum value equals 28 the equation of the curve is ....

(a) $y = 2 x^3 - 15 x^2$ (b) $y = 2 x^3 - 15 x^2 + 36 x$

(c) $y = 2 x^3 - 15 x^2 + 36 x + 28$ (d) $y = 6 x^2 - 30 x + 8$

14 The area of the region bounded by the two curves $y = x^2$, $y = x^3$ is .... square unit.

(a) 1 (b) $\frac{7}{12}$ (c) $\frac{1}{12}$ (d) 2

15 If $y = 1 + \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \cdots$ to ∞ , then $2 \ddot{y} + 3 \dot{y} - 4 y = \text{.............}$

(a) y (b) zero (c) $2 y$ (d) $9 y$

16 If $y = e^x \sin x$, then $\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + 2 y = \text{.............}$

(a) 0 (b) e^x (c) $e^x \sin x$ (d) $e^x \cos x$

17 $\int \frac{\sin^6 x}{\cos^8 x} \, dx = \text{.............}$

(a) $\tan^7 x + c$ (b) $\frac{1}{7} \tan^7 x + c$ (c) $\frac{1}{7} \tan 7x + c$ (d) $\sec^7 x + c$

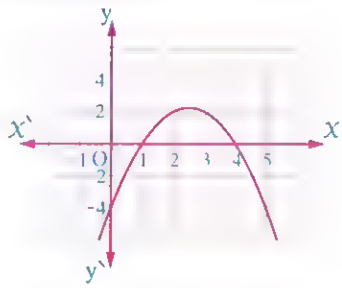
18 The volume of the solid generated by revolving the plane region bounded from the top by the curve $x^2 + y^2 = 4$ and from the bottom by the two straight lines $y = x$, $y = -x$ a complete revolution about x-axis equals .... cubic unit.

(a) $\frac{16\sqrt{2}}{3}\pi$ (b) $\frac{8\sqrt{2}}{3}\pi$ (c) $\frac{32\sqrt{2}}{3}\pi$ (d) $\frac{4\sqrt{2}}{3}\pi$

19 If the function $f : f(x) = x^3 - a x^2 + b$ has local minimum value at point $(2, 4)$, then the value of $a \times b = \text{.............}$

(a) -12 (b) zero (c) 12 (d) 24

20 The opposite figure represents $\acute{f}$, then the function f has a local maximum value at $x = \text{.............}$



(a) 1
(b) 4
(c) 0
(d) 2.5

21 A factory is producing electric appliances profits L.E. 30 in every appliance if it produces 50 appliances monthly. When the production increased than that , the profit in the appliance decreases by 50 piasters for every extra appliance produced , then the number of appliances produced monthly to get maximum profit = ... appliance.

(a) 52 (b) 55 (c) 60 (d) 65

22 If $\int (2x - 1) \ln x \, dx = yz - \int z \, dy$, then $yz = \text{.............}$

(a) $(2x - 1) \ln x$ (b) $\frac{2x - 1}{x}$ (c) $(x^2 - x) \ln x$ (d) $x - 1$

23 **In the opposite figure :**

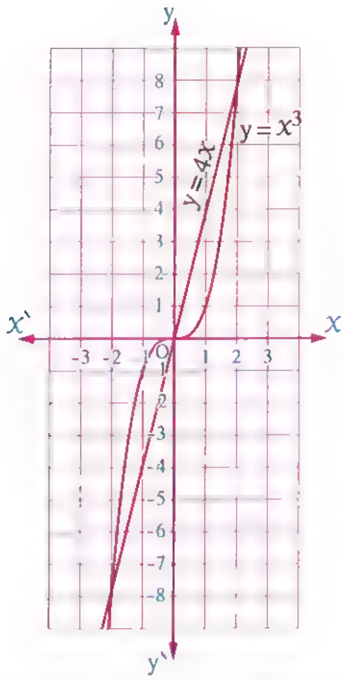
The area of the shaded region = ... square units

(a) 4

(b) 8

(c) 12

(d) 16



24 $\int \frac{1}{\cos x - 1}\, dx =$ .... + c

(a) $\sec x + \tan x$ (b) $-\csc x - \cot x$ (c) $\sin x - x$ (d) $\csc x + \cot x$

25 If $y = \sin x - \cos x$, then $\frac{d^{17} y}{d x^{17}} =$ ....

(a) $\sin x + \cos x$ (b) $\sin x - \cos x$

(c) $\cos x - \sin x$ (d) $-(\sin x + \cos x)$

26 If $\sin(x y) + \cos(x y) =$ zero , then $\frac{dy}{dx} =$ ....

(a) $x y$ (b) $-x y$ (c) $\frac{x}{y}$ (d) $\frac{-y}{x}$

27 Equation of the normal to the curve $y^2 (1 + x^2) = 8$ at the point $(-1 , 2)$ is ....

(a) $x + y - 1 = 0$ (b) $x - y + 3 = 0$ (c) $y + 2 = -(x - 1)$ (d) $y + 1 = x - 2$

28 If $y = \ln z$, $z = e^{3n}$, $n = \sin^2 x$, then $\frac{dy}{dx} =$ .... at $x = \frac{\pi}{3}$

(a) $\frac{3}{2}$ (b) $\frac{-3}{2}$ (c) $\frac{-3\sqrt{3}}{2}$ (d) $\frac{3\sqrt{3}}{2}$

29 A cylindrical tank the length of its base raduis 25 cm. and its height is 120 cm.

Oil is flow into the tank at a rate $\frac{5000}{\ell + 40}$ cm^3/sec. where ℓ is the oil height at any moment then the rate of increasing in the oil height inside the tank equals .... cm./sec. when it is half full.

(a) $\frac{4}{25}\pi$ (b) $\frac{1}{25\pi}$ (c) $\frac{2}{25\pi}$ (d) $\frac{8}{25\pi}$

30 $\int \frac{\sin 2x + 2\cos x}{\sin^2 x + 2\sin x + 1} \, dx =$ .... $+ c$

(a) $\sin x - \ln|\csc x + \cot x|$ (b) $\ln|1 + \sin x|$

(c) $2\ln|1 + \sin x|$ (d) $x + \cos x$

Practice Exams ? Exam 9

Answer the following questions :

1. If $\int_{1}^{4} f(x)\,dx + \int_{2b}^{8} f(x)\,dx = \int_{1}^{8} f(x)\,dx$, then $b =$ ....

(a) 2 (b) 4 (c) 1 (d) 8

2. If $y = a \cos(\ln x) + b \sin(\ln x)$, then $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} =$ ....

(a) 0 (b) y (c) $-x\,y$ (d) $-y$

3. $\lim\limits_{x \to 0} (1 + 3\tan^2 x)^{\cot^2 x} =$ ....

(a) e (b) e^3 (c) 3 e (d) $e^{\frac{1}{3}}$

4. If $x > 0$, then the smallest value of the expression $x + \frac{1}{x}$ equals ....

(a) zero (b) 1 (c) 2 (d) 4

5. The slope of the tangent to the curve of the function f at any point (x , y) on it is given by the relation $g(x) = x\,e^{3x}$, then the equation of the curve is .... given that it passes through the point $\left(\frac{1}{3} , 5\right)$

(a) $y = \frac{1}{3} x\,e^{3x}$
(b) $y = \frac{1}{9} x\,e^{3x} + 5 - \frac{e}{9}$
(c) $y = \frac{1}{3} x\,e^{3x} - \frac{1}{9} e^{3x}$
(d) $y = \frac{1}{3} x\,e^{3x} - \frac{1}{9} e^{3x} + 5$

6. The ratio between the slope of the tangent to the curve $y_1 = \ln 3\sqrt{x+1}$ and the slope of the tangent to the curve $y_2 = \ln 5\sqrt{x+1}$ at $x = a$ is ....

(a) 3 : 5 (b) 5 : 3 (c) 1 : 1 (d) ln 3 : ln 5

7. If $x = \ln t$, $y = \sin t$, then $\frac{dy}{dx} =$ ....

(a) cos t (b) t cos t (c) $t^2 \sin t$ (d) $t^2 \cos t$

8. The length of each of two equal sides in an isosceles triangle equals 6 cm. and the measure of the angle between them equals (x) , the rate of change of x is $\left(\frac{\pi}{90}\right)^{\text{rad.}}$ per minute , then the rate of change of its area at $x = 30°$ is ....

(a) $\frac{\sqrt{3}}{10}\pi$ (b) $\frac{\pi}{10}$ (c) $9\sqrt{3}$ (d) 9

9 If $f(x) = \ln\left(x + \sqrt{x^2+1}\right)$, then $f'(x) = \cdots\cdots$

ⓐ $\sqrt{x^2+1}$ ⓑ $\frac{x}{\sqrt{x^2+1}}$ ⓒ $1 + \frac{x}{\sqrt{x^2+1}}$ ⓓ $\frac{1}{\sqrt{x^2+1}}$

10 $\int (\sin^2 x + \cos^2 x + \cot^2 x)\, dx = \cdots\cdots + C$

ⓐ $\csc^2 x$ ⓑ $\cot x$ ⓒ $-\cot x$ ⓓ $x + \frac{1}{3}\cot^3 x$

11 If the function $f : f(x) = x^3 - 3x + 4$, then the function is decreasing on the interval $\cdots\cdots$

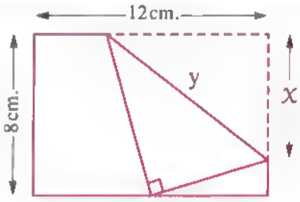
ⓐ $]-\infty , 0[$ ⓑ $]0 , -\infty[$ ⓒ $]-\infty , -1[,]1 , \infty[$ ⓓ $]-1 , 1[$

12 The normal equation to the curve $y = x\,|x|$ at the point $(-2 , -4)$ is $\cdots\cdots$

ⓐ $y + 4x + 12 = 0$ ⓑ $4y + x + 18 = 0$

ⓒ $4y + x + 14 = 0$ ⓓ $y + 4x - 4 = 0$

13 The top right corner of a piece of paper whose dimensions are 8 cm. , 12 cm. is folded to the lower edge as shown in the figure , then the value of x which makes y as small as possible = $\cdots\cdots$



ⓐ 6 ⓑ 4 ⓒ 2 ⓓ 8

14 $\pi \int_{-2}^{2} (4 - x^2)\, dx$, is the volume of $\cdots\cdots$

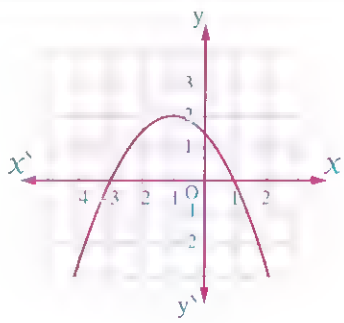
ⓐ A sphere whose radius is 4 units.

ⓑ A right circular cone whose height is 4 units.

ⓒ A sphere whose radius is 2 units.

ⓓ A right circular cylinder whose height is 4 units.

15 The opposite figure represents the curve $f'(x)$ of the function f , then the solution set of the inequality $f''(x) > 0$ is $\cdots\cdots$



ⓐ $]-1 , \infty[$

ⓑ $]1 , \infty[$

ⓒ $]-\infty , -1[$

ⓓ $]-\infty , 1[$

16 If $y = \sin(5x^3) \csc(5x^3)$, then $\frac{dy}{dx} =$ ....

(a) 0

(b) $-25 \cos(5x^3) \times \csc(5x^3) \cot(5x^3)$

(c) $15x^2 \cos(5x^3) - 15x^2 \csc(5x^3) \cot(5x^3)$

(d) 1

---

17 The area of the region bounded by the curve : $y = x^2 - 9$, the x-axis , the straight line $x = 4$ and above x-axis = ... square unit

(a) $\frac{44}{3}$ (b) 18 (c) $\frac{10}{3}$ (d) $\frac{98}{3}$

---

18 The gas leaks from a spherical balloon at a rate 20 cm^3/sec. , then the rate of change of the balloon external surface area at the moment which the radius length is 10 cm. equals .... cm^2/sec.

(a) -4 (b) 80π (c) -2 (d) -1

---

19 If $\sqrt{x} + \sqrt{y} = 1$, then $\frac{dy}{dx}$ at the point $\left(\frac{1}{4}, \frac{1}{4}\right)$ equals ....

(a) $\frac{1}{2}$ (b) 1 (c) -1 (d) 2

---

20 If f is a fifth degree polynomial , then the fifth derivative of the function f equals ....

(a) x (b) $5x$

(c) zero (d) non zero constant

---

21 A man his tall is 1.8 m. moves towards the base of a lamp post of height 9.6 m. at speed 2.6 m./sec. , then the rate of change of the man's shadow = ... m./sec.

(a) -0.4 (b) -0.6 (c) 0.2 (d) 0.8

---

22 $\int \frac{\ln x^5}{x \ln x^3} dx = $ .... + c

(a) $\frac{5}{3} \ln|x|$ (b) $\frac{3}{5} \ln|x|$ (c) $\ln(\ln|x|)$ (d) $\ln\left(\frac{5}{3}|x|\right)$

---

23 If the two curves of the functions f and g are touching at the point (2 , 4) and $f'(2) = 3$, then $g'(2) = $ ....

(a) 2 (b) 3 (c) 4 (d) 5

24 If $a^y = b^x$ where $a , b \in \mathbb{R}^+$, then $\frac{dy}{dx} = \dots\dots$

(a) $\log \frac{a}{b}$ (b) $\log_a b$ (c) $\log_b a$ (d) $\log \frac{b}{a}$

25 If the function $f : f(x) = k x^2 + (k + 5) x + k - 2$ has local maximum value at $x = 2$, then $k = \dots\dots$

(a) -2 (b) -1 (c) zero (d) 1

26 If $a \neq b$ and $\int_a^b (3 x^2 - 1) \, dx =$ zero , then $a^2 + b^2 = \dots\dots$

(a) $a b$ (b) $1 - a b$ (c) $a b + 1$ (d) $a + b$

27 If the perimeter of a circular sector is constant p , then the area has maximum value at $r = \dots\dots$

(a) $\frac{p}{2}$ (b) $\frac{2}{\sqrt{p}}$ (c) $\sqrt{p}$ (d) $\frac{p}{4}$

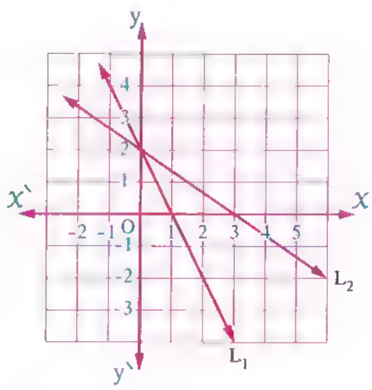
28 The absolute maximum value of the function f where $f(x) = 10 x e^{-x}$, $x \in$ [zero , 4] is $\dots\dots$

(a) $\frac{10}{e}$ (b) zero (c) 1 (d) e

29 If $f(x) = \frac{1 - \cot x}{1 + \cot x}$, then $f'\left(\frac{\pi}{4}\right) = \dots\dots$

(a) zero (b) 1 (c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

30 **In the opposite figure :**



The volume of the solid generated by revolving the shaded area a complete revolution about the y-axis = $\dots\dots$ cube units.

(a) $\frac{4}{3} \pi$

(b) $\frac{8}{3} \pi$

(c) 6π

(d) $\frac{16}{3} \pi$

Practice Exams ? Exam 10

Answer the following questions :

1. The curve $y - e^{xy} + x = 0$ has a vertical tangent at the point ....

ⓐ $(1 , 1)$ ⓑ $(0 , 0)$ ⓒ $(1 , 0)$ ⓓ $(2 , e^2)$

2. $\lim\limits_{x \to 0} \dfrac{(10)^{\sin x} - 1}{\tan x} =$ ....

ⓐ $\log 10$ ⓑ $\ln 10$ ⓒ $\ln \sin x$ ⓓ 1

3. $\int_0^6 |3 - x| \, dx =$ ....

ⓐ 9 ⓑ -9 ⓒ $\frac{9}{2}$ ⓓ $\frac{-9}{2}$

4. The absolute maximum value of the function $f : f(x) = \dfrac{x}{x^2 + 1}$, $x \in [0 , 2]$ equals ....

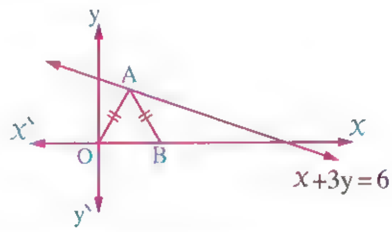
ⓐ 0 ⓑ $\frac{1}{2}$ ⓒ $\frac{2}{5}$ ⓓ 1

5. The height of a cylinder which has the greatest volume placed inside a sphere whose radius length (r) equals ....

ⓐ $\frac{2r}{\sqrt{5}}$ ⓑ $\frac{2r}{\sqrt{3}}$ ⓒ $2r$ ⓓ $2\sqrt{3}\, r$

6. **In the opposite figure :**

$A \in$ the line $x + 3y = 6$, then the greatest area of the isosceles triangle OAB = ... square unit.



ⓐ 2 ⓑ 3
ⓒ 6 ⓓ 9

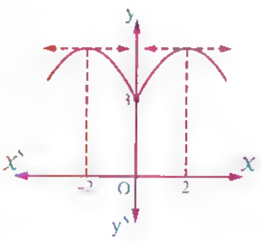
7. A point moves on a curve whose equation is $x^2 + y^2 - 4x + 8y - 6 = 0$, the rate of change of its x-coordinate with respect to time at point $(3 , 1)$ equals 4 units/sec. , then the rate of change of its y-coordinate with respect to time t at the same point equals ....

ⓐ $\frac{3}{5}$ ⓑ $\frac{-4}{5}$ ⓒ $\frac{-3}{5}$ ⓓ $\frac{4}{5}$

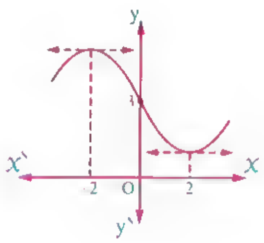
8. If $y = \ln\sqrt{\tan x}$, then $\frac{dy}{dx} =$ .... when $x = \frac{\pi}{4}$

(a) 1 (b) zero (c) $\frac{1}{2}$ (d) ∞

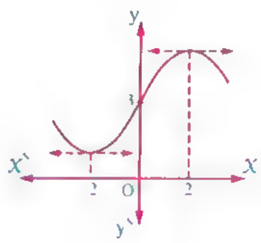
9. Which of the following figures represent the curve of the continuous function f in which $f(0) = 3$, $f'(2) = f'(-2) = 0$, $f'(x) > 0$ when $-2 < x < 2$, $f''(x) < 0$ when $x > 0$, $f''(x) > 0$ when $x < 0$?



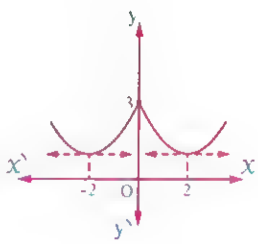
(a)



(b)



(c)



(d)

10. Find : $\int \frac{\ln x}{x}\, dx =$ .... + c

(a) $(\ln x)^2$ (b) $\frac{1}{2}(\ln x)^2$ (c) $\frac{1}{x^2}$ (d) $\ln x - 1$

11. If $x = a\sec^2\theta$, $y = a\tan^3\theta$, then $\frac{d^2y}{dx^2} =$ ....

(a) $\frac{3}{2}\sec^2\theta$ (b) $\frac{3\cot\theta}{4a}$

(c) $\frac{3a\cot\theta}{4}$ (d) $3a\sec^4\theta\tan\theta$

12. The current intensity I (Ampere) in a circuit for alternating current at any moment t (second) is given by the relation $I = 2\cos t + 2\sin t$, then the maximum value of the current in this circuit equals ....

(a) $2\sqrt{2}$ (b) $-2\sqrt{2}$ (c) $\frac{\pi}{4}$ (d) 8

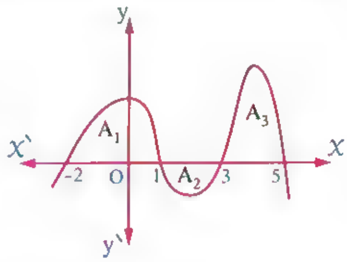
13. A ladder of constant length its upper end slides on a vertical wall at a rate k length unit/sec. , then the rate of increasing of the distance between the lower end and the wall when the ladder inclined to the vertical with an angle θ where $\csc\theta = \frac{5}{4}$ equals .... unit/sec.

(a) $\frac{3}{4}$ (b) $\frac{12}{25}$ (c) $\frac{12}{25}k^2$ (d) $\frac{3k}{4}$

14 $\int \frac{e^{-x}-1}{e^{-x}+x}\, dx = \cdots\cdots\cdots$

(a) $-\ln|e^{x}+x|+c$
(b) $\ln|e^{-x}+x|+c$
(c) $\ln|e^{x}+x|+c$
(d) $-\ln|e^{-x}+x|+c$

15 **In the opposite figure :**



If $A_1 = 5$ square unit , $A_2 = 2$ square unit , $A_3 = 8$ square unit

, then $\int_{-2}^{5} f(x)\, dx + \int_{-2}^{5} |f(x)|\, dx = \cdots\cdots\cdots$

(a) 15
(b) 20
(c) 22
(d) 26

16 If $f(x) = x - x \ln x$, then the slope of the tangent to the curve at $x = e$ equals $\cdots\cdots\cdots$

(a) zero
(b) -1
(c) 1
(d) e

17 The volume of the solid generated by revolving the plane region bounded by the curve $y = 2\sqrt{x-1}$ (where $x \geq 1$) and the tangent at the point (2 , 2) and the straight line $y = 0$ a complete revolution about x-axis equals $\cdots\cdots\cdots$ cubic unit.

(a) $\frac{\pi}{3}$
(b) $\frac{16\pi}{3}$
(c) $\frac{2\pi}{3}$
(d) $\frac{\pi}{2}$

18 If the normal to the curve $y = x \ln x$ is parallel to the straight line $2x - 2y + 3 = 0$, then the normal equation is $\cdots\cdots\cdots$

(a) $x - y = 3e^{-2}$
(b) $x - y = 6e^{-2}$
(c) $x - y = 3e^{2}$
(d) $x - y = 6e^{2}$

19 $\int \frac{\sec^2 x}{\tan x}\, dx = \cdots\cdots\cdots + c$

(a) $-\frac{1}{2}\tan^{-2} x$
(b) $\ln|\tan x|$
(c) $\ln|\sec^2 x|$
(d) $\frac{1}{3}\sec^3 x$

20 The derivative of $e^{\sin x}$ with respect to $\sin x$ equals $\cdots\cdots\cdots$

(a) $e^{\sin x}$
(b) $e^{\frac{1}{\sin x}}$
(c) $\cos x$
(d) $\sin x$

21. n is the number of sides in regular polygon its side length increases at a constant rate (a) cm./sec. , then the measure of its vertex angle ....

(a) increases at a constant rate (a) rad./sec.
(b) increases at a constant rate (na) rad./sec.
(c) increases at a non constant rate and unknown.
(d) remains constant.

22. The two curves $y = x^2 + a x + b$, $y = c x - x^2$ are touching at the point (1 , 0) , then $b + c - a =$ ....

(a) 0 (b) 2 (c) 3 (d) 6

23. If $\acute{f}(x) = x f(x)$, $f(3) = -5$, then $\ddot{f}(3) =$ ....

(a) -50 (b) -40 (c) 15 (d) 27

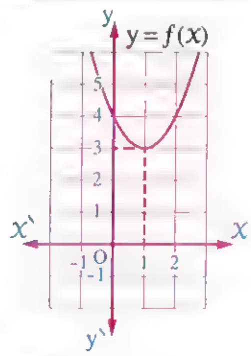
24. If $f(x) = \cot\left(\frac{\pi}{3} \sin x\right)$, then $\acute{f}\left(\frac{\pi}{6}\right) =$ ....

(a) $\frac{-2\sqrt{3}\pi}{3}$ (b) $\frac{-\sqrt{3}\pi}{2}$ (c) π (d) $\frac{\sqrt{3}\pi}{6}$

25. If the curve $y = x^3 + a x^2 + b x$ has an inflection point at (3 , -9) , then $a + b =$ ....

(a) 15 (b) 6 (c) -9 (d) -12

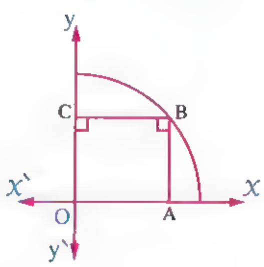
26. **In the opposite figure :**



$\int_0^1 \acute{f}(x) \cdot f(x) \cdot dx =$ ....

(a) $\frac{7}{2}$ (b) $\frac{5}{2}$
(c) $\frac{-5}{2}$ (d) $\frac{-7}{2}$

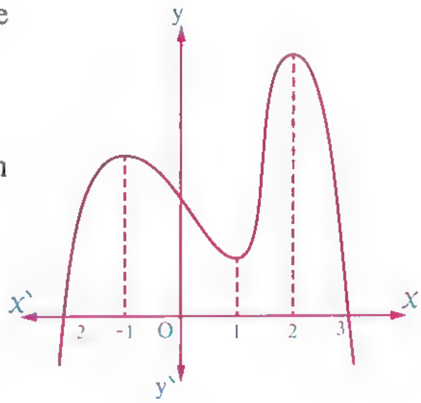
27. **In the opposite figure :**



The part is in the first quadrant from the circle $x^2 + y^2 = r^2$, then the greatest perimeter of the rectangle ABCO equals .... length unit.

(a) r (b) $\sqrt{2}$ r (c) 2 r (d) $2\sqrt{2}$ r

28 The opposite figure represents the curve of first derivative of the function $y = f(X)$, then all the following statements are true except ……………



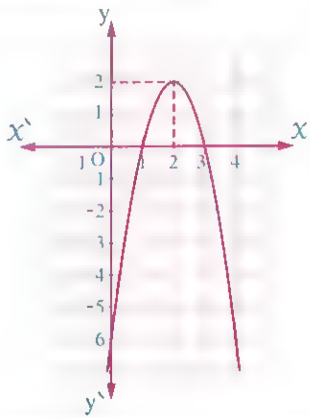
(a) At $X = -1$, there is an inflection point of the function $y = f(X)$

(b) At $X = 1$, there is a local minimum value of the function $y = f(X)$

(c) At $X = -2$, there is a local minimum value of the function $y = f(X)$

(d) At $X = 3$, there is a local maximum value of the function $y = f(X)$

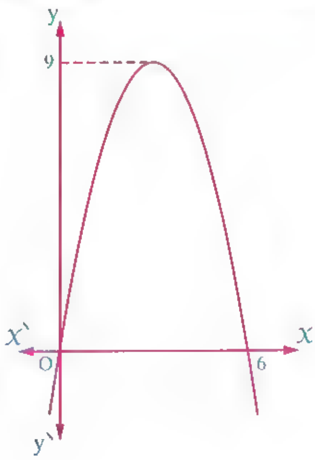
29 The opposite figure represents the curve $\grave{f}$, then the function f is increasing on the interval ……………



(a) $]-\infty , 2[$ (b) $]-\infty , 1[$

(c) $]1 , 2[$ (d) $]1 , 3[$

30 The opposite figure represents A quadratic function , its vertex is (k , 9) , then the area of the shaded region = …………… square units



(a) 6

(b) 9

(c) 12

(d) 18

Practice Exams — Exam 11

Answer the following questions :

1. $\lim\limits_{x \to \infty} \left(\frac{x+5}{x+3}\right)^{x} = \cdots\cdots$

   (a) e (b) e^2 (c) $\frac{1}{e}$ (d) $\frac{2}{e}$

2. The rate of change of tangent slope of a curve at any point (x , y) on it is $6(1 - 2x)$ and the curve has a critical point at $x = 1$ and the function has a local minimum value equals 4 , then the normal equation to the curve at $x = -1$ is $\cdots\cdots$

   (a) $12x + y + 3 = 0$ (b) $x - 12y + 109 = 0$

   (c) $12x + y - 3 = 0$ (d) $y - 3x^2 + 2x^3 + 4 = 0$

3. The measure of the angle which the tangent to the curve $\sin 2x = \tan y$ makes with the positive direction of x-axis at the point $\left(\frac{3\pi}{4} , \frac{3\pi}{4}\right)$ equals $\cdots\cdots$

   (a) zero (b) 135° (c) 45° (d) 26° 34̀

4. If $y = x^2 + 3x + 2$, $z = 3x^2 - 5x + 4$, then $\frac{d^2 y}{d z^2}$ at $x = 2$ equals $\cdots\cdots$

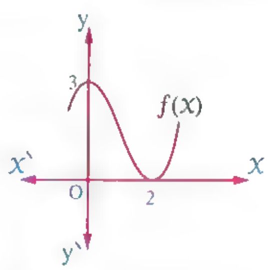
   (a) $\frac{-4}{7}$ (b) $\frac{-4}{49}$ (c) 8 (d) 56

5. **In the opposite figure :**

   $\int_0^2 [f(x)]^2 f`(x)\, dx = \cdots\cdots$

   (a) -9 (b) 9

   (c) 2 (d) 1



6. A rectangle whose perimeter is 40 m. , its area is as great as possible when its dimensions are $\cdots\cdots$ m.

   (a) 15 , 5 (b) 10 , 10 (c) 12.5 , 7.5 (d) 9 , 11

7. If $g(9) = 7$, $g(4) = 3$, then $\int_2^3 2x\, g(x^2)\, g`(x^2)\, dx = \cdots\cdots$

   (a) 10 (b) 20 (c) 5 (d) 2

8 ABC is right-angled triangle at $\angle C$, its area is constant and equals 24 cm^2 the rate of change of (b) equals 1 cm./sec. , then the rate of change of (a) at the instant when b equals 8 cm. equals .... cm./sec.

(a) $\frac{-3}{4}$ (b) $\frac{-3}{2}$ (c) $\frac{3}{8}$ (d) $\frac{3}{4}$

9 If $f : \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ where $f(x) = x^{2x}$, then $\dot{f}(e) =$ ....

(a) $4e^{2e}$ (b) 4 (c) $2e^{2e}$ (d) $4e$

10 If $f(x) = \begin{cases} 2x^3 + 3 & , \ x \leq -1 \\ 3x + 4 & , \ x > -1 \end{cases}$, then $\int_{-2}^{2} f(x)\, dx =$ ....

(a) 6 (b) 9 (c) 12 (d) 15

11 The area of the region bounded by the curve $y = x^2 - 9$ and x-axis and the straight line $x = 4$ and above x-axis equals .... square unit.

(a) $\frac{10}{3}$ (b) 36 (c) $\frac{98}{3}$ (d) 18

12 The normal equation to the curve $y = 3e^{x}$ at the point which lies on the curve and its x-coordinate is -1 is ....

(a) $ey = 3x$ (b) $3x + ey + 6 = 0$

(c) $y - ex - 4e = 0$ (d) $e^2 x + 3ey - 9 + e^2 = 0$

13 If f is a differentiable odd function in the interval $]-\infty , \infty[$ and $\acute{f}(3) = 2$, then $\acute{f}(-3) =$ ....

(a) zero (b) 1 (c) 2 (d) 4

14 If $y = \ln(\sin x)$, then $\frac{d^2 y}{dx^2} =$ ....

(a) $-\csc^2 x$ (b) $\sec x$ (c) $-\csc x \cot x$ (d) $\sec x \tan x$

15 $\int \frac{\cos^2 x}{1 - \sin x}\, dx =$ .... $+ c$

(a) $-\cos x$ (b) $1 - \cos x$ (c) $x + \cos x$ (d) $x - \cos x$

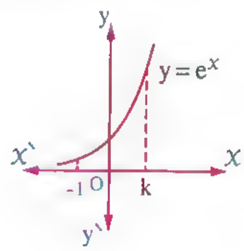
16. If $x \in [0, \pi]$, then the function $f : f(x) = x \sin x + \cos x$ has an absolute minimum value at $x =$ ....

(a) zero (b) $\frac{\pi}{2}$ (c) π (d) -1

17. A cuboid with square base, the sum of all its edges is 240 cm., then the dimensions (in centimetre) of the cuboid when its volume is maximum are ....

(a) 10, 10, 10
(b) 20, 20, 20
(c) 30, 30, 30
(d) 10, 20, 30

18. **In the opposite figure :**



The volume of a solid generated by revolving the shaded region a complete revolution about x-axis and the straight line $x = -1$, $x = k$ equals $\frac{\pi}{2}(e^{10} - e^{-2})$ cube unit, then $k =$ ....

(a) 5
(b) 10
(c) 2
(d) 1

19. The slope of the tangent to the curve $y = \sqrt{x + \sec x}$ at $x =$ zero equals ....

(a) -1 (b) zero (c) $\frac{1}{2}$ (d) 1

20. If $f(x) = 20\, x^{n-1}$ and $\dddot{f}(x) = c$, $c \in \mathbb{R}$, $n \in \mathbb{Z}^+$, then $n + c =$ ....

(a) 104 (b) 124 (c) 123 (d) 125

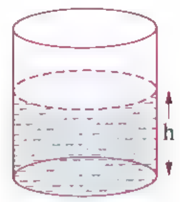
21. $\int \sec x \, dx =$ .... $+ c$

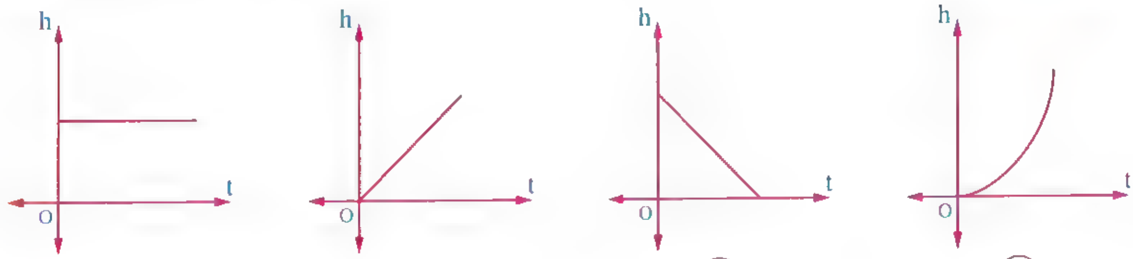
(a) $\sec x \tan x$
(b) $\ln |\sec x + \tan x|$
(c) $\cos^{-1} x$
(d) $\frac{1}{2} \sec^2 x$

22. If $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = \pi^x e^x$, then $\dot{f}(x) =$ ....

(a) $f(x) \ln \pi$
(b) $f(x)\, e^x$
(c) $f(x) \ln (\pi e)$
(d) $f(x)$

23 A water poured into a right circular cylinder at a constant rate as shown in the opposite figure which of the following figures represents the relation between the water height (h) in the cylinder and the time (t) ?





(a) (b) (c) (d)

24 If the equation of the normal to the common tangent of the two functions f and g at $x = 1$ is $y = \frac{-1}{3} x + \frac{3}{2}$, then $(f \times g)'(1) = \cdots\cdots$

(a) 4 (b) 7 (c) 2 (d) 10

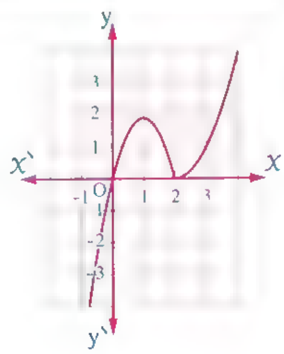
25 If f is an even function and continuous on $\mathbb{R}$ and the function has an inflection point at $x = -a$, then the sign of $f''(a^-) \times f''(a^+)$ is the same as $\cdots\cdots$

(a) $f'(a^-) \times f'(a^+)$
(b) $f(a^-) \times f(a^+)$
(c) $f''(-a)$
(d) $f''(-a^-) \times f''(a^-)$

26 $\int \frac{\ln x^4}{\ln x} \, d x = \cdots\cdots$

(a) $4 x + c$ (b) $\frac{x}{4} + c$ (c) $\frac{4}{x} + c$ (d) $4 x^2 + c$

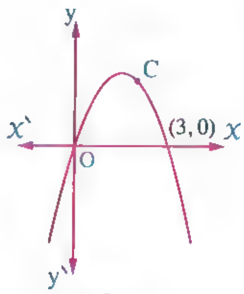
27 The opposite figure represents the curve of the function f , then f' is negative in the interval $\cdots\cdots$



(a) $]1 , 2[$
(b) $]0 , 3[$
(c) $\mathbb{R} - [1 , 2]$
(d) $]0 , 2[$

28 The opposite figure represents the curve of function f where $f(x) = 3x - x^2$

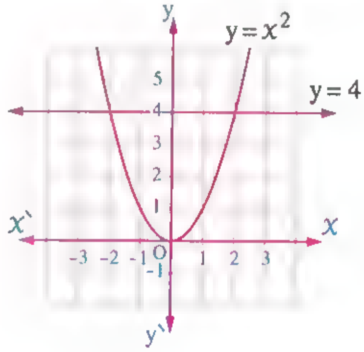
If the point C (a , b) lies on the curve , then the greatest value of the expression a + b is ....



(a) 6 (b) 5 (c) 4 (d) 3

29 **In the opposite figure :**

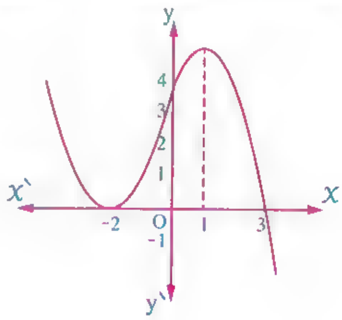
The volume of the solid generated by revolving the shaded region a complete revolution about the x-axis = ... cube units.



(a) $\frac{32}{5}\pi$ (b) $\frac{128}{5}\pi$

(c) $\frac{256}{5}\pi$ (d) $\frac{512}{5}\pi$

30 The opposite figure represents the curve of the first derivative of function $y = f(x)$, then all the following statements are true except ....



(a) f increases on $]-\infty , 3[$

(b) f decreases on $]3 , \infty[$

(c) $f(-2) > f(-3)$

(d) f decreases on $]-\infty , -2[$

Practice Exams

Exam 12

Answer the following questions :

1. If $\int_3^5 f(X)\, dX = 6$, then $\int_3^5 [4 f(X) - 1]\, dX =$ ....

(a) 18 (b) 22 (c) 23 (d) 26

2. If $f(X) = e^{\tan X}$, then $\lim\limits_{X \to \frac{\pi}{4}} \dfrac{f(X) - f\left(\frac{\pi}{4}\right)}{X - \frac{\pi}{4}} =$ ....

(a) e (b) 2 e (c) e^2 (d) $2 e^2$

3. The normal equation to the curve $y = \sin X$ at $(0 , 0)$ is ....

(a) $X = 0$ (b) $y = 0$

(c) $X + y = 0$ (d) $X - y = 0$

4. The minimum value of the function $f : f(X) = X \ln X$ equals ....

(a) e (b) $\frac{1}{e}$ (c) $\frac{-1}{e}$ (d) $-e$

5. In the curve equation $y = f(X)$ if $\dfrac{d^2 y}{d X^2} = a X + b$ where a , b are constant and the curve has an inflection point $(0 , 2)$ and local minimum value at the point $(1 , 0)$
, then $2a + b =$ ....

(a) 6 (b) 12 (c) -12 (d) -60

6. The local maximum value of the function : $y = \frac{1}{3} X^3 - 9X + 2$ equals ....

(a) 20 (b) -16 (c) 3 (d) -3

7. **In the opposite figure :**

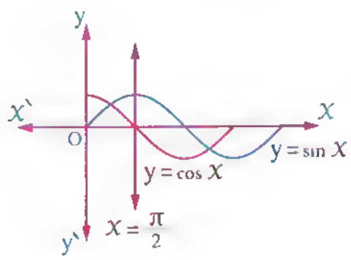
the area of the shaded part = ... square unit.

(a) 2

(b) $2\sqrt{2}$

(c) $2\sqrt{2} + 2$

(d) $2\sqrt{2} - 2$



8. If $f(x) = x^{2019}$, then the 2019th derivative of this function equals ....

(a) 2019 (b) $\underline{|2018}$ (c) $\underline{|2019}$ (d) zero

9. $\int e^{x}(\cot x - \csc^2 x)\, dx = \dots\dots + c$

(a) $e^{x}\cot x - 2e^{x}\csc^2 x$ (b) $e^{x}(-\csc x - \cot^2 x)$

(c) $e^{x}\cot x$ (d) $e^{x}\tan x - e^{x}\cot x$

10. A 5-metre rod is fixed by a hinge to the ground at its base. If its top rises up by a winch at a rate of 1 m./min. , then the rate of decreasing the projection length of the rod on the ground when the height of the top is 3 metres = ... m./min.

(a) $\frac{3}{4}$ (b) $\frac{3}{2}$ (c) $\frac{2}{3}$ (d) $\frac{4}{3}$

11. If $f(x) = (\cos x)^{\cos x}$, then $f'(0) = \dots\dots$

(a) -3 (b) -2 (c) -1 (d) zero

12. $\int_{-2}^{4} |x^2 - 3x|\, dx = \dots\dots$

(a) 9 (b) 15 (c) 18 (d) 24

13. The function $f : f(x) = x^3 + 4x + 8$ increases at $x \in \dots\dots$

(a) $]-4, \infty[$ only. (b) $]-\infty, \frac{-4}{3}[$ only. (c) $]\frac{-4}{3}, \infty[$ only. (d) $\mathbb{R}$

14. The area of the region bounded by the two curves $y^3 = x$, $y = x$ equals ....

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{-3}{4}$ (d) $\frac{-1}{2}$

15. If $y = \sec^n(x)$, then $\frac{dy}{dx} = \dots\dots$

(a) $n\, y \sec x$ (b) $n\, y \tan x$

(c) $n\, y \sec^2 x$ (d) $n\, y \tan^2 x$

16. A rectangle is drawn such that two adjacent vertices of the rectangle lie on the curve $y = x^2 - 12$ and the other two vertices lie on the curve $y = 12 - x^2$, then the maximum area of this rectangle = ... square unit.

(a) 96 (b) 64 (c) 8 (d) 112

17 The curve of the function f where $f(X) = \sqrt[3]{X-3}$ is convex upward in the interval ....

(a) $]3, \infty[$ (b) $]-\infty, 3[$ (c) $]-\infty, 0[$ (d) $]0, \infty[$

18 The volume of the solid generated by revolving the region bounded by the curve $y = \sqrt{X+5}$ and the straight lines $y = 0$, $X = 1$, $X = 3$ a complete revolution about X-axis = ... cubic unit.

(a) 14π (b) 5.3π (c) 19.5π (d) 32π

19 ABC is an equilateral triangle of side length 2ℓ, E is the midpoint of $\overline{BC}$, $D \in \overline{AB}$, $F \in \overline{AC}$ such that $\overline{DF} \parallel \overline{BC}$, then greatest area of the triangle DEF = ... the area Δ ABC

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) 2 (d) 4

20 A train starts its journey at 11 O'clock towards east with velocity 45 km./h., while another train began its journey at 12 O'clock from the same point towards south with velocity 60 km./h., then the rate of increasing of distance between the two trains at 3 O'clock afternoon = ... km./h.

(a) 180 (b) $52.5\sqrt{2}$ (c) $180\sqrt{2}$ (d) 75

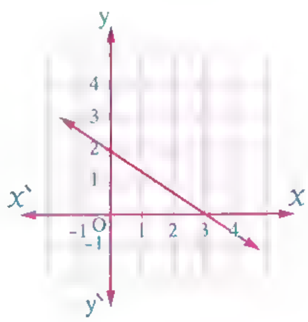
21 $\lim\limits_{X \to 0} \left(1 + \frac{X}{a}\right)^{\frac{a}{X}} = $ ....

(a) zero (b) $\frac{1}{a}$ (c) $\frac{1}{e}$ (d) e

22 $\int [(1 - \cot X)^2 + 2\cot X] \, dX = $ .... + c

(a) $\csc^2 X$ (b) $\cot X$ (c) $-\cot X$ (d) $\tan X$

23 The given figure represents the curve $f'(X)$, then the curve of $f(X)$ is convex upwards at $X \in$ ....



(a) $]-\infty, 0[$

(b) $]-\infty, 3[$

(c) $]0, \infty[$

(d) $]3, \infty[$

24 $\int_{-\pi}^{\pi} (4 + \pi \cos 2x)\, dx =$ ....

(a) π (b) 2π (c) 4π (d) 8π

25 The measure of the positive angle which the tangent to the curve : $y^2 + 2x^2 = 6$ at the point (1 , 2) makes with the positive direction of x-axis = ...°

(a) 45 (b) 135 (c) 120 (d) 150

26 If $y = e^{ax}$, then $\frac{d^4 y}{dx^4} =$ ....

(a) ay (b) $a^4 x$ (c) $a^4 y$ (d) $-a^4 y$

27 If $y = \frac{1}{x} \ln e^{x}$, then $\frac{dy}{dx} =$ ....

(a) 1 (b) e^{x} (c) zero (d) $\ln x$

28 The slope of the tangent to a curve at any point (x , y) on it equals $\frac{\sqrt{2y+1}}{\sqrt{3x-2}}$, then the equation of the curve given that it passes through (1 , 4) is ....

(a) $\sqrt{2y+1} = \sqrt{3x-2}$ (b) $2y + 1 = 3x$

(c) $\sqrt{2y+1} = \frac{2}{3}\sqrt{3x-2} + \frac{7}{3}$ (d) $\sqrt{2y+1} = 2\sqrt{3x-2} + 1$

29 $\frac{d}{dx}\left[(\csc x - \cot x)(\csc x + \cot x)\right] =$ ....

(a) zero (b) $\csc^2 x - \cot^2 x$

(c) $\csc x \cot x + \sec^2 x \tan x$ (d) $\csc x \cot x - \csc^2 x$

30 $\int \frac{dx}{\sqrt[3]{2x+9}} =$ .... + c

(a) $\frac{3}{2}(2x+9)^{\frac{2}{3}}$ (b) $\frac{3}{4}(2x+9)^{\frac{2}{3}}$

(c) $\frac{-3}{8}(2x+9)^{\frac{-4}{3}}$ (d) $\frac{1}{3}(2x+9)^{\frac{2}{3}}$

Practice Exams

Exam 13

Answer the following questions :

1. If $y = \frac{z+1}{z-1}$, $x = \frac{z-1}{z+1}$, then $\frac{dy}{dx} = \dots\dots\dots$ at $x = 2$

   (a) -9 (b) 4 (c) $\frac{1}{4}$ (d) $-\frac{1}{4}$

2. The height of the right cone which can be placed inside a sphere whose radius length is 9 cm. such that its volume is as great as possible equals .... cm.

   (a) 7 (b) 12 (c) 8 (d) 10

3. The tangent to the curve $x = e^{\theta} \cos \theta$, $y = e^{\theta} \sin \theta$ at the point at which $\theta = \frac{\pi}{4}$ makes with the positive x-axis an angle of measure ....

   (a) zero (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

4. $\int_0^2 \sqrt{4 - x^2}\, dx = \dots\dots\dots$

   (a) zero (b) 2 (c) π (d) $\frac{\pi}{2}$

5. If $f(2x - 1) = 2x^3 - 4x^2 + 6x + 7$ then $\acute{f}(3) = \dots\dots\dots$

   (a) 7 (b) 33 (c) 36 (d) 14

6. $\int \frac{\sec^2 x}{\tan x}\, dx = \dots\dots\dots$

   (a) $-\frac{1}{2} \tan^{-2} x + c$ (b) $\ln|\tan x| + c$
   (c) $\ln|\sec^2 x| + c$ (d) $\frac{1}{3} \sec^3 x + c$

7. $\lim\limits_{x \to \infty} \left(\frac{x+4}{x-2}\right)^{x+3} = \dots\dots\dots$

   (a) e^2 (b) e^{-6} (c) $-e^2$ (d) e^6

8. $\int \frac{x - \frac{1}{2}}{\sqrt{2x-1}}\, dx = \dots\dots\dots + c$

   (a) $\frac{1}{2}(2x-1)^{\frac{1}{2}}$ (b) $\frac{1}{3}(2x-1)^{\frac{3}{2}}$
   (c) $(2x-1)^{\frac{3}{2}}$ (d) $\frac{1}{6}(2x-1)^{\frac{3}{2}}$

9 If f is a function where $f(x) = x\,|x-2|$, then the function f is increasing on the interval ....

(a) $]1\ ,2[$ only
(b) $]-\infty\ ,1[\ ,\ ]2\ ,\infty[$ only
(c) $]-\infty\ ,1[$ only
(d) $]1\ ,\infty[$ only

10 The greatest value of the expression $\left(\sin x+\sqrt{3}\cos x\right)$ is at $x=$ .... where $x\in\left[0\ ,\frac{\pi}{2}\right]$

(a) $\frac{\pi}{3}$
(b) $\frac{\pi}{4}$
(c) $\frac{\pi}{6}$
(d) zero

11 In a closed electric circuit , V is the potential difference (Volt) , I is the current intensity (Ampere) R is the resistance (Ohm). If the potential difference increases at a rate of 1 Volt/sec. and the current intensity decreases at a rate of $\frac{1}{2}$ Ampere/sec. , then the rate of the resistance at the moment which V = 12 Volt and I = 2 Amperes equals .... ohm./sec. (where V = IR)

(a) 2
(b) 4
(c) 6
(d) 1

12 If $y = 2\sin x - x\cos x$, then : $\frac{d^2y}{dx^2}+y=$ ....

(a) $x\cos x$
(b) $\sin x$
(c) $2\sin x$
(d) $2\cos x$

13 If $\int (2x+3)\ln x\,dx = yz-\int z\,dy$, then $yz=$ ....

(a) $2x\ln x$
(b) $(2x+3)\ln x$
(c) $\frac{1}{2}(2x+3)\ln x$
(d) $x(x+3)\ln x$

14 If the area of the region bounded by the two curves $y=2x^2$, $y^2=4ax$ equals $\frac{2}{3}$ square unit , then $a=$ .... where $a>0$

(a) $\frac{2}{3}$
(b) 1
(c) $\frac{4}{9}$
(d) $\frac{9}{4}$

15 If $f(x)=(a-2)x^2+3x-5$, $x\in\mathbb{R}$, then the curve of the function f is concave downwards when ....

(a) $a>2$
(b) $a<2$
(c) $a=2$
(d) $a=0$

16 If $f(x)=2\sin\frac{x}{2}\cos\frac{x}{2}$, then the thousandth derivative of this function equals ....

(a) $\sin x$
(b) $\cos x$
(c) $-\sin x$
(d) $-\cos x$

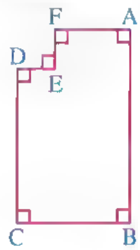
17 A factory is producing electric appliances with profit L.E. 50 in every appliance if it produces 80 appliances monthly. When the production increased than that , the profit of each appliance decreases by 50 piasters for every extra appliance produced.
, then the total number of appliances produced monthly if the profit is to be maximum = ... appliances.

(a) 80 (b) 90 (c) 100 (d) 70

18 The volume of the solid generated by revolving the region bounded by the curve $y = \frac{4}{X}$ and the straight line $y + X = 5$ a complete revolution about X-axis = ... cubic unit.

(a) 3π (b) 6π (c) 21π (d) 9π

19 **In the opposite figure :**



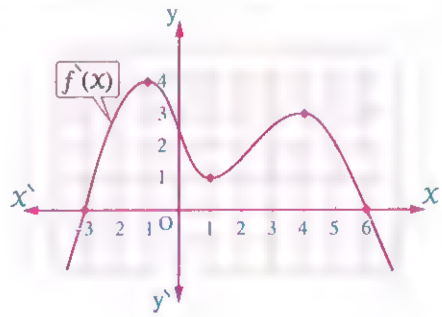
CD = 2AF , FE = ED , The perimeter of the figure ABCDEF = 40 cm.
, then the greatest area of the figure ABCDEF equals .... cm^2

(a) 90 (b) 95
(c) 89 (d) 91

20 If $\int_{0}^{\pi} \frac{\cos X}{1 + X^8} \, dX = k$, then $\int_{-\pi}^{\pi} \frac{2 \cos X}{1 + X^8} \, dX =$ ....

(a) 2 k (b) 4 k (c) 6 k (d) zero

21 The opposite figure represents the curve $f`(X)$, then the curve has maximum value at $X =$ ....
and minimum value at $X =$ ....



(a) – 3 , 6 (b) 6 , – 3
(c) 6 , 0 (d) 0 , 3

22 The sides of right-angled triangle changes , and its perimeter remains constant 40 cm. the rate of change of the hypotenuse is 7 cm./min. when the side lengths 8 , 15 , 17 cm. , then the rate of change of each side of the right-angle at this moment equals .... cm./min.

(a) – 32 , 25 (b) – 2 , 9 (c) 2 , – 9 (d) 32 , – 25

23. If $y = \ln x$, then $\frac{d^{10} y}{d x^{10}} = \dots\dots$

(a) $\frac{\lfloor 9}{-x^{10}}$ (b) $\frac{\lfloor 10}{-x^{9}}$ (c) $\frac{\lfloor 9}{x^{10}}$ (d) $\frac{\lfloor 10}{x^{9}}$

24. $\int (\sin x + \cot x)^9 (\cos x - \csc^2 x)\, d x = \dots\dots + c$

(a) $\frac{1}{2} (\cot x - \csc^2 x)^2$

(b) $\frac{1}{10} (\sin x + \cot x)^{10}$

(c) $(\sin x + \cot x)^{10}$

(d) $\frac{-1}{10} (\sin x + \cot x)^{10}$

25. If the curve of the function f lies above all tangents drawn from all points on the curve then the curve of the function is $\dots\dots$

(a) convex upwards.

(b) increasing.

(c) convex downwards.

(d) decreasing.

26. If the tangent to the curve $y = x^2$ passes through the point $(3, 5)$, then the equation of this tangent is $\dots\dots$

(a) $y = 6 x - 13$

(b) $y = 2 x - 1$ or $y = 10 x - 25$

(c) $y = -10 x + 25$ or $y = -2 x + 1$

(d) $y = x + 8$

27. $\int x^5 \left(1 + \frac{3}{x}\right)^5 d x = \dots\dots + c$

(a) $\frac{1}{6} (x + 3)^6$

(b) $\frac{1}{11} (x + 3)^{11}$

(c) $\frac{1}{18} (x + 3)^6$

(d) $\frac{1}{18} x^6$

28. If $\frac{dz}{dx} = 2 x - 3$, $\frac{dy}{dx} = x^2 + 1$, then $\frac{d^2 z}{dy^2}$ at $x = 1$ equals $\dots\dots$

(a) 1 (b) $\frac{3}{4}$ (c) $\frac{3}{2}$ (d) $\frac{4}{3}$

29. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan x}{x^2 + \cos x}\, d x = \dots\dots$

(a) $-\sqrt{3}$ (b) $\frac{-\sqrt{3}}{3}$ (c) zero (d) $\frac{\sqrt{3}}{3}$

30. The area of the region bounded by the curve $y = 6 - x^2$ and the straight line passing through the two points $(3, -3)$, $(-2, 2)$ equals $\dots\dots$ square units.

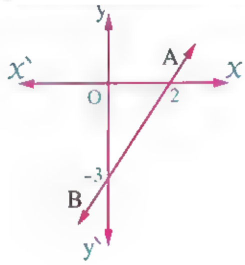
(a) $\frac{56}{3}$ (b) $\frac{55}{3}$ (c) $\frac{95}{6}$ (d) $\frac{125}{6}$

Answer the following questions :

1 The slope of the tangent to the curve $x y^3 = 3$ at the point $(3 , 1)$ equals ....

(a) $\frac{1}{9}$ (b) $-\frac{1}{9}$ (c) $\frac{2}{3}$ (d) $-\frac{2}{3}$

2 The opposite figure represents the first derivative of the function f and the function f has a local minimum equals -3 , then : $\int_0^3 f(x)\, dx =$ ....



(a) $\frac{27}{4}$ (b) -27

(c) $\frac{9}{4}$ (d) $-\frac{27}{4}$

3 The tangent to the curve $x = t^2 - 1$, $y = t^2 - t$ is parallel to x-axis at $t =$ ....

(a) zero (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{1}{2}$ (d) $-\frac{1}{\sqrt{3}}$

4 If the curve of a function which passes through the point $\left(\frac{\pi}{2} , \frac{\pi^2}{4} + 9\right)$ given that the slope of its tangent at any point on it (x , y) is given by the relation $m = 2x + \frac{1}{2}\sec^2\frac{x}{2}$, then the equation of this curve is ....

(a) $y = x^2 + \tan\frac{x}{2}$ (b) $y = x^2 + \tan x + 8$

(c) $y = x^2 + 2\tan\frac{x}{2} + 4$ (d) $y = x^2 + \tan\frac{x}{2} + 8$

5 $\int \frac{1 + \sin^2 x}{1 - \sin^2 x}\, dx =$ .... $+ c$

(a) $2\tan x - x$ (b) $2\sec^2 - 1$

(c) x (d) $\frac{1}{3}\sec^3 x - x$

6 $\lim\limits_{x \to 0} \frac{e^x - e^{\sin x}}{x - \sin x} =$ ....

(a) 1 (b) e (c) zero (d) $-e$

7 $\int_{-a}^{a} \frac{x}{x^4 + \cos x}\, dx =$ ....

(a) $-a$ (b) $2a$ (c) zero (d) $\frac{a^2}{a^5 + \sin a}$

8. $\int \frac{2x}{\sqrt{x^2}}\, dx = \dots\dots\dots + c$

(a) $\sqrt{x^2}$ (b) $\frac{1}{2}\sqrt{x^2}$ (c) $2\sqrt{x^2}$ (d) $3x^{\frac{3}{2}}$

9. The normal equation to the curve $2y = 3 - x^2$ at the point $(1, 1)$ is ....

(a) $x + y = 0$ (b) $x + y + 1 = 0$
(c) $x - y + 1 = 0$ (d) $x - y = 0$

10. The volume of the solid generated by revolving the region bounded by the curve $f(x) = x^2$ and x-axis and the two straight lines $x = -2$, $x = 2$ a complete revolution about x-axis equals ....

(a) $\frac{16\pi}{5}$ (b) $\frac{32\pi}{5}$ (c) $\frac{64\pi}{5}$ (d) 4π

11. A 3-metre wall is 3 metres away from a house , then the minimum length of the ladder that joined the ground and the house resting on the wall = ... metre.

(a) 3 (b) 6 (c) $3\sqrt{2}$ (d) $6\sqrt{2}$

12. If $\sin x = xy$, then : $x^2 (y + \dot{y}) + 2\cos x = \dots\dots\dots$

(a) 0 (b) $2x$ (c) $2y$ (d) y

13. The local minimum value of the function $f : f(x) = x + \frac{1}{x}$ is at $x = \dots\dots\dots$

(a) -1 (b) zero (c) 1 (d) 2

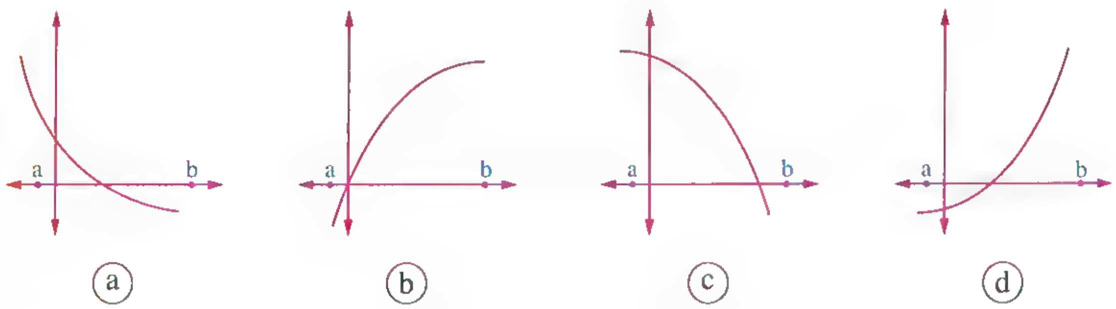
14. The area of the region under the curve $y = \sqrt{3x + 4}$ and above the x-axis. between the two straight lines $x = 0$, $x = 4$ equals .... square unit.

(a) 40 (b) $\frac{112}{9}$ (c) 40π (d) $\frac{112}{9}\pi$

15. If $y = x^x$, $x > 0$, then $\frac{dy}{dx} = \dots\dots\dots$

(a) $\ln x$ (b) $2 + \ln x$
(c) $x^x \ln x$ (d) $x^x (1 + \ln x)$

16 If $f'(X) < 0$, $f''(X) > 0$ for every $X \in [a , b]$, which of the following shown curves represents the curve of the function f in the interval $[a , b]$?



17 The function $f : f(X) = 2 \ln X - X^2$ is decreasing on the interval ....

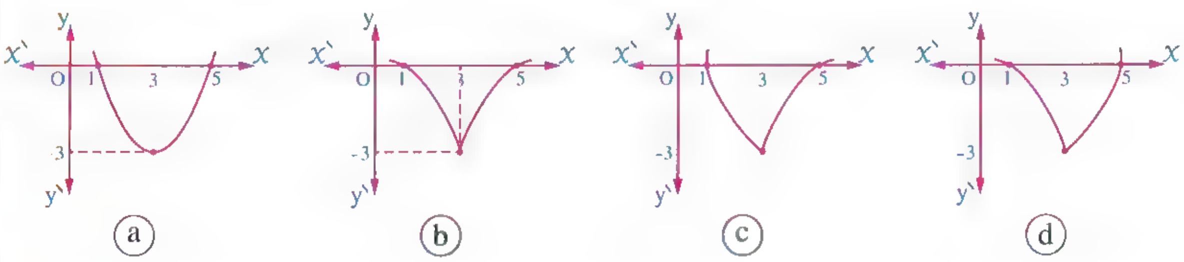
(a) $]0 , 1[$ (b) $]-\infty , 1[$ (c) $]1 , \infty[$ (d) $]0 , \infty[$

18 A cuboid whose dimensions at an instant are 3 , 4 , 12 cm. , the first dimension increases at a rate 2 cm./sec. , and the second dimension increases at a rate 1 cm./sec. and third dimension decreases at a rate 3 cm./sec. , then the rate of change of volume of the cuboid after 2 seconds = ... cm^3 /sec

(a) 528 (b) – 12 (c) 96 (d) 252

19 From the following figures which one represents the general shape of the curve of the continuous function of where $f(1) = f(5) =$ zero , $f(3) = -3$, $f'(X) < 0$ for every $X < 3$, $f'(X) > 0$ for every $X > 3$

$f''(X) < 0$ for every $X \neq 3$



20 ABC is a triangle , A (0 , 0) , B (5 , 0) , C (8 , 3) , then the volume of the solid generated by revolving this triangle a complete revolution about X-axis = ... cubic unit.

(a) 24π (b) 18π (c) 15π (d) 9π

21 25 metre rop passes over a pulley which is 12 m. high. One of its end tied to a heavy mass and the other end tied to a car moves on the ground with velocity 6 m./sec. away from the projection of the pulley on the ground , then the rate of change of height of the mass at the moment when the car at a distance 16 m. from the projection of the pulley = ... m./sec.

(a) 7 (b) 4.8 (c) 9.6 (d) 6

22 If $x = \sin y$, then $\frac{dy}{dx} =$ ....

(a) $\sqrt{1-x^2}$ (b) $\frac{1}{\sqrt{1-x^2}}$ (c) $\sqrt{x^2-1}$ (d) $\frac{1}{\sqrt{x^2-1}}$

23 The equation of the curve : $y = f(x)$. If the slope of the normal at any point on the curve is $(2y + 1) \csc x$ and the curve passes through the origin is ....

(a) $y^2 + y = \cos x - 1$ (b) $y^2 + y = -\cos x - 1$

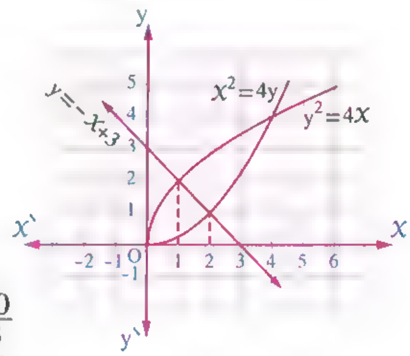
(c) $y^2 + y = \csc x \tan x$ (d) $y^2 + 1 = \cos x$

24 **In the opposite figure :**

Area of region located in the first quadrant and including between the curves :

$x + y = 3$, $x^2 = 4y$, $y^2 = 4x$

equals .... square unit.



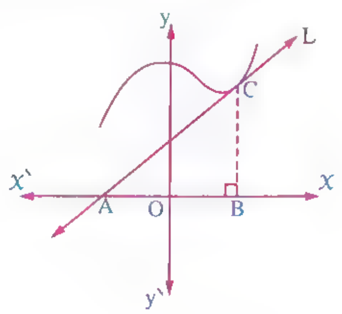
(a) $\frac{5}{4}$ (b) $\frac{10}{3}$

(c) $\frac{55}{12}$ (d) $\frac{13}{6}$

25 **In the opposite figure :**

The straight line L is a tangent to the function f at the point C and intersects the x-axis at the point A $(-4, 0)$, the coordinates of the point B $(4, 0)$ and $f(4) + f'(4) = 9$

, then the area of Δ ABC = ... square unit.



(a) 36 (b) 72 (c) 32 (d) 18

26 The equation of the tangent to the curve $y = \ln(e^{2x} + e^{x} + 1)$ at $x =$ zero is ....

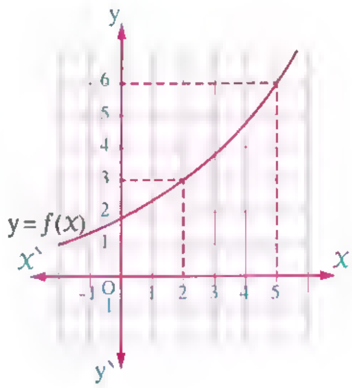
ⓐ $y = x + \ln 3$ ⓑ $y = x - \ln x$

ⓒ $3y = x + 3\ln 3$ ⓓ $y = 3x + \ln 3$

27 If $\tan(x^2 + y^2) =$ zero , then $\frac{dy}{dx} =$ ....

ⓐ $\frac{x}{y}$ ⓑ $\frac{y}{x}$ ⓒ $\frac{-x}{y}$ ⓓ $\frac{-y}{x}$

28 **From the opposite figure :**



$\int_2^5 \frac{\grave{f}(x)}{f(x)}\, dx =$ ....

ⓐ $\ln 2$

ⓑ $\ln 3$

ⓒ $\log_9 5$

ⓓ $\ln 6$

29 If $f(x) = x^{\sin x}$, then $\grave{f}\left(\frac{\pi}{2}\right) =$ ....

ⓐ -2 ⓑ -1 ⓒ zero ⓓ 1

30 If $\int_2^k \frac{dx}{4x} = \ln 2$, then $k =$ .... where $k > 2$

ⓐ 64 ⓑ 48 ⓒ 36 ⓓ 32

Practice Exams ?

Exam 15

Answer the following questions :

1. If $\int_1^k 3x^2\,dx = 7$, then k = ...

   (a) 2 (b) 1 (c) – 2 (d) – 1

2. The local maximum value of the function $f : f(x) = e^x (3 - x)$ equals ....

   (a) 2 (b) 0 (c) e^2 (d) $-e^2$

3. If $f(\sin x) = \sin^2 x$, then $\grave{f}(1) =$ ....

   (a) 1 (b) 2 (c) π (d) $\frac{\pi}{2}$

4. The curve $\left(\frac{x}{a}\right)^t + \left(\frac{y}{b}\right)^t = 2$ touches the straight line $\frac{x}{a} + \frac{y}{b} = 2$ at the point (a , b) when ....

   (a) t = 3 only (b) t = 2 only
   (c) for all values of t (d) not true for any t

5. The normal equation to the curve $y = e^{2x} \cos x$ at $x = 0$ is ....

   (a) $x + y = 2$ (b) $2y + x = 2$
   (c) $2x + y = 2$ (d) $x - y = 2$

6. A circular segment , the radius length of its circle is 10 cm. and measure of its central angle ($x°$) changes in the rate 3 $^{rad.}$ per minute , then the rate of increasing in the area of the circular segment at $x = 60°$ is .... cm^2/min.

   (a) 125 (b) 75 (c) 150 (d) 300

7. The function $f : f(x) = 3 - \ln x^2$ increases in the interval ....

   (a) $]-\infty , \infty[$ (b) $]-\infty , 0[$ (c) $]0 , \infty[$ (d) $]3 , \infty[$

8. $\lim_{h \to 0} \frac{\cot(x + h) - \cot(x)}{h} =$ ....

   (a) $-\csc^2 x$ (b) $\sec^2 x$
   (c) $-\cot^2 x$ (d) $\cot x \csc x$

9 The curve of tangent slope at any point on it equals a $\csc^2 x$ where a is a constant , and the curve passes through the two points $\left(\frac{\pi}{4}, 5\right)$, $\left(\frac{3\pi}{4}, 1\right)$, then the equation of the curve is ....

(a) $y = 2 \cot x + 3$
(b) $y = -2 \cot x - 3$
(c) $y = -\cot x$
(d) $y = 2 \cot x - 3$

10 A circular sector whose perimeter is 30 cm. , and its area is as great as possible , then the radius length of its circle = ...

(a) 15
(b) 30
(c) $\frac{15}{4}$
(d) 7.5

11 The curve $y = x^3 - 6x^2$ is convex downwards in the interval ....

(a) $\mathbb{R} -]0 , 4[$
(b) $]0 , 4[$
(c) $]2 , \infty[$
(d) $]-\infty , 2[$

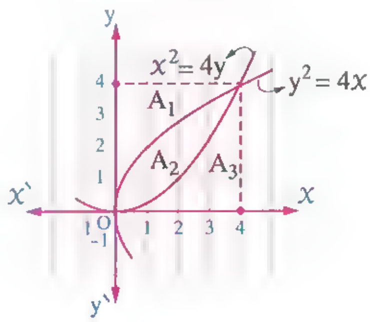
12 $\int \sec^{2017} x \tan x \, d x =$ .... + c

(a) $\frac{1}{2018} \sec^{2018} x$
(b) $\frac{1}{2016} \sec^{2016} x$
(c) $\frac{1}{2017} \sec^{2017} x$
(d) $\frac{1}{2015} \sec^{2015} x$

13 A 5 - metre water pipe with two ends A and B is leaning with its end A on a horizontal ground and with one of its points D against a 3 - metre vertical wall. If end A slides away from the wall at a rate $\frac{5}{4}$ m./min. , then the rate of sliding end B when the pipe reaches the edge of the wall = ... m./min.

(a) $-\frac{3}{5}$
(b) $-\frac{5}{4}$
(c) $\frac{-4}{5}$
(d) $\frac{-12}{25}$

14 **In the opposite figure :**

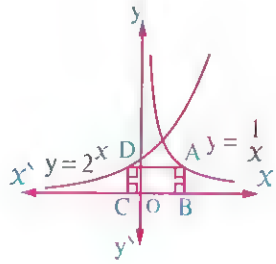


If A_2 is the region bounded by the two curves $y^2 = 4x$, $x^2 = 4y$, then $A_1 : A_2 : A_3 =$ ....

(a) 2 : 1 : 2
(b) 1 : 2 : 1
(c) 1 : 1 : 1
(d) 3 : 2 : 3

15 **In the opposite figure :**

The point A which make the area of the rectangle ABCD is maximum as possible equals ....



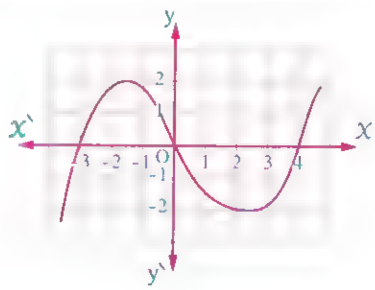
ⓐ $(e , \frac{1}{e})$ ⓑ $(\frac{1}{e} , e)$

ⓒ $(1 , 1)$ ⓓ $(2 , \frac{1}{2})$

16 The rate of change of $(x - \sin x)$ with respect to $(1 - \cos x)$ at $x = \frac{\pi}{3}$ equals ....

ⓐ $\frac{\sqrt{3}}{3}$ ⓑ $\sqrt{3}$ ⓒ $2\sqrt{3}$ ⓓ $\frac{2}{3}$

17 The opposite figure represents the curve $f''(x)$, the inflection is at $x =$ ....



ⓐ zero

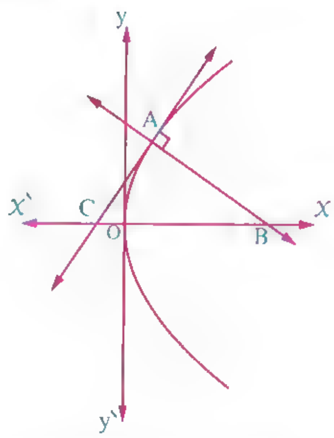
ⓑ -3

ⓒ 4

ⓓ all the previous.

18 **In the opposite figure :**

$\overleftrightarrow{AC}$ is a tangent to the curve $y^2 = 18x$ at the point A $(2 , 6)$ and $\overleftrightarrow{AB} \perp \overleftrightarrow{AC}$, then the length of $\overline{BC} =$ .... length unit.



ⓐ 9

ⓑ 11

ⓒ 2

ⓓ 13

19 If $y = f(x)$ represents a curve of polynomial function of third degree and $f''(x) < 0$ at $x < \frac{-2}{3}$, $f''(x) > 0$ at $x > \frac{-2}{3}$ and passes through the point $(1 , 6)$ and there is a critical point at $(-1 , 2)$, then the equation of the curve is ....

ⓐ $y = 2x^3 + 4$ ⓑ $y = 2x^3 + 4x^2$

ⓒ $y = x^3 + 2x^2 + x + 2$ ⓓ $y = x^3 + 2x^2 + 3$

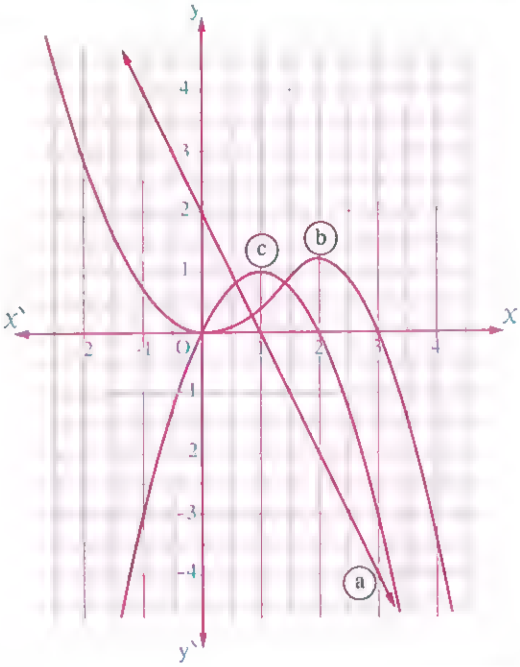
20 The maximum value of the expression ($\sin X + \cos X$) is ....

ⓐ 1 ⓑ 2 ⓒ $\sqrt{2}$ ⓓ $\frac{1}{\sqrt{2}}$

21 If $f(X) = X^2$, $g(X) = \cot X$ and $h(X) = (f \circ g)(X)$, then $h(\frac{\pi}{4}) =$ ....

ⓐ -4 ⓑ 4 ⓒ zero ⓓ -1

22 The opposite figure shows a graphical representation to the curves $f(X)$, $f'(X)$, $f''(X)$ where $f(X)$ is polynomial , then the curves a , b , c represent in order ....



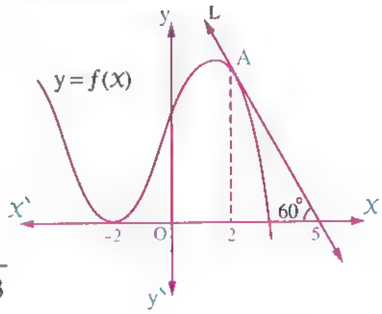
ⓐ $f(X)$, $f'(X)$, $f''(X)$
ⓑ $f'(X)$, $f''(X)$, $f(X)$
ⓒ $f''(X)$, $f(X)$, $f'(X)$
ⓓ $f''(X)$, $f'(X)$, $f(X)$

23 An engineer design a hotel in form of the curve whose equation $y = 4X - \frac{1}{2}X^2$ where X is in metres. If it is covered by glass, the cost of one square metre is L.E. 1200 , then the cost of the glass = L.E. ....

ⓐ $\frac{128}{3}$ ⓑ 715200 ⓒ 51200 ⓓ 15200

24 **In the opposite figure :**

If the straight line L is a tangent to the curve $y = f(X)$ at the point A (2 , k) , then $\int_{-2}^{2} [f''(X) + f'(X)]\, dX =$ ....



ⓐ $4\sqrt{3}$ ⓑ $2\sqrt{3}$
ⓒ 3 ⓓ 2

25 $\int \left(\csc^2 \frac{\pi}{4} + \cos \frac{\pi}{3}\right) d\,x = \dots\dots\dots\dots + c$

ⓐ $-\cot \frac{\pi}{4} + \sin \frac{\pi}{3}$
ⓑ $-4 \cot \frac{\pi}{4} - 3 \sin \frac{\pi}{3}$
ⓒ $2.5\,x$
ⓓ $\cot \frac{\pi}{4} - \sin \frac{\pi}{3}$

26 $\int e^{x} (f'(x) + f''(x))\, d\,x = e^{x} \times \dots\dots\dots\dots + c$

ⓐ x ⓑ $f(x)$ ⓒ $f'(x)$ ⓓ $f''(x)$

27 The function $f : f(x) = \begin{cases} 5 - x^2 & , \quad -3 \le x \le 2 \\ x^2 - 3 & , \quad 2 < x \le 3 \end{cases}$

then the function has an absolute minimum value = ...

ⓐ 5 ⓑ 1 ⓒ -5 ⓓ -4

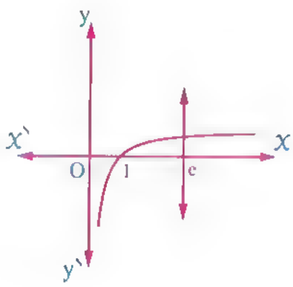
28 If f is a polynomial function of fifth degree , then the greatest possible number of inflection points is ....

ⓐ 2 ⓑ 3 ⓒ 4 ⓓ 5

29 The opposite figure represents the curve :

$y = \frac{\ln x}{\sqrt{x}}$ and the line $x = e$, then

the volume of the generated solid by revolving the shaded region a complete revolution about the x-axis = ... cube units.



ⓐ $\frac{1}{3}\pi$ ⓑ π ⓒ 3π ⓓ 9π

30 A man observes a plane flies horizontally at 3 km. hight exactly above him and with speed 480 km./h. , then the rate of change of the distance between the man and the plane after 30 sec. later = ...

ⓐ $\frac{320}{3}$ km./h.
ⓑ 384 km./h.
ⓒ 384 m./sec.
ⓓ $\frac{320}{3}$ cm./sec.

School Book Examinations

Differential & Integral calculus

School Book Examinations

Model 1

First Answer the following question

1 Choose the correct answer :

(1) Which of the following functions satisfies the relation $\frac{d^3 y}{d X^3} = y$?

ⓐ $\frac{1}{12}(X+1)^4$ ⓑ $\sin X$ ⓒ e^X ⓓ $\frac{X}{X-1}$

(2) If the radius length of a circle increases at a rate $\frac{1}{\pi}$ cm./sec. the circumference of the circle increases at a rate of .... cm./sec.

ⓐ $\frac{2}{\pi}$ ⓑ 2 ⓒ π ⓓ 2π

(3) The curve of the function f where $f(X) = X^3 - 3X^2 + 2$ is convex upwards when $X \in$ ....

ⓐ $]-\infty , 0[$ ⓑ $]-\infty , 1[$ ⓒ $]1 , 3[$ ⓓ $]1 , \infty[$

(4) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin X + \cos X) \, d X$ equals ....

ⓐ 4 ⓑ 2 ⓒ zero ⓓ π

(5) If f is a continuous function on $\mathbb{R}$, $\int_3^5 2 f(X) \, d X = 8$, $\int_3^4 3 f(X) \, d X = 9$, then $\int_4^5 5 f(X) \, d X$ equals ....

ⓐ zero ⓑ 1 ⓒ 3 ⓓ 5

(6) The area of the region bounded by the curve $y = \sqrt{16 - X^2}$ and X-axis measured in square units equals ....

ⓐ 16π ⓑ 12π ⓒ 8π ⓓ 4π

Second Answer three questions only of the following

2 [a] **Find :** $\int \sin X \cos^3 X \, d X$ « $-\frac{1}{4}\cos^4 X + c$ »

[b] If $e^{Xy} - X^2 + y^3 = 0$, **find :** $\frac{d y}{d X}$ when $X = 0$ « $\frac{1}{3}$ »

3 [a] Find the equation of the tangent to the curve : $X^2 - 3Xy - y^2 + 3 = 0$ at point $(-1 , 4)$

« $14X + 5y - 6 = 0$ »

[b] The lengths of the legs of the right-angle of a right-angled triangle at a moment , are 6 cm. and 30 cm. If the length of the first leg increases at a rate of $\frac{1}{3}$ cm./min. and the length of the second leg decreases at a rate of 1 cm./min. , **find :**

(1) The rate of increase in the area of the triangle after 3 minutes.

(2) The time at which the increase of the area of the triangle stops. « 1 cm²/min. , 6 »

---

4 **[a]** Determine the increasing and decreasing intervals to the function f where :

$f(X) = X + 2 \sin X \quad , \quad 0 < X < 2\pi$

[b] A rectangle is drawn such that two adjacent vertices of the rectangle lie on the curve $y = X^2 - 12$ and the other two vertices lie on the curve $y = 12 - X^2$, find the maximum area of this rectangle. « 64 »

---

5 **[a]** Find the volume of the solid generated by revolving the region bounded by the two curves $y = \frac{4}{X}$ and $y = (X - 3)^2$ a complete revolution about X-axis « 5.4 π cubic unit »

[b] Sketch the curve of the function f which satisfies the following properties :

(1) $f(1) = f(5) = 0 \quad , \quad f(2) = -3$

(2) $f'(X) < 0$ for each $X \neq 2$

(3) $f''(X) < 0$ for each $X < 2$

(4) $f''(X) > 0$ for each $X > 2$

School Book Examinations

Model 2

First Answer the following question

1 Choose the correct answer :

(1) The equation of the tangent to the curve of the function f where $f(x) = e^{2x+1}$ at point $\left(\frac{-1}{2}, 1\right)$ is ....

(a) $2y = x + 1$ (b) $y = 2x + 2$ (c) $y = 2x - 3$ (d) $2y = 3x + 1$

(2) If $y = 4n^3 + 4$, $z = 3n^2 - 2$, then the rate of change of z with respect to y equals ....

(a) $2n$ (b) 2 (c) $\frac{1}{2n}$ (d) 4

(3) The maximum value of the expression : $8x - x^2$ where $x \in \mathbb{R}$ is ....

(a) 8 (b) 16 (c) 32 (d) 64

(4) If the slope of the tangent to the curve of the function f at any point on it equals $\frac{1}{x-2}$ and the curve passes through point $(3, 0)$, then $f(e^2 + 2)$ equals ....

(a) 2 (b) 3 (c) ln 2 (d) ln 3

(5) If f is a continuous function on $\mathbb{R}$, $\int_1^2 f(x)\,dx = 9$ and $\int_6^2 f(x)\,dx = -7$, then $\int_1^6 f(x)\,dx$ equals ....

(a) 2 (b) 8 (c) 16 (d) -63

(6) The volume of the solid generated by revolving the region bounded by the curve $y = \sqrt{x+1}$ and the straight lines $y = 0$, $x = -1$ and $x = 1$ equals ....

(a) π (b) $\frac{3\pi}{2}$ (c) 2π (d) $\frac{5\pi}{2}$

Second Answer three questions only of the following

2 [a] Find : (1) $\int x(2x-1)^3\,dx$ (2) $\int x e^{-2x}\,dx$

[b] Find the rate of change for : $\sqrt{16 + x^2}$ with respect to $\frac{x}{x-2}$ when $x = -3$

3 [a] If $x \cos y + y \cos x = 1$, **find :** $\frac{dy}{dx}$

[b] Find the absolute extrema values of the function f in the interval $[-1 , 1]$

where $f(x) = 2x^3 + 6x^2 + 5$ « 13 , 5 »

---

4 [a] If $f(x) = \begin{cases} 2x + x^2 & \text{when} \quad x < 0 \\ 2x - x^2 & \text{when} \quad x \geq 0 \end{cases}$

Find : (1) The local maximum and minimum values of the function f

(2) $\int_{-1}^{3} f(x)\, dx$ « $-\frac{2}{3}$ »

[b] The volume of a cube increases regularly such that it keeps its shape at a rate of 27 cm^3/min. , find the increase of the area of its faces at the moment which its edge length is 3 cm. « 36 cm^2/min. »

---

5 [a] **Find the area of the region bounded by the two curves :**

$y = x^2$ and $y = 6x - x^2$ (in square units). « 9 square unit »

[b] If the function f where $f(x) = x^3 + a x^2 + b x$ has an inflection point at (2 , 2) , find the two values of the two constants a and b , then sketch the curve of the function. « – 6 , 9 »

School Book Examinations

Model 3

First Answer the following question

1 Choose the correct answer :

(1) The slope of the tangent to the curve of the circle : $X^2 + y^2 = 25$ when $X = 3$ equals ....

(a) $\frac{-4}{3}$ (b) $\frac{-3}{4}$ (c) $\frac{5}{12}$ (d) $\frac{4}{3}$

(2) If $f(X) = \frac{X}{X-2}$, then $\dot{f}(3) =$ ....

(a) -36 (b) -12 (c) 6 (d) 4

(3) If $\frac{dy}{dX} = \csc^2 X$, $y = 2$ and $X = \frac{\pi}{4}$, then y equals ....

(a) $-(2 + \cot X)$ (b) $-(3 + \cot X)$ (c) $2 - \cot X$ (d) $3 - \cot X$

(4) If $\int_2^4 f(X)\, dX = 7$, $\int_4^2 g(X)\, dX = 2$, then $\int_2^4 [2 f(X) - 3 g(X) - 5]\, dX$ equals ....

(a) -18 (b) -8 (c) 10 (d) 14

(5) The area of the region bounded by the straight lines :

$y = 2X - 3$, $y = X + 1$, $X = 2$ equals ....

(a) 2 (b) 3 (c) $\frac{9}{2}$ (d) 6

(6) The volume of the solid generated by revolving the region bounded by the two curves

$y = \tan X$, and $y = \sec X$ and two straight lines $X = \frac{\pi}{6}$, $X = \frac{\pi}{3}$

a complete revolution about X-axis approximated in cubic units equals ....

(a) $\frac{\pi^2}{6}$ (b) $\frac{\pi^2}{3}$ (c) $\frac{2\pi^2}{5}$ (d) $2\pi^2$

Second Answer three questions only of the following

2 [a] Find the derivative of y with respect to X where $y = X^2 \ln X$ « $X(2 \ln X + 1)$ »

[b] If $f(X) = \sqrt[3]{(X-4)^2}$, find the convexity intervals upwards and downwards and the inflection points (if existed) to the curve of the function f

3 [a] **Find : (1)** $\int x (x - 5)^3 \, dx$ (2) $\int 4 x e^{2x} \, dx$

[b] Find the absolute maximum values of the function f where :

$f(x) = x^4 - 4x^3$ on the interval $[0 , 4]$ « 0 »

4 [a] The volume of a solid of revolution generated by revolving the region bounded by the curve $y = x^3$ and the two straight lines $x = 0$ and $y = 1$ a complete revolution about x-axis is equal to the volume of a cylinder-like wire whose length is 42 units. What is the radius length of that wire ? « $\frac{1}{7}$ length unit »

[b] The two equal legs of the isosceles triangle with a constant base whose length is ℓ cm. decrease at a rate of 3 cm./min. What is the rate of decrease in the area when the triangle becomes an equilateral triangle ? « $\sqrt{3}\,\ell$ cm²/min »

5 [a] **Find the area of the region bounded by the two curves :** $x - y = 0$, $y = 4x - x^2$

« $\frac{9}{2}$ square unit »

[b] **Sketch the curve of the continuous function f which has the following properties :**

(1) $f(0) = 3$

(2) $f'(2) = f'(-2) = 0$

(3) $f'(x) > 0$ when $-2 < x < 2$

(4) $f''(x) < 0$ when $x > 0$, $f''(x) > 0$ when $x < 0$

School Book Examinations

Model 4

First — Answer the following question

1 Choose the correct answer :

(1) If $y = \frac{3x - 5}{x - 2}$, then at $x = 1$, $\frac{d^3 y}{d x^3}$ equals ....

ⓐ -12 ⓑ -6 ⓒ 6 ⓓ 12

(2) $\int \sec^3 x \tan x \, d x$ equals ....

ⓐ $\frac{1}{4} \sec^4 x + c$ ⓑ $\frac{1}{3} \sec^3 x + c$ ⓒ $\frac{1}{2} \tan^2 x + c$ ⓓ $-\frac{1}{2} \tan^2 x + c$

(3) The normal to circle $x^2 + y^2 = 12$ at any point in it passes through point ....

ⓐ $(2, 3)$ ⓑ $(1, 1)$ ⓒ $(0, 0)$ ⓓ $(-2, -2)$

(4) The curve of the function f where $f(x) = (x - 2) e^x$ is convex downwards on the interval ....

ⓐ $]-\infty, \infty[$ ⓑ $]-1, 2[$ ⓒ $]0, 2[$ ⓓ $]0, \infty[$

(5) $\int_{-1}^{3} 3x |x - 4| \, d x$ equals ....

ⓐ -27 ⓑ -20 ⓒ 20 ⓓ 27

(6) When the region bounded by the curve $x = \frac{1}{\sqrt{y}}$, $1 \le y \le 4$ and y-axis revolves a complete revolution about y-axis, then the volume of the solid generated measured in cubic units equals ....

ⓐ $\frac{2}{3} \pi$ ⓑ $3\sqrt{2}\,\pi$ ⓒ $2\pi \ln 2$ ⓓ $\frac{2}{3} \pi \log 3$

Second — Answer three questions only of the following

2 [a] **Find :** (1) $\int (3x^2 - 4e^{2x}) \, d x$ (2) $\int \frac{x - 1}{\sqrt{x + 3}} \, d x$

[b] If $\sin y + \cos 2x = 0$ **Prove that :** $\frac{d^2 y}{d x^2} - \left(\frac{d y}{d x}\right)^2 \tan y = 4 \cos 2x \sec y$

3 [a] If $\int_{1}^{4} f(x) \, d x = 7$, $\int_{4}^{1} g(x) \, d x = 3$

Calculate the value of : $\int_{1}^{4} [f(x) + 2 g(x) - 4] \, d x$

« – 11 »

[b] If the curve of the function f where $f(X) = a X^3 + b X^2 + c X + d$ has a local maximum value at $(2, 4)$ and an inflection point at $(1, 2)$, find the equation of the curve.

« $f(X) = -X^3 + 3X^2$ »

4 **[a] Find the area of the region bounded by the curve :**

$\sqrt{X} + \sqrt{y} = 1$ and the two straight lines $X = 0$, $y = 0$ « $\frac{1}{6}$ square unit »

[b] Graph the curve of the continuous function f which satisfies the following properties :

(1) $f(4) = 2 f(3) = 4$

(2) $f(2) = 0$

(3) $f'(X) < 0$ when $X > 4$ or $X < 2$

(4) $f''(X) < 0$ when $X > 3$, $f''(X) > 0$ when $X < 3$

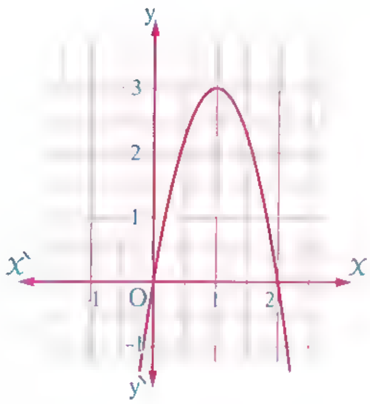
5 **[a]** Prove that the volume of the solid generated by revolving the region bounded by the two curves $y = \frac{4}{X}$ and $y = 5 - X$ just one revolution about X-axis equals 9π of the cubic units.

[b] If A is the area of the part bounded by two concentric circle whose radii lengths are r_1 and r_2 where $r_2 > r_1$, find the rate of change of A with respect to time at any moment at which $r_2 = 10$ cm. , $r_1 = 6$ cm. , if known that at this moment r_1 increases at a rate of 0.3 cm./s. and r_2 decreases at a rate of 0.2 cm./s. « -7.6π cm^2/sec. »

School Book Examinations

Model 5

First — Answer the following question



1 The opposite figure shows the curve $\grave{f}(X)$ of the function f where $f(X) = a X^3 + b X^2$, a , b are two constants.

Complete :

(1) The function f is decreasing for each $X \in$ ....

(2) The curve of f has critical points when $X \in$ ....

(3) The curve of f is convex upwards on the interval ....

(4) There is a local minimum value of the function f when $X =$ ....

(5) $f(1) =$ ....

(6) The area of the region bounded by the curve of the function f and the two straight lines $X = 2$, $X = 0$ and $y = 0$ in square units equals ....

Second — Answer three questions only of the following

2 [a] **Find :**

(1) $\int \csc^2\left(\frac{X+5}{2}\right) dX$ (2) $\int \frac{5X}{3X^2 - 1} dX$

[b] The function f where $f(X) = X^3 - 6X^2 + 9X - 1$

(1) Determine the increasing and decreasing intervals of function f

(2) Find the maximum values of the function f in the interval $[0 , 2]$

3 [a] If $f(X) = 4 + \cot X - \sec^2 X$, find the equation of the normal to the curve of the function f at a point lying on the curve and its X-coordinate equals $\frac{\pi}{4}$

« $4X - 24y - \pi + 72 = 0$ »

[b] An empty tank whose capacity is 10 cubic metres. If the water is poured gradually in that tank at a rate of $(2t + 3)$ m^3/min. where t time in minutes , find the time needed to fill the tank.

« 2 min. »

4 [a] **Find :** $\lim\limits_{X \to \infty} \left(\frac{2X-1}{2X+1}\right)^{2X}$ « $\frac{1}{e^2}$ »

[b] A rectangle - like poster contains 800 cm^2 of the printed material where the widths of both lower and upper margins are 10 cm. and the two side margins are 5 cm. what are the two dimensions of the posters which make its area as minimum as possible.

« 60 , 30 cm. »

---

5 [a] Find the volume of the solid generated by revolving the region bounded by the curve $y = 4 - X^2$ and the two positive parts of the axes of coordinates a complete revolution about X-axis. « $\frac{256}{15}\pi$ »

[b] If $f(X) = X^3 + a X^2 + b X + 4$ where a and b are two constants , find the two values of a and b if the function f has a local minimum value when $X = 2$ and an inflection point when $X = 1$, then sketch the curve of the function f

School Book Examinations

Model 6

First Answer the following question

1 In each of the following phrases , choose (a) if the phrase is true and (b) if the phrase is false :

(1) The local maximum value of the function is greater than its local minimum value. (a) (b)

(2) The rate of change of $\sqrt{n^2+3}$ with respect to $\frac{n}{n+1}$ is : $\frac{n\,(n+1)^2}{\sqrt{n^2+3}}$ (a) (b)

(3) If $\sqrt{y}-\sqrt{x}=2$, then $\frac{d^2y}{dx^2}=\frac{-1}{x\sqrt{x}}$ (a) (b)

(4) $\int \frac{x-4}{(x-2)^6}\,dx=\frac{7\,(x-4)^2}{2\,(x-2)^7}+c$ (a) (b)

(5) If $y=x\ln x-x$, then $\frac{dy}{dx}=\ln x$ (a) (b)

(6) If $\left(a\ ,f\,(a)\right)$ is an inflection point to the curve of the continuous function f , then $f''\,(a)=$ zero (a) (b)

Second Answer three questions only of the following

2 [a] **Find :**

(1) $\int \frac{7x^3}{2-5x^4}\,dx$ (2) $\int \left(3\,e^{-5x}+\frac{\pi}{x}\right)dx$

[b] If $y=a\,e^{x^2+1}$ **Prove that :** $\frac{d^3y}{dx^3}=4\,x\,y\,(3+2\,x^2)$

---

3 [a] **Find :** $\int \cot x\,\csc^3 x\,dx$

[b] If s is the distance between point (1 , 0) and point $(x\ ,y)$ lying on the curve $y=\sqrt{x}$, find the coordinates of point $(x\ ,y)$ at which s is as minimum as possible. " $\left(\frac{1}{2}\ ,\frac{1}{\sqrt{2}}\right)$ "

---

4 [a] Identify the absolute extrema values of the function f where $f\,(x)=|\,x\,|\,(x-4)$ in the interval $[-1\ ,3]$

[b] If the slope of the tangent to the curve $y = f(X)$ at any point on it equals $6X^2 + bX$ and $f(0) = 5$, $f(2) = -3$, find the value of the constant b , then sketch the curve of the function f « $b = -12$ »

5 [a] Find the rate of change of $\ln(9 + X^3)$ with respect to $X^2 + 3$ and $X = 1$ « $\frac{3}{20}$ »

[b] If A (0 , 3) , B (1 , 4) , C (2 , 0) **Find using integration :**

(1) The surface area of Δ ABC

(2) The volume of the solid generated by revolving Δ AOC a complete revolution about y-axis. « $\frac{5}{2}$ square unit , 4π cubic unit »

School Book Examinations

Model 7

First Answer the following question

1 In each of the following phrases , choose (a) if the phrase is true and (b) if the phrase is false :

(1) If $y^2 = 3x^2 - 7$, then $\frac{dy}{dx} = \frac{y}{3x}$ (a) (b)

(2) The function $f : f(x) = x^3 - 3x + 1$ has an inflection point which is $(0 , 1)$ (a) (b)

(3) $\frac{d}{dx}[\cot(\cos 3x)] = 3 \sin 3x \csc^2(\cos 3x)$ (a) (b)

(4) $\int (1 - \cos x)^4 \sin x \, dx = -\frac{1}{5}(1 + \cos x)^5 + c$ (a) (b)

(5) $\lim_{x \to \infty} \left(1 + \frac{5}{x}\right)^x = e^5$ (a) (b)

(6) $\int \left(\frac{2e}{x} + \frac{x}{e}\right) dx = 2e \ln|x| - \frac{x^2}{e} + c$ (a) (b)

Second Answer three questions only of the following

2 [a] Find :

(1) $\int x \sin x \, dx$

(2) $\int_{-1}^{1} \sqrt{x^4 + x^2} \, dx$

[b] Find the equation of the tangent to the curve $y = \ln\left(2 - \sqrt{2}\cos x\right)$ at the point lying on it and its x-coordinate equals $\frac{\pi}{4}$ « $x - y - \frac{\pi}{4} = 0$ »

---

3 [a] Identify the convexity intervals downwards and upwards and the inflection points (if existed) to the curve of the function f where $f(x) = (x - 1)^4 + 3$

[b] A cuboid of metal whose base is square. If the side length of the base increases at a rate of 0.4 cm./sec. and the height decreases at a rate of 0.5 cm./sec. , find the rate of change of the volume when the side length of the base is 6 cm. and the height is 5 cm.

« 6 cm^3/sec. »

4 [a] **Find :** $\int_0^3 x\sqrt{x+1}\, dx$ « $\frac{116}{15}$ »

[b] A rectangle - like playground in which two opposite sides end in a semi-circle outside the rectangle of a diameter length equal to the length of this side.

If the perimeter of the playground is 400 metres , prove that the surface area of the playground is as maximum as possible when the ground is a circle - like , then find its radius length. « $\frac{200}{\pi}$ »

5 [a] If $f(x) = x^3 - 3x + 3$, **find :**

(1) The absolute extrema value of the function f in the interval $f\,[0 , 2]$

(2) The area of the region bounded by the curve of the function f and the straight lines $x = 0$, $x = 2$, $y = 0$ « 4 square unit »

[b] Find the volume of the solid generated by revolving the region bounded by the curve $x\,y = 2$ and the two straight lines $x = 1$ and $x = 2$ about x-axis « 2π cubic unit »

School Book Examinations

Model 8

First **Answer the following question**

1 Complete the following :

(1) If $x^3 y^2 = 1$, then $\left[\frac{dy}{dx}\right]_{y=1} = \cdots\cdots$

(2) $\frac{d}{dx}\left[7 e^{\sec x}\right] = \cdots\cdots$

(3) The function $f : f(x) = x^3 - 3x - 1$ has an inflection point which is $\cdots\cdots$

(4) If f is a continuous function on the interval $[2, 7]$

, then $\int_2^7 f(x)\, dx + \int_7^4 f(x)\, dx = \cdots\cdots$

(5) The area of the region bounded by the two curves $y = x^2$ and $y = 4x$ equals $\cdots\cdots$ square units.

(6) If $y = x^2 \ln \frac{x}{a}$, $a \neq 0$, then $\left[\frac{d^3 y}{dx^3}\right]_{x=4} = \cdots\cdots$

Second **Answer three questions only of the following**

2 [a] **Find :**

(1) $\int \frac{(x+3)^3 - 27}{x}\, dx$ (2) $\int x^2 e^{-x}\, dx$

[b] Find the equation of the tangent to the curve of the function f where $f(x) = 2\tan^3 x$ at the point lying on the curve of the function f and its x-coordinate equals $\frac{\pi}{4}$

« $y = 12x - 3\pi + 2$ »

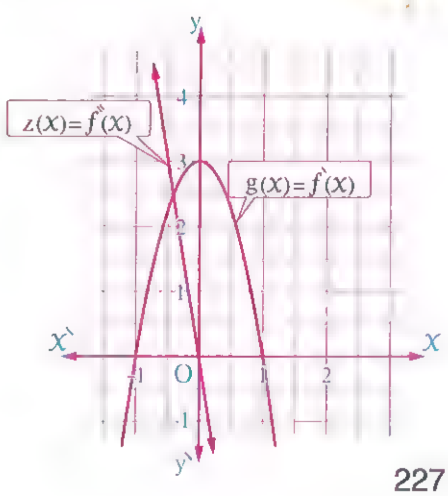
3 [a] **Find :** $\int_0^5 |x - 2|\, dx$ « $\frac{13}{2}$ »

[b] The opposite figure shows the two curves of the two functions g and z where :

$g(x) = f'(x)$, $z(x) = f''(x)$ and f is a polynomial function at the variable x

Sketch the curve of f knowing that it passes through the two points

$(-1, 0)$, $(1, 4)$



4 [a] Identify the absolute extrema values of the function f in the interval $[0 , 2]$

, where $f(X) = 3\sqrt{4 - X^2}$

[b] A five metre length rod is fixed by a hinge to the ground at its base. If its top rises up by a winch at a rate of 1 m./min. , find the rate of decreasing the projector length of the rod on the ground when the height of the top is 3 metres. « $\frac{3}{4}$ m./min. »

5 [a] If a trapezoid is drawn in a semi-circle such that its base its the diameter of the semi-circle , determine the measure of the angle of the trapezoid base such that its area is as maximum as possible. « $60°$ »

[b] If a is the region bounded by the curve $X y = 4 + X^2$ and the straight lines

$X = 1$, $X = 4$ and $y = 0$, **find :**

(1) The area of region a in square units to the nearest unit. « $4 \ln 4 + \frac{15}{2}$ square unit »

(2) The volume of the solid generated by revolving the region about X-axis.

« 57π cubic unit »

School Book Examinations

Model 9

First Answer the following question

1 Choose the correct answer :

(1) If $x = 2n^2 + 7$, $y = \sqrt{n^3}$, $n = 1$, then $\frac{dy}{dx}$ equals ....

ⓐ $\frac{3}{8}$ ⓑ $\frac{3}{4}$ ⓒ 2 ⓓ 6

(2) The curve of the function f is convex downwards on $\mathbb{R}$ if $f(x)$ equals ....

ⓐ $2 - x^2$ ⓑ $2 + x^3$ ⓒ $2 - x^4$ ⓓ $2 + x^4$

(3) If the curve of the function $f : f(x) = x^3 + k x^2 + 4$, $k \in \mathbb{R}$ has an inflection point when $x = 2$, then k = ...

ⓐ – 6 ⓑ – 3 ⓒ 6 ⓓ 9

(4) If f is a continuous function on $\mathbb{R}$, $\int_{-1}^{3} f(x)\, dx = 7$, $\int_{5}^{3} f(x)\, dx = -11$, then $\int_{-1}^{5} f(x)\, dx$ equals ....

ⓐ – 4 ⓑ 18 ⓒ – 18 ⓓ 77

(5) $\int_{-1}^{3} |x - 1|\, dx$ equals ....

ⓐ – 6 ⓑ 0 ⓒ 4 ⓓ 8

(6) The area of the region bounded by the curve $y = x^3$ and the two straight lines $y = 0$ and $x = 2$ equals ....

ⓐ 1 ⓑ 2 ⓒ 4 ⓓ 8

Second Answer three questions only of the following

2 [a] **Find :** (1) $\int \frac{3x}{x^2 - 1}\, dx$ (2) $\int 9x^2 e^{3x}\, dx$

[b] Find the measure of the positive angle which the tangent of the curve $y^3 = x^2$ makes with the positive direction of x- axis when $x = 8$ to the nearest minute. « 18° 26̀ »

3 [a] If $\sin X = X y$, **prove that :** $X^2 (y + \ddot{y}) + 2 \cos X = 2 y$

[b] If the curve $y = 2 X^3 + 3 X^2 + 4 X + 5$ has two parallel tangent , one of them touches the curve at point $(-1 , 2)$, find the equation of the other tangent. « $4 X - y + 5 = 0$ »

4 [a] A balloon rises up vertically at a constant rate of 28 m./min. If the balloon is observed by a ground observer distant 200 m. away from the site of launching the balloon , find the rate of change of the angle of elevation of the observer when the balloon is 200 m. up. « 0.07 rad./min. »

[b] If the slope of the tangent to the curve of the function f at any point (X , y) on the curve is $3 (X^2 - 1)$, find the local maximum and minimum values to the curve of the function f and the inflection points if existed known that the curve passes through the point $(-2 , -1)$, then sketch this curve.

5 The straight line $\overleftrightarrow{AB}$ intersects the curve of the function f at point $C (X , y)$, where $X > 0$, $A (0 , 2)$, $B (6 , 4)$ and $f (X) = \frac{9}{X}$, **find :**

(1) The equation of the straight line $\overleftrightarrow{AB}$ « $y = \frac{1}{3} X + 2$ »

(2) The coordinates of point C « $(3 , 3)$ »

(3) The equation of the normal on the curve of f at point C and prove that it passes through the origin point O « $X - y = 0$ »

(4) The volume of the solid generated by revolving the region bounded by the normal $\overleftrightarrow{OC}$ and the curve of the function and the straight line $X = 6$ and X-axis a complete revolution about X-axis. « $\frac{45}{2} \pi$ cubic unit »

School Book Examinations

Model 10

First Answer the following question

1 Complete :

(1) $\lim\limits_{X \to \infty} \left(1 + \frac{1}{X}\right)^{X+3} = \ldots\ldots\ldots\ldots$

(2) $\frac{d}{dX}(5 - 2 \cot X)^3 = \ldots\ldots\ldots\ldots$

(3) If the function $f : f(X) = k X^3 + 9 X^2$ has an inflection point when $X = -1$

, then k = ……………

(4) $\int_{-1}^{3} (4 X^3 - 6 X^2 + 5) \, dX = \ldots\ldots\ldots\ldots$

(5) If f is a continuous function on the interval $[1 , 4]$

, then $\int_{1}^{4} f(X) \, dX + \int_{4}^{1} f(X) \, dX = \ldots\ldots\ldots\ldots$

(6) The area of the region bounded by the two curves $y = X^4 + 1$ and $y = 2 X^2$

equals …………… square units.

Second Answer three questions only of the following

2 [a] **Find :** (1) $\int \tan(3X + 1) \, dX$ (2) $\int (1 - X^2)(3X - X^3)^5 \, dX$

[b] If the two parametric equations of the function f where $y = f(X)$ are :

$X = 2n^3 + 3$ and $y = n^4$, find each of the following when $n = 1$

(1) The equation of the tangent to the curve of the function f

(2) $\frac{d^2 y}{dX^2}$ « $2X - 3y - 7 = 0$, $\frac{1}{9}$ »

---

3 [a] Investigate the convexity of the curve of the function f where $f(X) = |X^3 - 1|$ and show the inflection points if existed.

[b] If $\int_{-2}^{3} f(X) \, dX = 9$, $\int_{5}^{3} f(X) \, dX = 4$

, **find the value of :** $\int_{-2}^{5} [3 f(X) - 6X] \, dX$ « -48 »

4 **[a] Find the area of the plane region bounded by the two curves :**

$y + x^2 = 6 \quad , \quad y + 2x - 3 = 0$ « $\frac{32}{3}$ square unit »

[b] A right circular cylinder-like container of internal height 9 cm. and the interior radius length of its base is 6 cm. A metal rod of length 16 cm. is placed in the container. If the rate of sliding the rod away from the edge of the cylinder is 2 cm./sec. , find the rate of sliding the rod on the cylinder base when the rod reaches the end of its base.

« $\frac{5}{2}$ cm./sec. »

---

5 **[a]** If the rate of change of the slope of the tangent to a curve at any point (x , y) on it is $6(1 - 2x)$ and the curve has a critical point when $x = 1$ and the function has a local minimum value equals 4

(1) Find the equation of the normal to the curve when $x = -1$

(2) Sketch the curve of the function and show the maximum and minimum values and the inflection points if existed. « $x - 12y + 109 = 0$ »

[b] Find the value of the solid generated by revolving the plane region bounded by the curves : $y = x^3 + 1 \quad , \quad y = 0$ and $x = 0 \quad , \quad x = 1$ a complete revolution about x-axis.

« $\frac{23}{14}\pi$ cubic unit »

Egypt Exams

(2017 : 2020 first and second sessions)

In

Differential & Integral calculus

Egypt exams

1^{st} session 2017

Answer the following questions :

1. If the function $f : f(X) = X + \frac{a}{X}$ has a critical point at $X = 2$, then the value of a = ...

(a) 4 (b) 3 (c) 2 (d) 1

2. If the curve of the function $f : f(X) = \cos X - a X^2$ has an inflection point at $X = \frac{\pi}{3}$, then the value of a = ...

(a) $\frac{1}{4}$ (b) $-\frac{1}{4}$ (c) $\frac{1}{2}$ (d) -1

3. The absolute maximum value of the function f such that : $f(X) = \sin X + \cos X$ in the interval $[0, 2\pi]$ is ....

(a) zero (b) $\frac{1}{\sqrt{2}}$ (c) 1 (d) $\sqrt{2}$

4. *Answer one of the following items :*

[a] Determine the local maximum values and the local minimum values (if there exist) for the function $f : f(X) = (2 - X) e^{X}$

[b] Find the absolute maximum value and the absolute minimum value of the function f such that : $f(X) = 3X^4 - 4X^3$ in the interval $[-1, 2]$

5. $\int 2\cos^2 X \, dX =$ ....

(a) $X + \frac{1}{2}\sin 2X + c$ (b) $X + 2\sin 2X + c$

(c) $X - \frac{1}{2}\sin 2X + c$ (d) $X - \sin 2X + c$

6. In the orthogonal coordinate plane , the straight line $\overleftrightarrow{AB}$ is drawn passing through the point C (3 , 2) , cutting the positive part of X-axis at the point A and the positive part of y-axis at the point B , find the smallest area for Δ AOB such that O is the origin point.

7. If $f(X) = |X|$, then $\int_{-2}^{2} f(X) \, dX =$ ....

(a) 4 (b) 2 (c) 0 (d) -1

8 Find the area of the region bounded by the two curves : $y = x^2$, $y = 5x$

9 Find the volume of the solid generated by revolving the region bounded by the two curves : $y = x^2$, $y = 3x$ a complete revolution about the x-axis

10 *Answer one of the following items :*

[a] Find : $\int \frac{x}{x+1} \, dx$

[b] Find : $\int x^2 \ln x \, dx$

11 If $f(x) = a e^x$, then $\acute{f}(-2) = \dots\dots$

ⓐ $-f(2)$ ⓑ $-\acute{f}(2)$ ⓒ $-f(-2)$ ⓓ $f(-2)$

12 $\int \frac{\ln x^2}{\ln x} \, dx = \dots\dots$

ⓐ $\frac{x}{2} + c$ ⓑ $\frac{1}{x} + c$ ⓒ $2x + c$ ⓓ $\ln|x| + c$

13 $\int \cot x \, dx = \dots\dots$

ⓐ $\ln|\sin x| + c$ ⓑ $\ln|\cos x| + c$

ⓒ $-\ln|\sin x| + c$ ⓓ $\ln|\csc x| + c$

14 Find the equation of the normal to the curve $y = 3e^x$ at the point lying on it and its x-coordinate equals -1

15 If $y = \cot\left(\frac{\pi}{6} t\right)$, $t = 3\sqrt{x}$, then $\left(\frac{dy}{dx}\right)_{x=1} = \dots\dots$

ⓐ $\frac{-\pi}{4}$ ⓑ $\frac{-\pi}{9}$ ⓒ $\frac{-\pi}{6}$ ⓓ $\frac{\pi}{4}$

16 The slope of the tangent to the curve $x y^2 = 3$ at the point $(3 , 1) = \dots\dots$

ⓐ -6 ⓑ -3 ⓒ $\frac{-1}{6}$ ⓓ $\frac{1}{3}$

17 If $x = \frac{z+1}{z-1}$, $y = \frac{z-1}{z+1}$, find : $\frac{d^2y}{dx^2}$ at $z = 0$

18 If a stone fell in a settle water lake , then a circular wave is formed whose radius increases at a rate of 4 cm./sec. Find the rate of increasing of the surface area of the wave at the end of 5 seconds.

Egypt exams

2^{nd} session 2017

Answer the following questions :

1. $\int \sec^4 x \tan x \, dx = \dots\dots$

(a) $\frac{1}{5} \sec^5 x + c$ (b) $\frac{1}{4} \sec^4 x + c$ (c) $\frac{1}{3} \tan x + c$ (d) $\frac{-1}{3} \tan^3 x + c$

2. Find the maximum area for the isosceles triangle that could be drawn inscribed in a circle whose radius equals 12 cm.

3. If $f(x) = \sin^3 x$, then $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(x) \, dx = \dots\dots$

(a) 4 (b) 2 (c) zero (d) -1

4. Find the area of the region bounded by the two curves : $y = x^2$, $y = 4x$

5. Find the volume of the solid generated by revolving the region bounded by the two curves : $y = x^2$, $y = 2x$ a complete revolution about x-axis.

6. *Answer one of the following items :*

[a] Find : $\int \frac{x}{3x^2 + 1} \, dx$ [b] Find : $\int \frac{x}{e^{2x}} \, dx$

7. If $y = \sec x$, then $\dot{y}\left(\frac{\pi}{3}\right) = \dots\dots$

(a) $2\sqrt{3}$ (b) 6 (c) 8 (d) 14

8. If $x = 2t^2 + 3$, $y = \sqrt{t^3}$, then $\left(\frac{dy}{dx}\right)_{t=1} = \dots\dots$

(a) $\frac{3}{8}$ (b) 5 (c) $\frac{8}{3}$ (d) 6

9. If $y = x \sin x$, prove that : $x \frac{d^3 y}{dx^3} + x \frac{dy}{dx} + 2y = 0$

10. A rectangle of length 24 cm. and width 10 cm. , if its length shrinks at a rate of 2 cm./sec. while its width increases at a rate of 1.5 cm./sec. Find the rate of change of its area at the end of 4 seconds , after how many seconds does the area stop increasing ?

11 $\lim\limits_{x \to 0} \frac{2^{x}-1}{3x} = \dots\dots\dots$

(a) $3 \ln 2$ (b) $\frac{1}{3} \ln 2$ (c) $\ln \frac{2}{3}$ (d) $2 \ln 3$

12 $\int 4x e^{x^2+1} \, dx = \dots\dots\dots$

(a) $e^{x^2+1} + c$ (b) $4 e^{x^2+1} + c$

(c) $\frac{1}{2} e^{x^2+1} + c$ (d) $2 e^{x^2+1} + c$

13 $\int \frac{\ln x^2}{x \ln x^3} \, dx = \dots\dots\dots$

(a) $x \ln \frac{1}{x} + c$ (b) $\frac{2}{3 \ln x} + c$

(c) $\frac{2}{3} \ln |x| + c$ (d) $\frac{2}{3 x \ln x} + c$

14 If $y = (x^3 + 5)^x$, find $\frac{dy}{dx}$

15 If $f :]-1, 4[\longrightarrow \mathbb{R}$, $f(x) = x^3 - 3x$, then the number of the critical points for the function f equals $\dots\dots\dots$

(a) zero (b) 1 (c) 2 (d) 3

16 If the curve $y = x^3 + a x^2 + b x$ has an inflection point at $(3, -9)$, then $a + b = \dots\dots\dots$

(a) 15 (b) 6 (c) -9 (d) -12

17 The maximum value for the expression : $4x - x^2$, where $x \in \mathbb{R}$ is $\dots\dots\dots$

(a) 4 (b) 2 (c) 3 (d) 6

18 ***Answer one of the following items :***

[a] Determine the maximum and the minimum local values for the function f such that : $f(x) = x^3 - 3x^2 - 9x$, then determine the inflection point (if exists) for the function.

[b] **Find the absolute extrema values of the function f such that :**

$f(x) = 10 x e^{-x}$, $x \in [0, 4]$

Egypt exams

1^{st} session 2018

Answer the following questions :

1. If $a^y = b^x$ such that $a , b \in \mathbb{R}^+$, $a \neq b$, then $\frac{dy}{dx} = \dots\dots$

   (a) $\log \frac{a}{b}$ (b) $\log_a b$ (c) $\log_b a$ (d) $\log \frac{b}{a}$

2. If $\int_{-2}^{3} f(x)\,dx = 12$, $\int_{-2}^{5} f(x)\,dx = 16$, then $\int_{3}^{5} f(x)\,dx = \dots\dots$

   (a) -28 (b) -4 (c) 4 (d) 28

3. *Answer one of the following items :*

   [a] **Find :** $\int x^3 (x^2+1)^6\,dx$

   [b] **Find :** $\int (x-3)\,e^{2x}\,dx$

4. $\int \tan\theta\,d\theta = \dots\dots$

   (a) $-\ln|\cos\theta| + c$ (b) $-\ln\cos\theta + c$

   (c) $\ln\cos\theta + c$ (d) $|\ln\cos\theta| + c$

5. $\int_{-\pi}^{\pi} \frac{2x - \sin x}{x^2 + \cos x}\,dx = \dots\dots$

   (a) $-\pi$ (b) zero (c) π (d) 2π

6. *Answer one of the following items :*

   [a] Find the local maximum values and the local minimum values of the function $f : f(x) = x^3 - 3x - 2$, and the inflection points of the curve of the function (if exists)

   [b] Find the absolute extrema values of the function $f : f(x) = x(x^2 - 12)$ in the interval $[-1 , 4]$

7. If $f'(x) = x f(x)$ and $f(3) = -5$, then $f''(3) = \dots\dots$

   (a) -50 (b) 4 (c) 15 (d) 27

8. The curve of the function $f : f(x) = (x-2)\,e^x$, is convex upwards in the interval $\dots\dots$

   (a) $]-1 , 2[$ (b) $]-\infty , 0[$ (c) $]0 , \infty[$ (d) $]0 , 2[$

9 **Find the equations of the tangent and the normal to the curve :**

$X = \sec\theta$, $y = \tan\theta$ at $\theta = \frac{\pi}{6}$

10 If $\sin y + \cos 2X = 0$, **prove that :** $\frac{d^2y}{dX^2} - \left(\frac{dy}{dX}\right)^2 \tan y = 4\cos 2X \sec y$

11 If $X = 2t^3 - 15t^2 + 36t + 1$, $y = t^2 - 8t + 11$, then this curve has a vertical tangent at t = ...

(a) 4 (b) 3 or 2 (c) 6 (d) 8

12 For the function f such that $f''(X) = -2X + 6$, then all of the following statements are correct except ....

(a) the curve of the function f convex upwards in the interval $]-\infty , \infty[$

(b) the function f has a local minimum value at $X = 3$

(c) the curve of the function f has no inflection points.

(d) $f(X)$ is decreasing in the interval $]3 , \infty[$

13 If $y = aX^b$ such that a and b are constants , **prove that :** $\frac{1}{y} \times \frac{dy}{dt} = \frac{b}{X} \times \frac{dX}{dt}$

14 Find the volume of the solid generated by revolving the region bounded by the curve $y = X^2 + 2$, the X-axis and the two straight lines $X = -2$, $X = 2$ a complete revolution about the X-axis.

15 $\lim\limits_{X \to 0} \left(\frac{2^X - 1}{3X}\right) =$ ....

(a) 3 ln 2 (b) $\frac{1}{3}$ ln 2 (c) ln $\frac{2}{3}$ (d) 2 ln 3

16 If $f(X) = X(a - \ln X)$ such that a is constant , the curve of the function has a critical point at $X = e$, then a = ...

(a) 1 (b) 0 (c) e (d) 2

17 A metalic circular sector whose area is 4 cm^2. Find the radius length of the sector's circle which makes its perimeter as minimum as possible.
What is the measure of its angle then ?

18 Find the area of the region bounded by the curve $y = 4 - X^2$ and the straight line $y = X + 2$

Egypt exams

2nd session 2018

Answer the following questions :

1. If $f(x) = \sqrt{\sin 2x - \csc x}$, then $f'\left(\frac{\pi}{4}\right) =$ ....

(a) $\sqrt{2}$ (b) 1 (c) zero (d) -1

2. If the curve : $y = (2x - a)^3 + 4$ has an inflection point at $x = 5$, then $a =$ ....

(a) 2 (b) 4 (c) 5 (d) 10

3. A lake infected by bacteria has been treated by an antibacterial. If the number of bacteria z in 1 cm^3 after n day is given by the relation $z(n) = 20\left(\frac{n}{12} - \ln\left(\frac{n}{12}\right)\right) + 30$ such that $1 \le n \le 15$

(1) When the number of bacteria be minimum during this interval ?

(2) What is the least number of bacteria during this interval ?

4. Find the volume of the solid generated by revolving the region bounded by the two curves $y = x^2$ and $y = 3x - 2$ a complete revolution about the x-axis.

5. If $y = e^{(1 + \ln x)}$, then $\frac{dy}{dx} =$ ....

(a) x (b) $e\,x$ (c) e (d) 1

6. $\int_1^1 \frac{x^3}{x^4 + \cos x}\,dx =$ ....

(a) -1 (b) zero (c) 1 (d) 4

7. *Answer one of the following items :*

[a] Find : $\int x(x+2)^6\,dx$ [b] Find : $\int (x+5)\,e^x\,dx$

8. $\int \frac{x+2}{x+1}\,dx =$ ....

(a) $1 + \ln(x+1) + c$ (b) $x - \ln|x+1| + c$

(c) $x + \ln(x+1) + c$ (d) $x + \ln|x+1| + c$

9 $\int_0^{\frac{\pi}{4}} \sec^2 x \tan x \, dx =$ ....

(a) zero (b) $\frac{1}{2}$ (c) 1 (d) 2

10 *Answer one of the following items :*

[a] Find the local maximum and minimum values (if found) of the function f :

$f(x) = x^4 - 2x^2$

[b] Find the absolute extrema values of the function $f : f(x) = \frac{4x}{x^2 + 1}$

in the interval $[-1 , 3]$

11 $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{3x} =$ ....

(a) 1 (b) 3 (c) e (d) e^3

12 If the curve of the function $f(x) = a x^2 + 12x + 1$ has a critical point at $x = 2$

, then a = ...

(a) 12 (b) -3 (c) -1 (d) 3

13 Find the equations of the tangent and the normal to the curve : $y = 3 + \sec x$ at the point which lies on the curve and its x-coordinate equals $\frac{2\pi}{3}$

14 Find the area of the region bounded by the curve $y = \sqrt{2x}$ and the straight line $y = x$

15 If $y = 2t^3 + 7$, $z = t^2 - 4$, then the rate of change for y with respect to z equals ....

(a) 2 t (b) 3 t (c) 6 (d) 12

16 The curve of the function $f : f(x) = (x - 2) e^x$ is convex downwards in the interval ....

(a) $]-\infty , \infty[$ (b) $]-1 , 2[$ (c) $]0 , 2[$ (d) $]0 , \infty[$

17 If $\sin x = x y$, **prove that :** $x^2 (y + \ddot{y}) + 2 \cos x = 2y$

18 If $x e^y = 2 - \ln 2 + \ln x$ and $\frac{dx}{dt} = 6$ at $x = 2$, $y = 0$, find $\frac{dy}{dt}$

Egypt exams

1st session 2019

Answer the following questions :

1. The volume of the solid generated by revolving the region enclosed by the curve $y = 2\,x^2$ and the line $y = 8\,x$ a complete revolution about the x-axis is equal to ....

(a) $\pi \int_0^8 (8\,x - 2\,x^2)^2 \, dx$ (b) $\pi \int_0^4 (8\,x - 2\,x^2)^2 \, dx$

(c) $\pi \int_0^4 (64\,x^2 - 4\,x^4) \, dx$ (d) $\pi \int_0^4 (4\,x^4 - 64\,x^2)^2 \, dx$

---

2. The area of the region bounded by the curve $y = x^3$ and the straight lines : $y = 0$ and $x = 2$ equals .... unit of area.

(a) 8 (b) 4 (c) 2 (d) 1

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3. *Answer one of the following two questions :*

[a] Use integration by parts to find : $\int x^3 \sqrt{4 - x^2} \, dx$

[b] Find : $\int \sin^3 x \, dx$

---

4. The function $f : f(x) = x^4 - 4\,x^2$ has ....

(a) one local minimum value and two local maximum values.

(b) two different local minimum values and one local maximum value.

(c) two local minimum values and no local maximum values.

(d) two equal local minimum values and one local maximum value.

---

5. Let f be the function , defined by : $f(x) = \frac{x}{\ln x}$, then the local minimum value of f is ....

(a) e (b) $\frac{1}{e}$ (c) ln e (d) – e

---

6. *Answer only one of the following two questions :*

[a] Find the values of a and b such that the curve of the function $y = x^3 + a\,x^2 + b\,x$ has an inflection point at $(3 , - 9)$, then determine the local maximum and local minimum values of the function.

[b] Find the absolute extrema values of the function f , where $f(x) = 2x^2 e^x$, $x \in [-3, 1]$

7. $\lim\limits_{x \to 0} \frac{a^{2x} - 1}{x} = \dots\dots\dots$

(a) a^2 (b) $2a$ (c) $2 \ln a$ (d) $2 \ln a^2$

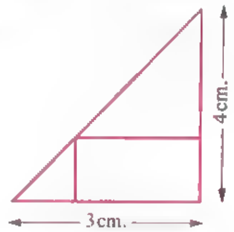
8. If $y = (e^{-x} \ln x)$, then $\frac{dy}{dx} = \dots\dots\dots$

(a) $e^{-x}\left(\frac{1}{x} - \ln x\right)$
(b) $e^{x}\left(\frac{1}{x} - \ln x\right)$
(c) $\frac{e^{-x}}{x} - \ln x$
(d) $e^{-x}\left(\frac{1}{x} + \ln x\right)$

9. **Find the equation of the tangent to the curve :**

$y = 3x^2 - \ln x$ at the point $(1, 3)$ which lies on it.

10. Determine the dimensions of the rectangle of largest area that can be inscribed in the right-angled triangle shown in the figure.



11. If $y = \sec^n x$, then $\frac{dy}{dx} = \dots\dots\dots$

(a) $n \sec^{n-1} x \tan x$
(b) $ny \tan x$
(c) $ny \cot x$
(d) ny

12. The slope of the tangent to the curve : $\cos\left(\sqrt{\pi y}\right) = 3x + 1$ at the point $\left(\frac{-1}{3}, \frac{\pi}{4}\right)$, equals $\dots\dots\dots$

(a) $\frac{-3x}{4}$ (b) 0 (c) 3 (d) -3

13. As a spherical raindrop falls , it reaches a layer of dry air and begins to evaporate at a rate that is proportional to its surface area. Show that the radius of the raindrop decreases at a constant rate.

given that : the area $(A) = 4\pi r^2$, the volume $(v) = \frac{4}{3}\pi r^3$

14 If $y = \frac{10 - \cos x}{x}$, **prove that :** $x \frac{d^2 y}{d x^2} + 2 \frac{d y}{d x} = \cos x$

15 $\int \frac{\ln x^3}{\ln x} \, d x =$ ....

(a) $3 x + c$ (b) $\frac{x}{3} + c$ (c) $\frac{3}{x} + c$ (d) $3 x^2 + c$

16 Let f be the function given by : $f(x) = \frac{x^4 + 1}{x^2}$, then the function f is decreasing in ....

(a) $]-\infty , -1[$ only (b) $]-1 , 0[$ and $]1 , \infty[$

(c) $]0 , 1[$ only (d) $]-\infty , -1[$ and $]0 , 1[$

17 If the slope of the tangent to a curve at any point (x , y) on it is $(a \csc^2 x)$, where a is constant , find the equation of this curve given that the curve passes through the two points $\left(\frac{\pi}{4} , 5\right)$, $\left(\frac{3\pi}{4} , 1\right)$

18 **Find :** $\int_0^6 | x - 4 | \, d x$ (write your steps)

Egypt exams

2nd session 2019

Answer the following questions :

1. If $y = \sec \frac{x}{4} + \sec \frac{\pi}{4}$, then $\frac{dy}{dx} =$ ....

   (a) $\frac{1}{4} \sec \frac{x}{4} \tan \frac{x}{4}$ (b) $4 \sec \frac{x}{4} \tan \frac{x}{4}$

   (c) $\frac{1}{4} \sec \frac{x}{4} \tan \frac{x}{4} + \sqrt{2}$ (d) $\frac{1}{4} \sec \frac{x}{4} \tan \frac{x}{4} + \frac{1}{4} \sec \frac{\pi}{4} \tan \frac{\pi}{4}$

2. The derivative of $(x - \sin x)$ with respect to $(1 - \cos x)$ at $x = \frac{\pi}{3}$ equals ....

   (a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{2}$ (c) $\sqrt{3}$ (d) $\frac{\sqrt{3}}{2}$

3. ABC is a triangle , in which AC = 7 cm. , BC = 3 cm. , AB = x cm. and m ($\angle$ ABC) = θ

   If $\frac{d\theta}{dt} = 1.3$ rad.min. when $\theta = \frac{\pi}{3}$, find $\frac{dx}{dt}$ at this instant.

4. If $y = \sec x$, **prove that :** $y \frac{d^2 y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = y^2 (3 y^2 - 2)$

5. $\lim\limits_{x \to 0} \frac{2^x - 1}{3x} =$ ....

   (a) 3 ln 2 (b) $\frac{1}{3}$ ln 2 (c) log $\frac{2}{3}$ (d) 2 ln 3

6. If $y = \ln (1 + e^{2x})$, then $\frac{dy}{dx} =$ ....

   (a) $\frac{1}{1 + e^{2x}}$ (b) $\frac{e^{2x}}{1 + e^{2x}}$ (c) $\frac{2}{1 + e^{2x}}$ (d) $\frac{2 e^{2x}}{1 + e^{2x}}$

7. Find the equation of the tangent to the curve $y = \ln [2 - \sqrt{2} \cos x]$ at the point which lies on it and its x-coordinate is $\frac{\pi}{4}$

8. Find the positive number for which the sum of its multiplicative inverse and four times its square is the smallest possible.

9 $\int \tan x \, d x =$ ....

(a) $\ln |\cos x| + c$
(b) $-\ln |\sec x| + c$
(c) $\sec^2 x + c$
(d) $\ln |\sec x| + c$

10 Let f be the function given by : $f(x) = (x^2 - 4)^{\frac{2}{3}}$, then the function f is decreasing in ....

(a) $]-\infty , -2[$ and $]0 , 2[$
(b) $]-2 , 0[$ and $]2 , \infty[$
(c) $]-\infty , -2[$ only
(d) $]0 , 2[$ only

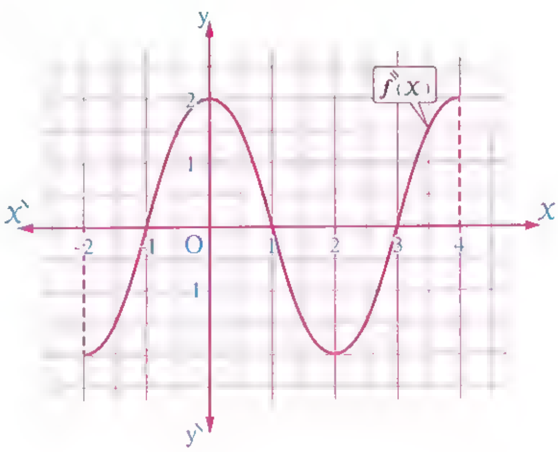
11 If the slope of the tangent to a curve at any point (x , y) on it is $\left(x\sqrt{x+1}\right)$, find the equation of this curve given that the curve passes through the point $\left(0 , \frac{11}{15}\right)$

12 If $f : f(x) = \begin{cases} 2x + x^2 & , \text{ at } x < 0 \\ 2x - x^2 & , \text{ at } x \geq 0 \end{cases}$, find $\int_{-1}^{3} f(x) \, d x$ (write your steps)

13 If the function $f : f(x) = 2a x^2 + b x + 3$ has a local extrema at $(1 , 2)$, then $a + b =$ ....

(a) -1
(b) $\frac{5}{2}$
(c) $-\frac{3}{2}$
(d) $\frac{3}{2}$

14 If the graph of $f''(x)$ (the second derivative of f) is shown in the given figure for $-2 \leq x \leq 4$, then the graph of the function f is convex upwards in ....



(a) $-1 < x < 1$
(b) $0 < x < 2$
(c) $-2 < x < -1$ only
(d) $-2 < x < -1$ and $1 < x < 3$

15 ***Answer one of the following two questions :***

[a] If the curve of the function :

$y = a x^3 + b x^2$ has an inflection point at $(1 , 4)$, determine the values of a and b , then determine the local maximum and local minimum values of the fuction.

[b] Find the values of the absolute extrema of the function $f(x) = 2 x e^x , x \in [-3 , 1]$

16 The volume of the solid generated by revolving the region enclosed by the curve $y = x^2$ and the line $y = 3x$ a complete revolution about the x-axis is equal to ....

(a) $\pi \int_0^3 (3x - x^2)^2 \, dx$
(b) $\pi \int_0^3 (9x^2 - x^4) \, dx$
(c) $\pi \int_0^3 (x^4 - 9x^2) \, dx$
(d) $\pi \int_0^3 (x^2 - 3x)^2 \, dx$

17 The area of the region bounded by the straight lines : $y = x$, $x = 2$ and $y = 0$ equals .... unit of area.

(a) $\frac{1}{2}$
(b) 1
(c) 2
(d) 4

18 ***Answer one of the following two questions :***

[a] Use integration by parts to find $\int x^2 \ln x \, dx$

[b] Find $\int x \sin(2x^2) \, dx$

Egypt exams

1^{st} session 2020

Answer the following questions :

1. $\int \frac{6x+9}{x^2+3x}\,dx =$ ....

(a) $\ln|x^2+3x|+c$ (b) $3\ln|x^2+3x|+c$

(c) $\frac{1}{3}\ln|x^2+3x|+c$ (d) $3\log|x^2+3x|+c$

2. The curve of the function $f : f(x) = x^3 - 9x^2 - 120x + 6$ is convex downwards at $x \in$ ....

(a) $]10, \infty[\cup]-\infty, 4[$ (b) $]-4, 10[$ (c) $]3, \infty[$ (d) $]-\infty, 3[$

3. Find the equation of the curve passes through the point $(1, 0)$ and the slope of its tangent at any point on it equals $x\,e^y$

4. Find $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin x + \cos x)^2\,dx$ (write your steps)

5. If the curve of the function $f : f(x) = x^3 + k\,x^2 + 4$, $k \in \mathbb{R}$ has an inflection point at $x = 2$, then $k =$ ....

(a) -6 (b) -3 (c) 6 (d) 9

6. The function f such that $f(x) = \frac{x}{x-1}$, $x \in [2, 4]$ has ....

(a) an absolute maximum value at $x = 4$ (b) an absolute minimum value at $x = 2$

(c) an absolute minimum value at $x = 1$ (d) an absolute maximum value at $x = 2$

7. ***Answer only one of the following two questions :***

[a] Identify the increasing and decreasing intervals and the local maximum and local minimum values of the function $f : f(x) = 2x^3 - 9x^2 + 12x$

[b] Identify the convexity intervals upwards and downwards and the inflection points (if exists) of the function $f : f(x) = x^4 - 6x^2 + 16$

8. If $x = \tan\theta$, $y = \sec\theta$, then $\frac{dy}{dx} =$ ....

(a) $x\,y$ (b) $\frac{x}{y}$ (c) $\frac{y}{x}$ (d) $\frac{x^2}{y^3}$

9. $\frac{dy}{dx} = \sec^2 x$, $y = 3$ at $x = \frac{\pi}{4}$, then $y =$ ....

(a) $2 - \tan x$ (b) $1 + \tan x$ (c) $3 + \tan x$ (d) $2 + \tan x$

10. ***Answer only one of the following two questions :***

[a] Find : $\int (x^2 + 1)\sqrt{x+2}\, dx$

[b] Find : $\int x \sin x \, dx$

11. If $\frac{d}{dx}[(\sec x - 1)(\sec x + 1)] =$ ....

(a) $\sec^2 x \tan^2 x$ (b) $2 \sec^2 x \tan x$

(c) $2 \sec^2 x$ (d) $\sec^2 x \tan x$

12. If $x y^2 + 2 x y = 8$, then the value of y at the point (1 , 2) equals ....

(a) $-\frac{5}{2}$ (b) $-\frac{1}{2}$ (c) -1 (d) $-\frac{4}{3}$

13. A plane fly horizontally at a height of 3000 m. from the surface of the ground and with velocity 480 km./h. to pass directly above an observer on the ground.
Find the rate of change of the distance between the plane and the observer after 30 sec.

14. If $y = a e^{x^2 + 1}$ such that a is constant , prove that : $\frac{d^3 y}{dx^3} = 4 x y (3 + 2 x^2)$

15. $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{x+3} =$ ....

(a) e (b) e^3 (c) $\frac{1}{3}$ e (d) 3 e

16. If $y = x^{\sin x}$, then $\frac{dy}{dx} =$ ....

(a) $y \sin x \cos x$ (b) $(\sin x)(x)^{\sin x - 1}$

(c) $y\left(\frac{\sin x}{x} + \ln x \cos x\right)$ (d) $\frac{y}{x} \sin x \cos x$

17. Find the equations of the two tangents of the curve : $y = x^3 + 3x - 2$ which are perpendicular to the straight line : $x + 6y = 1$

18. Find the greatest volume of the cuboid whose base is a square and its total surface area equals 150 cm^2

Egypt exams

2^{nd} session 2020

Answer the following questions :

1 $\lim\limits_{x \to \infty} \left(1 + \frac{5}{x}\right)^{x} =$ ....

ⓐ $5e$ ⓑ e^5 ⓒ e^{-5} ⓓ $e^{\frac{1}{5}}$

---

2 If $y = \ln|\sin x|$, then $\frac{dy}{dx} =$ ....

ⓐ $\tan x \ln 10$ ⓑ $\tan x$ ⓒ $\cot x \log e$ ⓓ $\cot x$

---

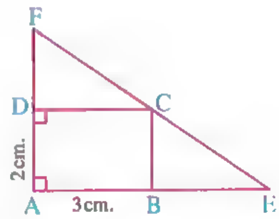
3 Find the equation of the two tangents to the curve : $x^2 + y^2 = 8$ in which the tangents are perpendicular to the straight line $y = 4 - x$

---

4 **In the given figure :**

ABCD is a rectangle in which : AB = 3 cm. , BC = 2 cm.

, a straight line is drawn passes throught the point C and intersects $\overrightarrow{AB}$ in E and $\overrightarrow{AD}$ in F.



Find the smallest area of Δ AEF

---

5 $\int (\cos x \, e^{\sin x} + 3x^2) \, dx =$ ....

ⓐ $e^{\cos x} + x^3 + c$ ⓑ $-e^{\cos x} + x^3 + c$

ⓒ $e^{\sin x} + x^3 + c$ ⓓ $-e^{\sin x} + x^3 + c$

---

6 The function $f : f(x) = x^3 + 3x^2 - 9x$ has ....

ⓐ a local a minimum value at the point (0 , 0)

ⓑ an inflection point at (1 , – 5)

ⓒ an inflection point at (– 1 , 11)

ⓓ a local a minimum value at the point (– 3 , 27)

---

7 Find the equation of the curve passes through the point (1 , 2) and the slope of its tangent at any point on it equals $\frac{1 + x}{x y}$, $x \neq 0$, $y \neq 0$

8 **Find :** $\int_{-\pi}^{\pi} (4 + \pi \cos 2x)\, dx$ (write your steps)

9 If the curve of the function $f : f(x) = x^3 + k x^2 + 4$, has an inflection point at $x = 1$, then k equals ....

(a) 3 (b) 6 (c) -3 (d) -6

10 The function $f : f(x) = x + \frac{1}{x}$, $x \in \left[\frac{1}{2}, 3\right]$, has ....

(a) an absolute minimum value at $x = 1$
(b) an absolute minimum value at $x = \frac{1}{2}$
(c) an absolute maximum value at $x = -1$
(d) an absolute minimum value at $x = 3$

11 ***Answer only one of the following two questions :***

[a] Identify the increasing and decreasing intervals and the local maximum and local minimum values of the function $f : f(x) = x^3 + 3x^2 - 9x - 7$

[b] Identify the convexity intervals upwards and downwards and the inflection points (if exists) of the function $f : f(x) = x^3 - x^2$

12 If $x = a \sec^2 \theta$, $y = a \tan^3 \theta$, a is constant , then $\frac{dy}{dx} =$ ....

(a) $\tan \theta$ (b) $\frac{3}{2} \tan \theta$ (c) $\frac{3}{2} \tan \theta \sec \theta$ (d) $\frac{3}{2} a \tan \theta$

13 If $\int_2^5 f(x)\, dx = 4$, then $\int_2^5 \left(3 f(x) - 1\right) dx =$ ....

(a) 12 (b) 11 (c) 9 (d) -8

14 ***Answer only one of the following two questions :***

[a] Find : $\int x (x + 2)^6\, dx$

[b] Find : $\int x \ln |x|\, dx$

15 If $y = \sec x + \tan x$, then : ....

(a) $\grave{y} + y \sec x = 0$
(b) $\grave{y} - y \sec x = 0$
(c) $\grave{y} + y \csc x = 0$
(d) $\grave{y} - y \csc x = 0$

16 If $x^2 + xy + y^3 = 0$, then $\frac{dy}{dx} = \dots\dots\dots\dots$

(a) $-\frac{2x+y}{x+3y^2}$ (b) $-\frac{x+3y^2}{2x+y}$ (c) $-\frac{2x+y}{x+3y^2-1}$ (d) $\frac{-2x}{x+3y^2}$

17 A car moves from a fixed point in the north direction with velocity 30 km./h. and after one hour another car moves from the same point in the west direction with velocity 80 km./h. **Find the rate of change of the distance between the two cars after one hour from the movement of the second car.**

18 If $y = e^{3x} + x^2$ Prove that : $\frac{d^2y}{dx^2} = 9(y - x^2) + 2$

Al-Azhar Exams

(2019 , 2020 first and second sessions)

In

Differential & Integral calculus

Al-Azhar exams

1^{st} session 2019

Answer the following questions :

1 Choose the correct answer from the given ones :

(1) If $f(X) = X \ln X$, then $\int_1^e \hat{f}(X)\, dX = \dots\dots$

(a) 1 (b) 2 (c) $\frac{e^2 - e - 1}{2}$ (d) $\frac{1-e}{e}$

(2) The curve of the function $f : f(X) = (X - 2)\, e^X$ is convex downwards on the interval $\dots\dots$

(a) $]-\infty, \infty[$ (b) $]-1, 2[$ (c) $]0, 2[$ (d) $]0, \infty[$

(3) A point moves on a curve whose equation is $y^2 = 16X$ If the rate of change of its X-coordinate with respect to time at $y = 2$ equals $\frac{5}{4}$ cm./sec.

, then the rate of change of its y-coordinate with respect to time at the same point $= \dots\dots$ cm./sec.

(a) 5 (b) 10 (c) $\frac{4}{5}$ (d) $\frac{5}{16}$

(4) The volume of the solid generated by revolving the region bounded by the curve $y = X^2$ and the straight line $y = 2X$ complete revolution about X-axis $= \dots\dots \pi$ cube unit.

(a) $\frac{32}{5}$ (b) $\frac{32}{15}$ (c) $\frac{64}{15}$ (d) $\frac{64}{5}$

(5) If $\frac{2}{\sqrt{X} + \sqrt{y}} = 9$, then $\left(\frac{dy}{dX}\right)^2 = \dots\dots$

(a) $\frac{X}{y}$ (b) $\frac{y}{X}$ (c) $\frac{2y}{X}$ (d) $\frac{1}{2\sqrt{Xy}}$

(6) If $X = 2t^3 + 3$, $y = t^4$, then $\frac{d^2y}{dX^2} = \dots\dots$ at $t = 1$

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) 4 (d) $\frac{1}{9}$

Answer only three questions from the following :

2 [a] Find the two equations of the tangent and the normal to the curve $X \sin 2y = y \cos 2X$ at the point $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

[b] Calculate the area of the region bounded by the curve of the function $f : f(x) = 3x^2 + 1$, x-axis and the two straight lines $x = -1$ and $x = 2$

---

3 **[a]** If $x^2 y = a \ln x$, prove that : $x^2 y'' + 5 x y' + 4 y = 0$ where a is constant.

[b] If the slope of the perpendicular to a curve at any point (x , y) lies on it equals $\frac{-1}{a \csc^2 x}$ where a is constant , find the equation of the curve given that it passes through the two points $\left(\frac{\pi}{4} , 3\right)$ and $\left(\frac{3\pi}{4} , -1\right)$

---

4 **[a]** In a perpendicular coordinate plane , $\overleftrightarrow{AB}$ is drawn to pass through the point C $(3 , 2)$ and intersects the positive coordinate axes at point A and point B

Prove that the minimum area of triangle AOB equals 12 squared units where O is the origin point $(0 , 0)$

[b] Find :

(1) $\int_0^{\ln 2} (e^{2x} + e^{x}) \, dx$ (2) $\int x^2 \ln x \, dx$

---

5 **[a]** If the point $(1 , 12)$ is the inflection point to the curve of the function f where $f(x) = a x^3 + b x^2$, find the values of a and b , then determine the absolute extrema values of the function f on the interval $[-1 , 3]$

[b] Find :

(1) $\int \frac{x + 1}{\sqrt{x - 1}} \, dx$ (2) $\int_0^2 \sqrt{4 - x^2} \, dx$

Al-Azhar exams

2^{nd} session 2019

Answer the following questions :

1 Choose the correct answer :

(1) The volume of the solid generated by revolving the region bounded by the two curves : $y = x^2$, $y = 1$ one complete revolution about y-axis is ....

(a) π (b) $\frac{1}{2}\pi$ (c) $\frac{1}{4}\pi$ (d) $-\pi$

(2) $\int_{e}^{e^3} \frac{1}{x-1}\, dx =$ ....

(a) $\ln(e-1)$ (b) $\ln(e^2+e+1)$ (c) $\ln(e^2+e)$ (d) $\ln(e^3-1)$

(3) $\lim\limits_{x \to 1} \frac{\ln x}{x-1} =$ ....

(a) 0 (b) e (c) 1 (d) e^{-1}

(4) If $x = (1-y)(1+y)(1+y^2)(1+y^4)$, then $\frac{dy}{dx} =$ ....

(a) $\frac{1}{8}y^7$ (b) $-8y^7$ (c) $-\frac{1}{8}y^{-7}$ (d) $-\frac{1}{4}y^4$

(5) If $y = \cot\left(\frac{\pi}{6} z\right)$, $z = 3\sqrt{x}$, then $\frac{dy}{dx} =$ .... at $x = 1$

(a) $-\frac{\pi}{3}$ (b) $\frac{\pi}{36}$ (c) $-\frac{\pi}{4}$ (d) $-\frac{\pi}{36}$

(6) If the surface area of a sphere increases at a constant rate 6 cm^2/sec. when the radius length of the sphere equals 30 cm. , then the rate of increasing of the volume of sphere at this moment = ... cm^3/sec.

(a) -18 (b) 140 (c) 90 (d) 90π

Answer only three questions from the following :

2 [a] Find the two equations of the tangent and the normal to the curve : $x = \sec^2\theta - 1$, $y = \tan\theta$ at $\theta = \frac{\pi}{4}$

[b] Find the area of the region bounded by the curve of the function $f : f(x) = \sqrt[3]{2x+2}$, the straight line $x = 3$ and lies above x-axis.

3 **[a]** If the perimeter of a circular sector is 30 cm. and its area is as maximum as possible , find the radius length of its circle.

[b] If $\int_{-2}^{3} f(x)\, dx = 9$, $\int_{5}^{3} f(x)\, dx = 4$, find the value of $\int_{-2}^{5} [3 f(x) + 6 x]\, dx$

4 **[a]** Find the local maximum , local minimum and the inflection points (if exists) of the function $f : f(x) = x \ln x$

[b] If the slope of the tangent to a curve at any point (x , y) lying on it is equal $(5 - 2 \sec^2 2x)$, find the equation of the curve known that the curve passes through the point $\left(\frac{\pi}{8} , \frac{5\pi}{8}\right)$

5 **[a]** If $y = a e^{\frac{b}{x}}$, prove that $x y y'' + 2 y y' - x y'^2 = 0$, where a and b are two non-zero constants.

[b] **Find :**

(1) $\int 3x\sqrt{2x+3}\, dx$

(2) $\int \frac{\ln 5x}{x}\, dx$

Al-Azhar exams

1^{st} session 2020

Answer the following questions :

1 Choose the correct answer :

(1) If $f(X) = 4X + \int 6\cos^3 X \, dX$, then $\grave{f}(o) = \cdots\cdots$

ⓐ 10 ⓑ 4 ⓒ -2 ⓓ 2

(2) If the slope of the tangent to the curve at any point (X, y) equals $4e^{2X}$, $f(0) = 2$, then $f(-2) = \cdots\cdots$

ⓐ 4 ⓑ $4e^{-4}$ ⓒ $2e^{-4}$ ⓓ $2e$

(3) If $y = X^n$ where n is a natural number, $\frac{d^3 y}{d X^3} = 120 X^{n-3}$, then $n = \cdots\cdots$

ⓐ 7 ⓑ 10 ⓒ 6 ⓓ 5

(4) The curve of the function f is convex downwards in $\mathbb{R}$ if $f(X) = \cdots\cdots$

ⓐ $3 + X^4$ ⓑ $3 - X^2$ ⓒ $3 - X^3$ ⓓ $3 - X^4$

(5) A cuboid whose base dimensions are 9 cm. and 12 cm., if the rate of increase of its volume is 27 cm^3/min., then the rate of change of its height = $\cdots\cdots$ cm./min.

ⓐ 4 ⓑ $\frac{1}{4}$ ⓒ 2 ⓓ $\frac{1}{2}$

(6) $\int \frac{2}{X \ln 3X} dX = \cdots\cdots + c$

ⓐ $2 \ln|\ln 3X|$ ⓑ $\frac{2}{3} \ln|\ln 3X|$

ⓒ $6 \ln|\ln 3X|$ ⓓ $\frac{2}{3} \ln|3X|$

Answer only three questions form the following :

2 [a] Find the two equations of the tangent and the normal to the curve $X = \sec\theta$, $y = \tan\theta$ at $\theta = \frac{\pi}{3}$

[b] Find : $\int_0^5 |X - 3| \, dX$

---

3 [a] Find the equation of the curve $y = f(X)$, slope of the normal at any point on it is $(4y + 3)\sec X$ known that the curve passes through the origin point.

[b] If $y = a e^{\frac{b}{X}}$, prove that : $X y \ddot{y} + 2 y \grave{y} - X (\grave{y})^2 = 0$

4 [a] An open field is bounded by a straight river from one of its sides. Determine how to place a fence around the other sides of the rectangle-like piece of land to surrounded as maximum area as possible by a 800 meter fence. What is the area of the land then ?

[b] If $\int_1^4 f(X)\, dX = 7$, $\int_4^1 g(X)\, dX = 3$

, calculate the value of $\int_1^4 [f(X) + 2\,g(X) - 4]\, dX$

---

5 [a] If $f(X) = (X - 1)^4 + 3$, determine the increasing and decreasing intervals of the function , then find the local maximum values , local minimum values and the inflection points if it exist.

[b] **Find :**

(1) $\int X \sec^2 X\, dX$

(2) $\int X \sqrt[3]{X - 1}\, dX$

Al-Azhar exams ? 2^{nd} session 2020

Answer the following questions :

1 Choose the correct answer :

(1) If $y^2 - 2\sqrt{x} = \text{zero}$, then $\frac{dy}{dx} = \dots\dots$

(a) $\frac{2y}{\sqrt{x}}$ (b) $\sqrt{x}$ (c) $\frac{x}{y^2}$ (d) $\frac{1}{y^3}$

(2) If the curve of the function f has an inflection point when $x = 2$ where $f(x) = x^3 + k x^2 + 4$, then the value of $k = \dots\dots$

(a) -6 (b) -3 (c) 3 (d) 6

(3) If $\int_2^5 f(x)\, dx = 4$, then $\int_2^5 [3 f(x) - 1]\, dx = \dots\dots$

(a) 9 (b) 11 (c) 12 (d) -8

(4) The sum of two positive integer is 5 and the sum of cubic of smaller number and double of square of the other is as minimum as possible , then the two numbers are represented by the set of elements $\dots\dots$

(a) $\{2 , 1\}$ (b) $\{2 , 3\}$ (c) $\{4 , 1\}$ (d) $\{\frac{7}{2} , \frac{3}{2}\}$

(5) If the radius length of circle increase at rate $\frac{1}{\pi}$ cm./sec. , the circumference of the circle increase at a rate of $\dots\dots$ cm./sec.

(a) $\frac{2}{\pi}$ (b) 2 (c) π (d) 2π

(6) If $x = \sin y$, then $\frac{d^2y}{dx^2}$ at $y = \frac{\pi}{4}$ equals $\dots\dots$

(a) 1 (b) undefined (c) $\frac{1}{2}$ (d) 2

Answer only three questions form the following :

2 [a] Find the two equations of the tangent and the normal to the curve of the function : $y = 3 - \cot^2 x$ at $x = \frac{\pi}{4}$

[b] **Find each of the following :**

(1) $\int_0^1 \frac{3e^{x} - 2e^{2x}}{2e^{x}}$ (2) $\int \frac{(3x-1)^2}{3x}\, dx$

3 **[a]** If $y^2 + 2xy = 8$, prove that : $(x + y)\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^2 = \text{zero}$

[b] The slope of tangent to a curve at any point (x , y) on it equals $\frac{2x+3}{x}$

Find the equation of the curve if it passes through the point $(e , 2e + 5)$

---

4 **[a]** A balloon rises vertically from point A on the ground surface. An apparatus is placed to follow up the motion of the balloon at point B at the same horizontal plane of point A and distance 200 meters from it. At a moment the apparatus observed the elevation angle of the balloon to find it $\frac{\pi}{4}$, and it increases at a rate of 0.12 rad/min. Find the rate of the balloon elevation at this moment.

[b] **Find each of the following :**

(1) $\int x^2 \ln x \, dx$

(2) $\int_0^5 |x - 2| \, dx$

---

5 **[a]** If $f(x) = \frac{1}{3}x^3 - 9x + 3$ find the intervals of increasing and decreasing , then find the local maximum and minimum values.

[b] **Find following integrations :**

(1) $\int \frac{\cos^3 x - 5}{1 - \sin^2 x} \, dx$

(2) $\int \frac{\ln 5x}{x} \, dx$

Notes

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